

# Proofs and Supplementary Materials

## Appendix A Summary of Major Notation

Table EC.1: Summary of major notation

Notation	Description
<b>Sets</b>	
$J$	The set of teams;
$T$	The set of periods;
$K$	The type space of workers;
$P$	The path set over a time-expanded network;
$\mathcal{M}_j$	The feasible set of team $j$ 's intra-team schedules;
$\mathcal{S}(R, \gamma)$	Solution set of QVI parameterized by the contest scheme $(R, \gamma)$ .
<b>Variables of intra-team coordination problem</b>	
$U_j$	The utility of team $j$ (\$);
$R_j$	The revenue of team $j$ (\$);
$q_j^t$	Service output of team $j$ in period $t$ (requests/hour);
$q^t$	Served demand in period $t$ (requests/hour);
$N_j^{kt}$	Supply of $k$ -type active workers from team $j$ in period $t$ (/hour);
$N_j^t$	Supply of active workers from team $j$ in period $t$ (/hour), vector $\mathbf{N}^t = (N_j^t)_{j=1}^{J_0}$ ;
$f_j^{kp}$	Flow along path $p$ of $k$ -type workers from team $j$ , vector $\mathbf{f}_j = (f_j^{kp})_{k \in K, p \in P}$ ;
$u_j^k$	Utility of a $k$ -type worker in team $j$ (\$).
<b>Variables of contest scheme design problem</b>	
$R$	The winner's reward (\$);
$\gamma^t$	Attraction weight of period $t$ , vector $\boldsymbol{\gamma} = (\gamma^t)_{t=1}^{T_0}$ ;
$\lambda_j^k$	Lagrangian multiplier for constraint (6b), vector $\boldsymbol{\lambda}_j = (\lambda_j^k)_{k=1}^{K_0}$ ;
$\mathbf{f}$	$(\mathbf{f}_j)_{j=1}^{J_0}$ , the scheduling plans of all teams.
<b>Parameters</b>	
$Q_0^t$	Potential demand in period $t$ (requests/hour);
$N_j^k$	The number of $k$ -type workers in team $j$ , vector $\mathbf{N}_j^0 = (N_j^k)_{k=1}^{K_0}$ ;
$c^{kt}$	The service cost of $k$ -type workers in period $t$ (\$/hour), vector $\mathbf{c}^k = (c^{kt})_{t=1}^{T_0}$ ;
$p^t$	Per unit service profit in period $t$ (\$/request);
$w^t$	Workers' earning per completed service order (\$/request);
$h^p$	Working hours along path $p$ (hour);
$c_h$	Cost parameter for amounting working hours;
$\nu$	Workers' degree of aversion to working duration;
$\delta_t^p$	Parameter indicating whether period $t$ lies in path $p$ ;
$u_0^k$	A $k$ -type worker's utility under self-scheduling (\$).
<b>Other vectors</b>	
$\mathbf{N}_{-j}^t$	$(N_j^t)$ , supply from all teams other than $j$ in period $t$ , vector $\mathbf{N}_{-j} = (\mathbf{N}_{-j}^t)_{t=1}^{T_0}$ ;
$\mathbf{N}$	$(\mathbf{N}^t)_{t=1}^{T_0}$ , supply of active workers during the entire planning horizon;
$L_j(\mathbf{f}_j)$	The derivative of $-U_j$ with respect to $\mathbf{f}_j$ .
<b>Acronyms</b>	
OSPs	On-demand service platforms;
GNE	Generalized Nash equilibrium.
QVI	Quasi-variational inequalities;
VI	Variational inequalities;
MPEC	Mathematical programming with equilibrium constraints.

## Appendix B Supplementary Materials for Section 3

### B.1 Discussion on an alternative labor cost function

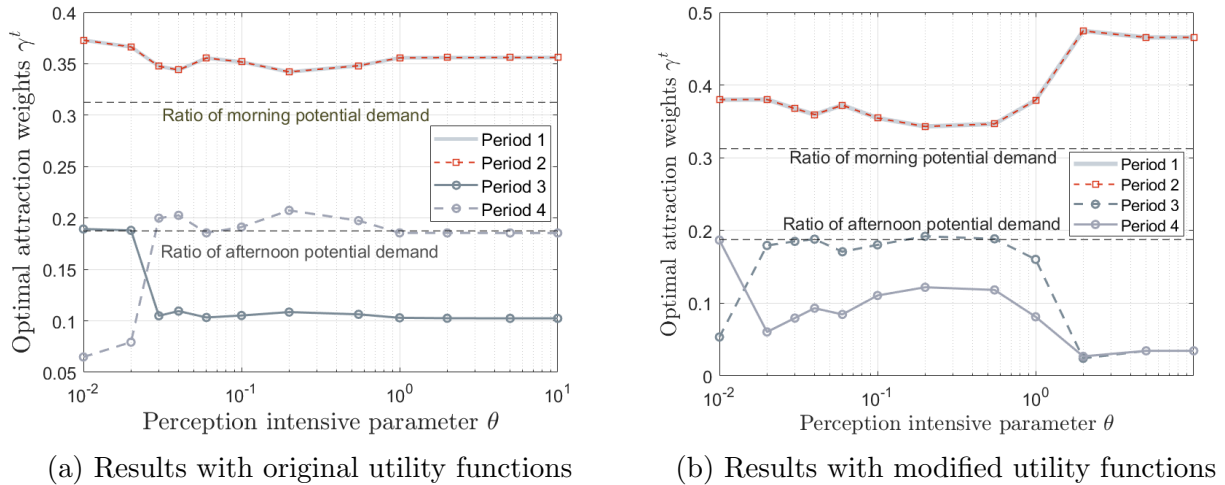
In equation (5), we use the function  $\frac{c_h}{N_j^k} \sum_{p \in P} f_j^{kp} (h^p)^\nu$  to calculate workers' ex-post average costs concerning their working duration. An alternative is to compute the ex-ante costs which consider workers' expected average working hours and can be expressed as  $c_h \left( \frac{\sum_{p \in P} f_j^{kp} h^p}{N_j^k} \right)^\nu$ . Specifically, in team  $j$ , a  $k$ -type worker's probability of being assigned to path  $p$  is equal to  $\frac{f_j^{kp}}{N_j^k}$ . All  $k$ -type workers from team  $j$  are expected to experience the same average working hours because they follow the same strategy profile. Considering all possible work schedules, the average working hour and the corresponding costs of  $k$ -type workers are  $\sum_{p \in P} \frac{f_j^{kp}}{N_j^k} \cdot h^p$  and  $c_h \cdot \left( \frac{\sum_{p \in P} f_j^{kp} h^p}{N_j^k} \right)^\nu$ , respectively. This leads to the following modified utility functions for teams and individuals:

$$U_j(\mathbf{f}_j, \mathbf{N}_{-j}, R) = R_j(\mathbf{N}, R) - \sum_{k \in K} \sum_{t \in T} c^{kt} \cdot N_j^{kt} - c_h N_j^k \cdot \left( \frac{\sum_{p \in P} f_j^{kp} h^p}{N_j^k} \right)^\nu, \quad \forall j \in J, \quad (\text{EB.1})$$

$$u_j^k(\mathbf{f}_j, \mathbf{N}_{-j}, R | F_m) = \hat{r}_j^k(\mathbf{N}, R | F_m) - \frac{1}{N_j^k} \left( \sum_{t \in T} c^{kt} N_j^{kt} + c_h \cdot \left( \frac{\sum_{p \in P} f_j^{kp} h^p}{N_j^k} \right)^\nu \right), \quad \forall j \in J, k \in K. \quad (\text{EB.2})$$

Replacing equations (4) and (5) with (EB.1) and (EB.2) results in a modified model, which inevitably impacts market equilibrium and the platform's contest scheme design. To determine if using this ex-ante cost function influences our major insights, we compare the numerical results using the original and modified utility functions:

1. **Impacts on the platform's optimal contest scheme design:** Under symmetric two-team contests, the optimal winner's reward remains zero regardless of the changes in the utility functions. For the optimal attraction weights, our previous results indicate that their values are consistent with market demand. Peak morning hours have higher attraction weights than non-peak afternoon hours. Figure EC.1 shows such a conclusion still holds when using the modified utility functions.



**Figure EC.1** The optimal attraction weights under symmetric two-team contests

2. **Impacts on the profits of the platform and workers** Figure EC.2a and Figure EC.3a show that team contests have undetermined impacts on the platform. Because the trade-off between intra-team coordination and inter-team competition remains, the same impacts are observed with modified utility functions (Figure EC.2). In contrast, workers as a whole obtain a lower profit when the ex-ante cost function is applied (Figure EC.2b and Figure EC.3b). Nevertheless, workers are still better off than under the fully self-scheduling benchmark.

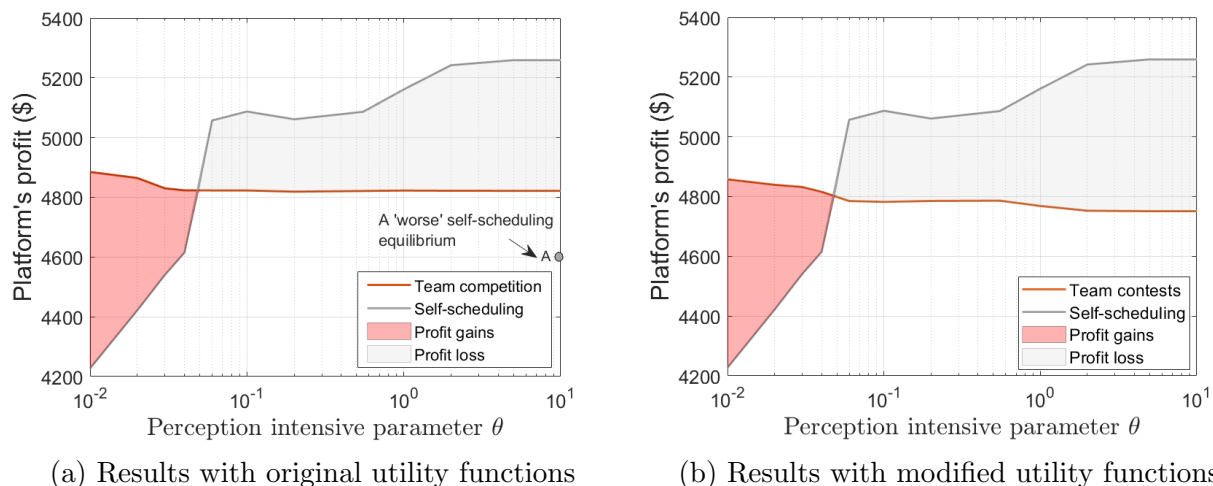


Figure EC.2 Platform's profit under symmetric two-team contests

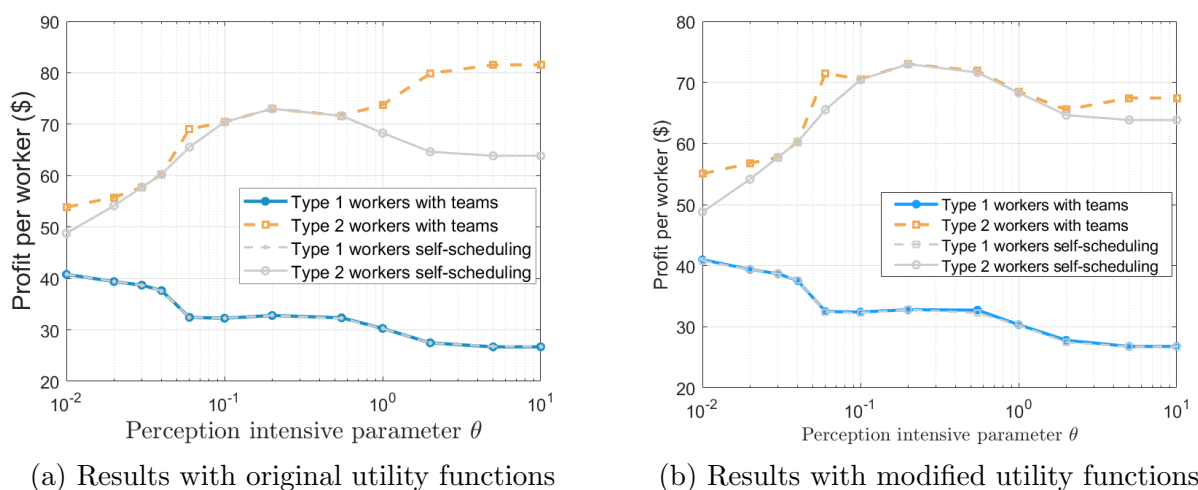
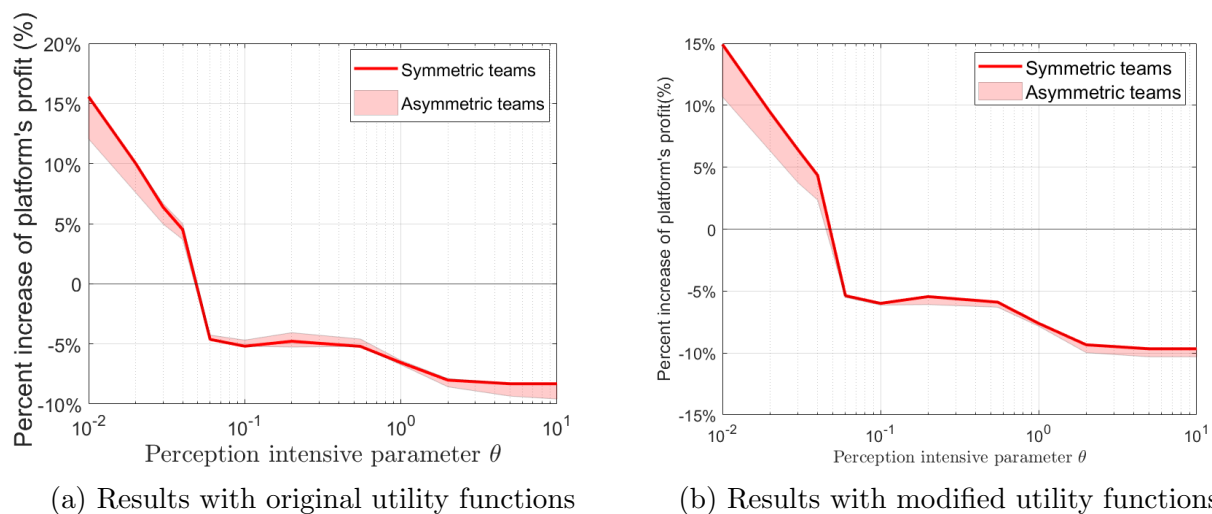


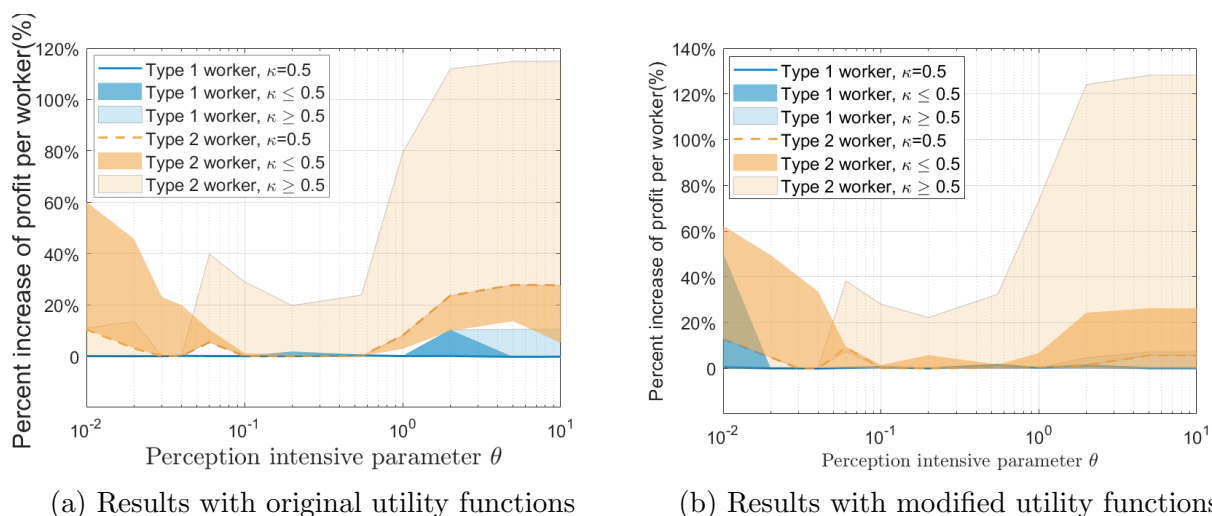
Figure EC.3 Profit per worker under symmetric two-team contests

3. **Impacts on the asymmetric two-team contests** Lastly, we examine how the way labor costs are defined affects contest outcomes when teams differ in their composition. Again, the changes to the utility functions have marginal impacts on the platform's profit. Previous results indicate that the impacts of team

compositions on workers are type-dependent. Figure EC.5b shows that the same variation in the profit per worker regardless of the changes in the utility functions.



**Figure EC.4** Platform's profit under asymmetric two-team contests



**Figure EC.5** Platform's profit under asymmetric two-team contests

## B.2 Algorithm for solving market equilibrium under team contests

### B.3 Contest participation constraints and the relaxation of exogenous self-scheduling utility

The intra-team coordination problem (6) (see Section 3.2) takes the self-scheduling utility  $u_0^k$  as input for contest participation constraints (6d). There, we assume that  $u_0^k$  is an exogenous parameter determined by a fully self-scheduling benchmark where all workers are self-scheduled, and no teams contests are implemented. Such an assumption greatly simplifies our market equilibrium analysis because a worker's decision for joining a team is independent of others' team joining decisions.

**Algorithm 1:** A penalty-duality-based algorithm for market equilibrium**Result:** Scheduling plans of all teams at market equilibrium  $\mathbf{f}^*$ ;Set iteration counter  $m = 0$ , initialize parameters  $\mu_j^k$ ,  $\rho$  and an error tolerance  $\epsilon$ ;Solve optimization problem (10) for the optimal schedule  $\mathbf{f}^{*(0)}$ ;**while** *there exists*  $(k, j)$  such that  $g_j^k(\mathbf{f}_j^{*(m)}, \mathbf{N}_{-j}^{t(m)}, R, F_m) > \epsilon$  **do**    Update  $\mu_j^k$  and  $\rho$  by setting  $\mu_j^{k(m+1)} = [\mu_j^{k(m)} + \rho^{(m)} \cdot g_j^k]^+$  and  $\rho^{(m+1)} > \rho^{(m)}$ ;    Solve optimization problem (10) for the optimal solution  $\mathbf{f}^{*(m+1)}$ ;    Set  $\mathbf{f}^{*(m+1)} \rightarrow \mathbf{f}^{*(m)}$ ,  $\mu_j^{k(m+1)} \rightarrow \mu_j^{k(m)}$ ,  $\rho^{(m+1)} \rightarrow \rho^{(m)}$ ;**end**Output the converged scheduling plans of all teams  $\mathbf{f}^{*(m)}$ .

**Team assignment problem.** Relaxing the assumption on exogenous self-scheduling utility could lead to combinatorial team assignment problems. Consider the case where the self-scheduling utility  $u_0^k$  is calculated as the utility earned by a focal worker who decides to work independently while all others stick to their team joining decisions. Let  $N_j^k$  be the number of  $k$ -type workers in team  $j$ . By the definition of the above "endogenous" self-scheduling utility,  $u_0^k$  is calculated from a new game where teams have compositions  $\{N_j^k - 1, 1, N_{-j}^k\}_{j \in J, k \in K}$  and it is equal to the focal worker's utility at equilibrium (denoted as  $u_0^k(\{N_j^k - 1, 1, N_{-j}^k\})$  for indicating its relations with team composition). Then, the resultant contest participation constraints (6d) become  $u_j^k \geq u_0^k(\{N_j^k - 1, 1, N_{-j}^k\})$  for any team  $j \in J$  and worker type  $k \in K$ . Note that  $u_0^k(\{N_j^k - 1, 1, N_{-j}^k\})$  depends on teams' coordinated schedules corresponding to the team composition  $\{N_j^k - 1, 1, N_{-j}^k\}$ , it further depends on  $u_0^k(\{N_j^k - 2, 1, 1, N_{-j}^k\})$  by virtue of contest participation constraints. Continuing the above process finally leads to a team assignment problem, which involves finding pairs of self-scheduling workers such that all workers are willing to work together with their teammates.

**An approximation method.** The above team assignment problem arises because of the discretization of team workers and the interdependence between team compositions and self-scheduling utility. In view of these two reasons, we make the following two assumptions for approximating workers' endogenous team joining decisions. First, we assume that the supply of team workers is infinitesimal so that the impacts of a worker's decision on the market state are negligible. Second, we suppose that a worker would join the team if she could get a utility no less than that of being self-scheduled *given that other workers participate in contests and stick to their team schedules*. In this way, the self-scheduling utility  $u_0^k$  becomes a function of teams' supply decision  $\mathbf{N}$  ( $\mathbf{N} = (\mathbf{N}^t)_{t=1}^{T_0}$ ) corresponding to the current team composition  $\{N_j^k\}_{\{j \in J, k \in K\}}$ . Specifically, given the per-period number of active workers  $\mathbf{N}^t$ , a self-scheduling worker's utility  $u_k^p$  along each path  $p$  can be expressed as the following formula:

$$u_k^p(\mathbf{N}) = \sum_{t \in T} w^t \frac{q^t(\mathbf{N}^t)}{\sum_{j \in J} N_j^t} \cdot \delta_t^p - \sum_{t \in T} c^{kt} \cdot \delta_t^p - c_h (h^p)^\nu, \quad \forall k \in K, \quad (\text{EC.1})$$

where the first term represents a worker's total revenue if the worker decides to work independently and chooses the schedule defined by path  $p$ ; and the last two terms describe the corresponding labor costs. With

$u_k^p$ , the approximated self-scheduling utility  $u_0^k$  can be derived by applying individual workers' scheduling strategies. Consider, for example, a self-scheduled worker who chooses the path that maximizes her perceived utility, as in the fully self-scheduling benchmark (see Section 5). Then, given the perception intensive parameter  $\theta$ , the expected utility  $u_0^k$  of a self-scheduled k-type worker can be expressed as the following formula:

$$u_0^k(\mathbf{N}|\theta) = \sum_{p \in P} \frac{\exp(\theta \cdot u_k^p(\mathbf{N}))}{\sum_{p' \in P} \exp(\theta \cdot u_k^{p'}(\mathbf{N}))} \cdot u_k^p(\mathbf{N}), \quad \forall k \in K, \quad (\text{EC.2})$$

where the term  $\frac{\exp(\theta \cdot u_k^p(\mathbf{N}^t))}{\sum_{p' \in P} \exp(\theta \cdot u_k^{p'}(\mathbf{N}^t))}$  describes the probability that a k-type self-scheduled worker chooses path  $p$  (see Appendix D for details). Then, for each team  $j \in J$ , the contest participation constraints (6d) are modified as follows:

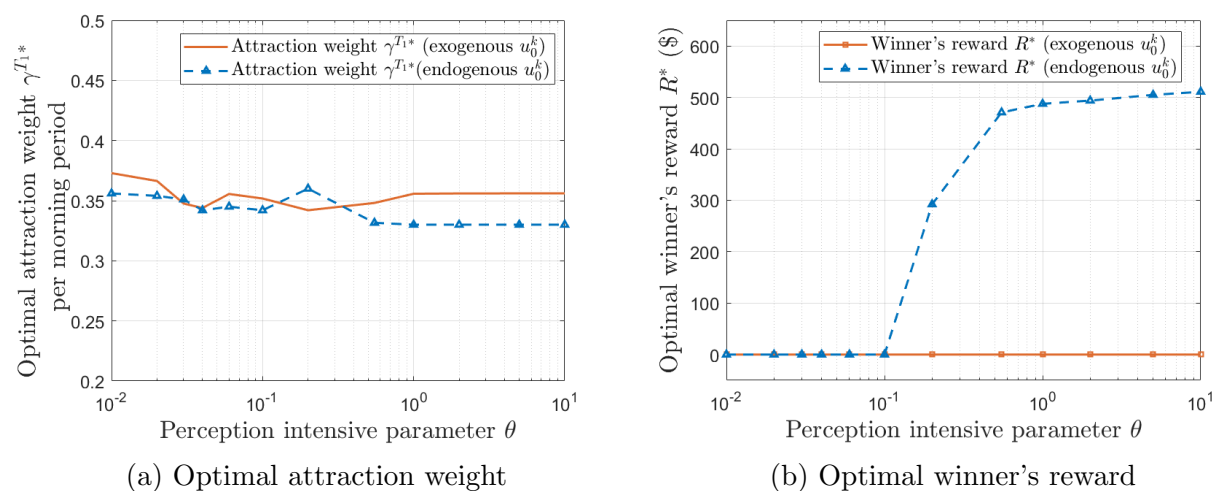
$$u_j^k(\mathbf{f}_j, \mathbf{N}_{-j}, R|F_m) \geq \sum_{p \in P} \frac{\exp(\theta \cdot u_k^p(\mathbf{N}))}{\sum_{p' \in P} \exp(\theta \cdot u_k^{p'}(\mathbf{N}))} \cdot u_k^p(\mathbf{N}), \quad \forall k \in K. \quad (\text{EC.3})$$

The above approximation endogenizes workers' contest participation constraints because each team leader must consider the impact of team schedules on workers' joining decisions and ensure that team members are better off than if they were independently self-scheduled.

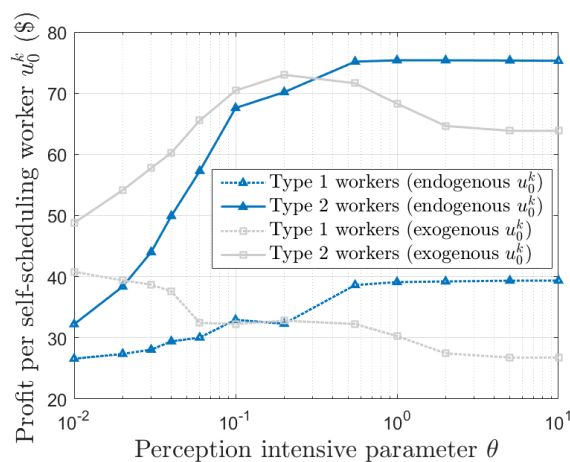
**Numerical experiments.** In numerical experiments, we consider the contest scheme design for the situation in which the designed scheme and resulting team schedules ensure the participation of all workers. The contest scheme design problem and the underlying market equilibrium model are reformulated by replacing (6d) with the modified contest participation constraints (EC.3) and are solved using the algorithm introduced in Section 4.2 and Section 3.4, respectively. Other numerical parameters, such as team compositions and market potential demand, remain the same as in Section 5.1. With symmetric two-team contests, numerical experiments examine the optimal platform-centric contest scheme and the corresponding market equilibrium under different values of perception-intensive parameter  $\theta$ . The results are shown in Figure EC.6, Figure EC.7, and Figure EC.8. There, we use "exogenous  $u_0^k$ " and "endogenous  $u_0^k$ " to differentiate whether self-scheduling utility  $u_0^k$  are exogenous parameters or endogenously determined by the above approximation method, respectively.

Figure EC.6 compares the optimal attraction weights per morning period and the optimal winner's reward based on different approaches for calculating self-scheduling utility. Figure EC.6a indicates that whether  $u_0^k$  is determined endogenously has marginal impacts on the optimal attraction weights. In contrast, it does affect the value of the optimal winner's reward (Figure EC.6b). When workers perceive their utility inaccurately, the optimal winner's reward remains zero despite the differences in calculating self-scheduling utility. When workers perceive their utility accurately, however, endogenizing self-scheduling utility induces a positive winner's reward.

The positive winner's reward could be explained by the higher value of self-scheduling utility when  $u_0^k$  is endogenously determined and could be accurately perceived by workers (Figure EC.7). In this case, the high self-scheduling utility is expected because workers can freely choose to join the team or not to maximize true utility. An external reward is hence necessary to attract workers to participate in contests.



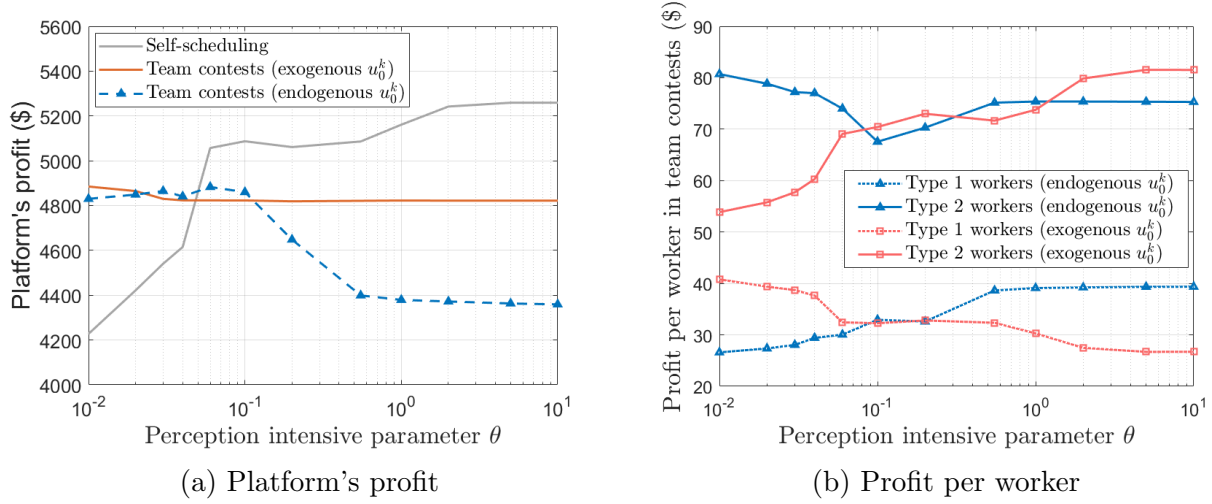
**Figure EC.6** The platform's optimal contest scheme per different settings



**Figure EC.7** Worker's self-scheduling utility

Figure EC.8 compares the profit of the platform and the profit per worker under different market settings. Figure EC.8a shows that the "dual effects" hold despite different assumptions regarding self-scheduling utility and workers' contest participation constraints. When workers exhibit low perception accuracy towards their utility, relaxing the assumption on exogenous self-scheduling utility has a marginal effect on the platform's profit. The profit discrepancy increases when workers perceive their scheduling utilities accurately, which can be attributed to the increasing value of the optimal winner's reward. Figure EC.8b shows that the impacts of using endogenous self-scheduling utility differ among worker types. If the self-scheduling utility is endogenously determined, Type 2 workers who prefer to provide service in the morning periods are more willing to stay in teams when their perception accuracy is low. In contrast, the opposite holds for Type 1 workers who prefer to provide service in the afternoon periods. Overall, Type 2 workers are less sensitive to the ways of calculating self-scheduling utility than Type 1 workers.

To summarize, the impact of (approximately) endogenizing self-scheduling utility varies based on workers' perception accuracy and differs among the platform and workers. When workers perceive their utility less



**Figure EC.8 Profit of the platform and workers per different settings**

accurately, using the endogenous self-scheduling utility has marginal impacts on the optimal platform-centric contest scheme design and the platform's profit. However, when workers perceive their utility accurately, endogenizing self-scheduling utility would incur a positive winner's reward and reduce the platform's profit. Depending on time preferences, workers of different types differ in their sensitivity to the changes in self-scheduling utility.

## Appendix C Supplementary Materials for Section 4

### C.1 Feasibility of the platform-centric contest scheme design problem

**PROPOSITION 1.** [*Feasibility of the platform-centric contest scheme design problem*] *There exists at least one feasible contest scheme  $(R, \gamma)$  to problem (13), under which the market equilibrium exists.*

Proving Proposition 1 is equivalent to proving that there exists at least one non-negative solution  $(R, \gamma)$  such that the normalization constraint (13b) is satisfied and the QVI's solution set  $\mathcal{S}(R, \gamma)$  is non-empty. In this proof, we first remove contest participation constraints (6d) from intra-team coordination problem (6) and transform the QVI defined by (8) into the following VI:

$$\sum_j L_j(\mathbf{f}_j^*)^T (\mathbf{f}_j - \mathbf{f}_j^*) \geq 0, \quad \forall \mathbf{f}_j \in \mathcal{M}_j. \quad (\text{E1})$$

where the feasible set of each team's schedules is  $\mathcal{M}_j = \{\mathbf{f}_j | \sum_{p \in P} f_j^{kp} = N_j^k, f_j^{kp} \geq 0\}$  and is independent of other teams' decisions. After proving the solution existence to the above VI (Lemma EC.1), we complete the proof by showing that there is a contest scheme  $(R, \gamma)$  that satisfies constraint (13b) and that  $\mathcal{S}(R, \gamma)$  coincides with the solution set of the above VI.

**LEMMA EC.1.** *There exist solutions to the VI problem (E1).*

The feasible set  $\mathcal{M}$  that defines the VI problem (E1) is the Cartesian product of the feasible set for each team  $j \in J$ , i.e.,  $\mathcal{M} = \prod_{j=1}^{J_0} \mathcal{M}_j$ . Because  $\mathcal{M}_j$  is a nonempty, closed, and bounded polyhedron, the set  $\mathcal{M}$  is

compact. Furthermore, because the solutions to the VI also belong to  $\mathcal{M}$ , the VI problem (E1) essentially defines a mapping from  $\mathcal{M}$  to the set itself.

Next, we demonstrate that the function  $L_j(\mathbf{f}_j)$  that defines the VI problem (E1) is continuous with variables  $\mathbf{f}_j$ . For each team  $j \in J$ , the partial derivative of team utility  $U_j$  with respect to a specific path flow  $f_j^{kp}$  is as follows:

$$\frac{\partial U_j(\mathbf{f}_j, \mathbf{N}_{-j}, R)}{\partial f_j^{kp}} = \sum_{t \in T} w^t \cdot \frac{\partial q_j^t}{\partial N_j^t} \cdot \delta_t^p + R \cdot \Phi_j - \sum_{t \in T} c^{kt} \cdot \delta_t^p - c_h \cdot (h^p)^\nu, \quad \forall k \in K, p \in P, \quad (\text{E2})$$

where the team utility  $U_j(\mathbf{f}_j, \mathbf{N}_{-j}, R)$  is specified by equation (4); the term  $\Phi_j$  comes from the Tullock contest success function in equation (3) and is derived as follows:

$$\Phi_j = \frac{1}{\sum_{t \in T} q^t} \cdot \left( \sum_{t \in T} \frac{\partial q_j^t}{\partial N_j^t} \cdot \delta_t^p \right) - \frac{\sum_{t \in T} q_j^t}{(\sum_{t \in T} q^t)^2} \cdot \left( \sum_{t \in T} \sum_{i \neq j} \frac{\partial q_i^t}{\partial N_j^t} \cdot \delta_t^p \right), \quad \forall j \in J. \quad (\text{E3})$$

By equation (E2),  $\partial U_j(\mathbf{f}_j, \mathbf{N}_{-j}, R) / \partial f_j^{kp}$  is a continuous function of  $f_j^{kp}$  because the demand function  $F_q(\cdot)$  is twice differentiable (equation (1)) and the link flow  $N_j^t$  continuously changes with the path flow  $f_j^{kp}$ . Therefore, the function  $L_j(\mathbf{f}_j) = -\nabla_{f_j^{kp}} U_j(\mathbf{f}_j, \mathbf{N}_{-j}, R)$  defines a continuous mapping from  $\mathcal{M}_j$  into  $\mathbb{R}^{(|K| \times |P|)}$  for each team  $j \in J$ .

Based on Brouwer's fixed-point theorem and Theorem 3.1 from Harker and Pang (1990), the above conclusions on compact  $\mathcal{M}$  and continuous  $L_j(\mathbf{f}_j)$  imply that there exist solutions to the VI defined by (E1).

□

Next, we show that there exists a feasible solution  $(R, \gamma)$  to the platform-centric contest scheme design problem (13). For simplicity, we select an attraction weight  $\gamma^t = 1/|T|$  for period  $t \in T$ , where  $|T|$  is the cardinality of the period set  $T$ . The normalization constraint (13b) is automatically satisfied.

Following equation (12), a  $k$ -type worker's utility under the platform-centric scheme  $(R, 1/|T|)$  is as follows:

$$u_j^k(\mathbf{f}_j, \mathbf{N}_{-j}, R | \frac{1}{|T|}) = \frac{\sum_{t \in T} N_j^{kt}}{\sum_{k \in K} \sum_{t \in T} N_j^{kt}} \cdot \frac{R_j}{N_j^k} - \frac{1}{N_j^k} \cdot \left( \sum_{t \in T} c^{kt} N_j^{kt} + c_h \sum_{p \in P} f_j^{kp} \cdot (h^p)^\nu \right), \quad \forall j \in J, k \in K. \quad (\text{E4})$$

where the team revenue  $R_j$  is given by

$$R_j = \sum_{t \in T} w^t q_j^t(N_j^t, \mathbf{N}_{-j}^t) + R \cdot \frac{\sum_{t \in T} q_j^t}{\sum_{t \in T} q^t(\mathbf{N}^t)}, \quad \forall j \in J. \quad (\text{E5})$$

Fixing workers' schedules  $(\mathbf{f}_j)_{j \in J}$  and active workers' supply  $(\mathbf{N}^t)_{t \in T}$  at one solution of the VI defined by (E1), a  $k$ -type worker's utility  $u_j^k$  continually increases with the winner's reward  $R$  if  $\sum_{t \in T} N_j^{kt} > 0$ . Thus, there must exist a non-negative reward  $\hat{R}$  such that the contest participation constraints  $u_j^k(\mathbf{f}_j, \mathbf{N}_{-j}, \hat{R}) > u_0^k$  hold for each worker type  $k \in K$  and each team  $j \in J$ . With that, contest participation constraints (6d) could be safely dropped in deriving the optimal solutions to intra-team coordination problem (6). Consequently, the QVI defined by (8) becomes the VI as shown in (E1), giving back  $(\mathbf{f}_j)_{j \in J}$  as a solution in the set  $\mathcal{S}(R, \gamma)$ . Therefore, the contest scheme  $(\hat{R}, 1/|T|)$  is a feasible solution to the contest scheme design problem (13). □

## C.2 Market equilibrium and contest scheme design for the one-period special case

PROPOSITION 2. [*Supply of active workers with the number of teams*] If both the average revenue of workers  $w \frac{q(\sum_{j \in J} f_j^1)}{\sum_{j \in J} f_j^1}$  and the marginal revenue  $w \frac{\partial q(\sum_{j \in J} f_j^1)}{\partial f_j^1}$  decrease with the number of active workers, then the total supply of active workers increases with the number of competing teams  $|J|$  at market equilibrium.

Without causing confusion, the following proof removes the superscripts  $k$  and  $t$  from notations, as this one-period special case considers one type of worker and one period. With team contests, each team maximizes team utility by optimizing the number of active workers  $f_j^1$ . For a given winner's reward  $R$ , the intra-team coordination problem (6) reduces to the following optimization problem:

$$\max_{f_j^1} U_j(f_j^1, \mathbf{f}_{-j}^1, R) = \frac{f_j^1}{\sum_{i \in J} f_i^1} \cdot \left( w \cdot q(\sum_{i \in J} f_i^1) + R \right) - (c + c_h) \cdot f_j^1, \quad (\text{EC.4a})$$

$$\text{s.t.} \quad 0 \leq f_j^1 \leq \frac{N}{|J|}, \quad (\text{EC.4b})$$

$$U_j(f_j^1, \mathbf{f}_{-j}^1, R) \geq 0. \quad (\text{EC.4c})$$

where constraint (EC.4c) is derived from contest participation constraints (6d) with the self-scheduling utility being zero. As  $f_j^1 = 0$  already constitutes a feasible solution to problem (EC.4), constraint (EC.4c) is satisfied automatically. Therefore, the following will focus on the case where  $f_j^{1*} > 0$  and constraint (EC.4c) is inactive at optimality. Further, the constraint (EC.4b) can also be assumed to be inactive at optimality because  $N$  is large enough.

Because teams are identical, team leaders are expected to adopt the same scheduling strategies. Applying the first-order optimality conditions to problem (EC.4) hence leads to the following formula:

$$\frac{|J| - 1}{|J|} \cdot \frac{w \cdot q + R}{|J| \cdot f_j^{1*}} + \frac{1}{|J|} \cdot w \frac{\partial q}{\partial f_j^1} \Big|_{f_j^1 = f_j^{1*}} = (c + c_h), \quad (\text{EC.5})$$

$$\frac{w \cdot q + R}{|J| \cdot f_j^{1*}} > c + c_h. \quad (\text{EC.6})$$

where (EC.5) states that the weighted average of the marginal revenue and the average revenue of workers is a constant at optimality. Inequality (EC.6) results from constraint (EC.4c) and indicates that the average revenue of workers is higher than the service costs.

By (EC.5) and (EC.6), the inequality  $w \frac{\partial q}{\partial f_j^1} < (c + c_h) < \frac{w \cdot q + R}{|J| \cdot f_j^{1*}}$  holds at the optimal solution. Therefore, for given winner's reward  $R$  and schedule  $f_j^1$ , the left-hand side of equation (EC.5) increases with the number of teams  $|J|$ . Considering that both the marginal revenue and workers' average revenue decrease with the number of active workers,  $f_j^{1*}$  increases with  $|J|$  for sustaining equation (EC.5). Therefore, the supply of total active workers increases with the number of teams.

PROPOSITION 3. [*The optimal contest scheme and market equilibrium*] If the marginal revenue satisfies  $w \frac{\partial q}{\partial f} \geq w \frac{q(f)}{f}$  at self-scheduling equilibrium, then the optimal winner's reward  $R^* \geq 0$  and the number active workers at team contest market equilibrium satisfies  $\sum_{j \in J} f_j^{1*} \geq f_0$ . Otherwise, the inequality  $\sum_{j \in J} f_j^{1*} < f_0$  holds at market equilibrium when  $R^* = 0$ .

By assumption, the number of workers  $N$  is large enough and the utility of workers equals zero at the self-scheduling equilibrium. Therefore, the supply of active workers at the self-scheduling equilibrium  $f_0$  satisfies the following formula:

$$w \cdot \frac{q(f_0)}{f_0} = (c + c_h), \quad (\text{EC.7})$$

That is, the average revenue of workers is equal to their service costs.

When  $w \frac{\partial q}{\partial f} \geq w \frac{q(f)}{f}$  holds at the self-scheduling equilibrium, equation (EC.7) indicates that the inequality  $w \frac{\partial q}{\partial f} \geq (c + c_h)$  holds at  $f_j^1 = f_0/|J|$ . Then, if  $w \frac{\partial q}{\partial f} = (c_0 + c_h)$  holds at  $f_j^1 = f_0/|J|$ , the winner's reward  $R^* = 0$  constitutes an optimal solution to the contest scheme design problem and  $f_j^{1*} = f_0/|J|$  at market equilibrium. If  $w \frac{\partial q}{\partial f} > (c_0 + c_h)$  holds at  $f_j^1 = f_0/|J|$ , for (EC.5) and (EC.6) to be sustained, the supply of active workers at equilibrium satisfies  $\sum_i f_j^{1*} < f_0$  because the marginal revenue  $w \frac{\partial q}{\partial f}$  decreases with  $f$ . As a result, the optimal winner's reward  $R^* > 0$ .

Otherwise, when  $w \frac{\partial q}{\partial f} < w \frac{q(f)}{f}$  holds at  $f_0$  and  $R^* = 0$ , the number of active workers should satisfy  $\sum_i f_j^{1*} < f_0$  for sustaining workers' participation constraint (EC.6).

### C.3 Global optimization for the optimal contest scheme design

Bayesian optimization (BO) is a powerful approach for solving black-box derivative-free global optimization problems (Frazier 2018). In the absence of an exact objective function form, BO approximates it with a probabilistic surrogate model and then performs the optimization. More specifically, BO treats the objective function as a random function and applies a prior measure to it. With each sample drawn from the objective function, a posterior distribution is then computed to better approximate the objective. To determine the next sampling point, an acquisition function is optimized based on the posterior distribution. The acquisition function combines exploration and exploitation in its search for a new point. There, exploration means searching toward unexplored regions with high predicted uncertainty. By contrast, exploitation focuses on sampling where the surrogate model predicts a favorable outcome. Typically, Bo applies to optimization problems with simple feasible sets and dimensions less than 20.

To solve the optimal contest scheme with BO, we use the commonly adopted Gaussian process as the surrogate model. The sampling points are determined by an acquisition function called ‘‘expected-improvement-plus’’ function. This acquisition function evaluates a point based on the expected improvement in the objective function value. It avoids regions from being over-exploited to escape a local optimum and is a typical choice of BO. In programming, we set the exploration ratio at 0.6. As a stop criterion, BO is set to evaluate the objective function no more than 50 times. Throughout numerical experiments, the function ‘‘bayesopt’’ in Matlab is used to implement BO.

## Appendix D Supplementary Materials for Section 5

**Fully self-scheduling benchmark.** Under fully self-scheduling, each worker makes scheduling decisions independently to maximize perceived utility. Thus, a  $k$ -type worker chooses schedule  $p$  only if  $\theta u_k^p + \xi^p \geq \theta u_k^{p'} + \xi^{p'}$  for all  $p' \in P$ . Here,  $\theta$  measures the dispersion among workers regarding their perceived utility. When  $\theta$  is small, the random term  $\xi^p$  and the deterministic term  $\theta \cdot u_k^p$  are comparable, indicating a large

**Algorithm 2:** Solution algorithm for the optimal platform-centric contest scheme

**Result:** The global optimal platform-centric contest scheme  $(R^*, \gamma^*)$  and the scheduling plans of all teams at market equilibrium  $\mathbf{f}^*$ ;

Initialize suggested searching points  $(R^{(0)}, \gamma^{(0)})$ , penalty parameter  $\eta$ , and an error tolerance  $\epsilon$ ;

**while** BO does not reach the maximum number of objective function evaluations **do**

    Set iteration counter  $m = 0$ , initialize parameters  $\mu_j^k$  and  $\rho$ ;

    Solve problem (14) for a local optimal solution  $(R^{*(m)}, \gamma^{*(m)}, \mathbf{f}^{*(m)})$ ;

**while**  $z_{VI}^{*(m)} > \epsilon$  **do**

        Update  $\mu_j^k$  and  $\rho$  by setting  $\mu_j^{k(m+1)} = [\mu_j^{k(m)} + \rho^{(m)} \cdot g_j^k]^+$ ,  $\rho^{(m+1)} > \rho^{(m)}$ ;

        Solve problem (14) for a local optimal solution  $(R^{*(m+1)}, \gamma^{*(m+1)}, \mathbf{f}^{*(m+1)})$ ;

        Set  $\mathbf{f}_j^{*(m+1)} \rightarrow \mathbf{f}_j^{*(m)}$ ,  $\mu_j^{k(m+1)} \rightarrow \mu_j^{k(m)}$ ,  $\rho^{(m+1)} \rightarrow \rho^{(m)}$ ;

**end**

    Output the local optimal solution  $(R^{*(m)}, \gamma^{*(m)}, \mathbf{f}^{*(m)})$  and the objective function value of problem (14);

    Seek the next suggested searching point  $(R_0, \gamma_0)$  via BO;

**end**

Output the best local solution so far  $(R^*, \gamma^*, \mathbf{f}^*)$ ;

variance in workers' perceptions. By contrast, a large value of  $\theta$  makes the deterministic term a dominant one, which implies a small perception variance among workers. Using the discrete choice model, the probability that a  $k$ -type worker would choose schedule  $p$  is given by  $\mathbb{P}^{kp} = \mathbb{P}[\theta u_k^p + \xi^p \geq \theta u_k^{p'} + \xi^{p'}, \forall p' \in P]$ . Assume that the random terms follow identical and independent Gumbel distributions, we have  $\mathbb{P}^{kp} = \frac{\exp(\theta \cdot u_k^p)}{\sum_{p' \in P} \exp(\theta \cdot u_k^{p'})}$ . At equilibrium, no worker can gain a higher perceived utility by unilaterally changing the working schedule. Plugging in the above scheduling choice model, the equilibrium flow  $\mathbf{f}$  can be obtained by solving the following fixed-point problem:

$$f^{kp} = \mathbb{P}^{kp}(\mathbf{f}) \cdot \sum_{j \in J} N_j^k, \quad \mathbb{P}^{kp}(\mathbf{f}) = \frac{\exp(\theta \cdot u_k^p(\mathbf{f}))}{\sum_{p' \in P} \exp(\theta \cdot u_k^{p'}(\mathbf{f}))}, \quad \forall p \in P, k \in K. \quad (\text{E6})$$

When  $\theta \rightarrow 0$ , workers will take each possible schedule with an equal probability. At equilibrium, workers are evenly distributed over all paths. When  $\theta \rightarrow +\infty$ , all workers choose the schedule yielding the highest real utility  $u_k^p$ . For a given value of  $\theta$ , the self-scheduling utility is calculated as  $u_0^k = \sum_{p \in P} \mathbb{P}^{kp} \cdot u_k^p$  for each worker type  $k \in K$ .