Electronic Companion

EC.1. Proof of Proposition 1

Proof. The set of constraints in ALP (9) is equivalent to

$$(TV)(\boldsymbol{x},\ell) \geq V(\boldsymbol{x},\ell), \quad \boldsymbol{x} \in \mathcal{X}', \ \ell \in \mathcal{L}, \tag{EC.1}$$

where T is a monotonic dynamic programming operator. Additionally, T is a contraction with fixed point V^*. Therefore, for any feasible $\tilde{V}$ that satisfies constraints (EC.1), we have that

$$\tilde{V} \leq T\tilde{V} \leq T^2\tilde{V} \leq \cdots \leq V^*,$$

from which the result follows. □

EC.2. Proof of Lemma EC.1

The following Lemma EC.1 is needed to prove Proposition 2.

LEMMA EC.1. *For a given $n \in \mathcal{N}$ and $x_n \in \mathcal{D}$,*

$$\bigcup_{\boldsymbol{x}_{-n}\in\mathcal{X}_{-n}} \mathcal{G}(\boldsymbol{x}) = \bigcup_{\boldsymbol{x}_{-n}\in\mathcal{X}_{-n}} \bigcup_{\boldsymbol{a}_{-n}\in\mathcal{A}_{-n}} Q(n,\boldsymbol{x},\boldsymbol{a}_{-n}). \tag{EC.2}$$

For example, if $N=3$ and we fix $n=1$ and $x_1 \in \mathcal{D}$, the left-hand side of (EC.2) is

$$\bigcup_{\boldsymbol{x}_{-1}\in\mathcal{X}_{-1}} \mathcal{G}(\boldsymbol{x}) = \bigcup_{(x_2,x_3)\in\mathcal{D}^2} \mathcal{G}((x_1,x_2,x_3)).$$

Similarly, for the right-hand side of (EC.2) we have

$$\bigcup_{\boldsymbol{x}_{-1}\in\mathcal{X}_{-1}} \bigcup_{\boldsymbol{a}_{-1}\in\mathcal{A}_{-1}} Q(1,\boldsymbol{x},\boldsymbol{a}_{-1}) = \bigcup_{(x_2,x_3)\in\mathcal{D}^2} \bigcup_{(a_2,a_3)\in\{0,1\}^2} Q(1,(x_1,x_2,x_3),(a_2,a_3)).$$

The proof of Lemma EC.1 follows.

Proof. For a given n and $x_n \in \mathcal{D}$, first assume

$$(\boldsymbol{x}',\ell',\boldsymbol{a}') \in \bigcup_{\boldsymbol{x}_{-n}\in\mathcal{X}_{-n}} \mathcal{G}(\boldsymbol{x}),$$

which is equivalent to the following:

(i) $\hat{x}_n(x'_n,\ell',a'_n) = x_n$, and

(ii) $\hat{\boldsymbol{x}}_{-n}(\boldsymbol{x}'_{-n},\ell',\boldsymbol{a}'_{-n}) \in \mathcal{X}_{-n}$.

Relation (ii) holds since $\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}$ and $\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}$. Thus,

$$(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \bigcup_{\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}} \mathcal{Q}(n, \boldsymbol{x}, \boldsymbol{a}_{-n}),$$

and, subsequently,

$$\bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \mathcal{G}(\boldsymbol{x}) \subseteq \bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \bigcup_{\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}} \mathcal{Q}(n, \boldsymbol{x}, \boldsymbol{a}_{-n}).$$

Next, assume

$$(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \bigcup_{\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}} \mathcal{Q}(n, \boldsymbol{x}, \boldsymbol{a}_{-n}),$$

which is equivalent to the following:

(a) $\hat{x}_n(x'_n, \ell', a'_n) = x_n$,

(b) $\boldsymbol{x}'_{-n} \in \mathcal{X}_{-n}$,

(c) $\boldsymbol{a}'_{-n} \in \mathcal{A}_{-n}$.

For any ℓ' that satisfies (a), and from (b) and (c), we have $\hat{\boldsymbol{x}}_{-n}(\boldsymbol{x}'_{-n}, \ell', \boldsymbol{a}'_{-n}) \in \mathcal{X}_{-n}$. Thus,

$$(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \mathcal{G}(\boldsymbol{x}),$$

and, subsequently,

$$\bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \bigcup_{\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}} \mathcal{Q}(n, \boldsymbol{x}, \boldsymbol{a}_{-n}) \subseteq \bigcup_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \mathcal{G}(\boldsymbol{x}).$$

Therefore, the result holds. □

EC.3. Proof of Proposition 2

Proof. It suffices to show that every solution z that satisfies constraints (11b) and (11c) also satisfies (10b) and (10c). Hence, consider a z that satisfies the set of constraints (11b), i.e.,

$$\sum_{\boldsymbol{a} \in \mathcal{A}} z(\boldsymbol{x}, \ell, \boldsymbol{a}) - \lambda \sum_{(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \mathcal{G}(\boldsymbol{x})} p_{\ell'\ell}\, z(\boldsymbol{x}', \ell', \boldsymbol{a}') = \frac{1}{Ld^N}, \quad (\boldsymbol{x}, \ell) \in \Gamma.$$

Next, for a given (n, x_n, ℓ), sum both sides of the equality over all possible $\boldsymbol{x}_{-n}$ in the set $\mathcal{X}' \backslash \{\mathcal{D}\}$. Then, we have

$$\sum_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \sum_{\boldsymbol{a} \in \mathcal{A}} z(\boldsymbol{x}, \ell, \boldsymbol{a}) - \lambda \sum_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \sum_{(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \mathcal{G}(\boldsymbol{x})} p_{\ell'\ell}\, z(\boldsymbol{x}', \ell', \boldsymbol{a}') = \sum_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \frac{1}{Ld^N},$$

which is equivalent to

$$\sum_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \sum_{\boldsymbol{a} \in \mathcal{A}} z(\boldsymbol{x}, \ell, \boldsymbol{a}) - \lambda \sum_{\boldsymbol{x}_{-n} \in \mathcal{X}_{-n}} \sum_{\boldsymbol{a}_{-n} \in \mathcal{A}_{-n}} \sum_{(\boldsymbol{x}', \ell', \boldsymbol{a}') \in \mathcal{Q}(\boldsymbol{x})} p_{\ell'\ell}\, z(\boldsymbol{x}', \ell', \boldsymbol{a}') = \frac{1}{Ld}. \tag{EC.3}$$

Therefore, for all (n, x_n, ℓ), z satisfies the equality (EC.3). □

EC.4. Proof of Theorem 1

Proof. Suppose there exists a DLP and DALP optimal solution satisfying condition (12). This assumption implies that the DLP and DALP optimal objective function values are equal. Since DALP is a relaxation of DLP (Proposition 2), every optimal solution of DLP is also optimal for DALP.

Now, suppose π^* is the optimal policy for the Bellman equations. By Theorem 6.9.1 of Puterman (2014), we can define an optimal solution to DLP denoted by z^*. Hence, z^* is also a DALP optimal solution that corresponds to the optimal policy π^*. This deterministic policy can be represented equivalently as the following set

$$\Pi := \{(\boldsymbol{x}, \ell, \boldsymbol{a}) : z^*(\boldsymbol{x}, \ell, \boldsymbol{a}) > 0\}.$$

By complementary slackness, the ALP constraints corresponding to triples in the set Π are binding. Hence, for each triple $(\boldsymbol{x}, \ell, \boldsymbol{a})$ in Π, $\boldsymbol{a}$ is a greedy optimal action at state $(\boldsymbol{x}, \ell)$ with respect to the approximate value function obtained by $\boldsymbol{v}^*$. □

EC.5. Proof of Lemma 1

Proof. By the constraints in DALP, we have

$$\sum_{(\boldsymbol{x}_{-n}, \boldsymbol{a}) \in \mathcal{X}_{-n} \times \mathcal{A}} y(\boldsymbol{x}, \ell, \boldsymbol{a}) - \lambda \sum_{(\boldsymbol{x}_{-n}, \boldsymbol{a}_{-n}) \in \mathcal{X}_{-n} \times \mathcal{A}_{-n}} \sum_{(x'_n, k, a'_n) \in \mathcal{Q}(n, x_n)} p_{k\ell}\, y(\boldsymbol{x}', k, \boldsymbol{a}') = \frac{1}{Ld}, \quad (n, \ell, x_n) \in \Xi,$$

By summing both sides of the equality constraints over all $n \in \mathcal{N}$, $\ell \in \mathcal{L}$, $x_n \in \mathcal{X}'$, we obtain

$$N \sum_{(\boldsymbol{x}, \ell, \boldsymbol{a}) \in \Theta} y(\boldsymbol{x}, \ell, \boldsymbol{a}) - N\lambda \sum_{(\boldsymbol{x}, \ell, \boldsymbol{a}) \in \Theta} y(\boldsymbol{x}, \ell, \boldsymbol{a}) = \frac{NLd}{Ld}.$$

After some simple algebra, we obtain the equality

$$\sum_{(\boldsymbol{x}, \ell, \boldsymbol{a}) \in \Theta} y(\boldsymbol{x}, \ell, \boldsymbol{a}) - \lambda \sum_{(\boldsymbol{x}, \ell, \boldsymbol{a}) \in \Theta} y(\boldsymbol{x}, \ell, \boldsymbol{a}) = 1,$$

i.e.,

$$\sum_{(\boldsymbol{x}, \ell, \boldsymbol{a}) \in \Theta} y(\boldsymbol{x}, \ell, \boldsymbol{a}) = \frac{1}{1 - \lambda},$$

and the proof is complete. □

EC.6. Proof of Lemma 2

Proof. By contradiction, suppose there exists an optimal solution $\boldsymbol{y}^*$ to DALP and $\ell' \notin \mathcal{L}^=(\boldsymbol{v}^*)$ such that

$$\sum_{(\boldsymbol{x},\boldsymbol{a})\in\mathcal{X}'\times\mathcal{A}} y^*(\boldsymbol{x},\ell',\boldsymbol{a}) > 0.$$

This implies the existence of at least one pair $(\boldsymbol{x}',\boldsymbol{a}')$ such that $y^*(\boldsymbol{x}',\ell',\boldsymbol{a}') > 0$. The feasible region of DALP is bounded due to the nonnegativity of y^* and equation (13) in Lemma 1. Therefore, the constraint in the ALP that corresponds to $(\boldsymbol{x}',\ell',\boldsymbol{a}')$ holds as equality, which contradicts $\ell' \notin \mathcal{L}^=(\boldsymbol{v}^*)$. Therefore, it must be the case that $\ell' \in \mathcal{L}^=(\boldsymbol{v}^*)$. □

EC.7. Proof of Proposition 3

Proof. We assume $\mathcal{L} \neq \mathcal{L}^=(\boldsymbol{v}^*)$. Then, there exists an $\ell' \in \mathcal{L}\backslash\mathcal{L}^=(\boldsymbol{v}^*)$, and by Lemma 2,

$$\sum_{(\boldsymbol{x},\boldsymbol{a})\in\mathcal{X}'\times\mathcal{A}} y^*(\boldsymbol{x},\ell',\boldsymbol{a}) = 0.$$

Since $y^*(\boldsymbol{x},\ell',\boldsymbol{a}) \geq 0$, $y^*(\boldsymbol{x},\ell',\boldsymbol{a}) = 0$ for all $\boldsymbol{x} \in \mathcal{X}'$ and $\boldsymbol{a} \in \mathcal{A}$. Therefore, condition (12) cannot hold in this case since, by definition of the optimal deterministic policy π^*, for any $(\boldsymbol{x},\ell')$ there exists an action, $\boldsymbol{a}'$, for which $z^*(\boldsymbol{x},\ell',\boldsymbol{a}') > 0$. □

EC.8. Proof of Theorem 2

Proof. Assume $\alpha(\boldsymbol{x},\ell) > 0$ for all $(\boldsymbol{x},\ell)$, then we have

$$\mathbb{E}(V) - \mathbb{E}(\tilde{V}) = \sum_{(\boldsymbol{x},\ell)\in\Gamma'} \alpha(\boldsymbol{x},\ell)[V(\boldsymbol{x},\ell) - \tilde{V}(\boldsymbol{x},\ell)] \geq \min_{(\boldsymbol{x},\ell)\in\Gamma'}\{\alpha(\boldsymbol{x},\ell)\} \cdot \|V - \tilde{V}\|. \tag{EC.4}$$

The last inequality holds since we have already shown that $V(\boldsymbol{x},\ell) \geq \tilde{V}(\boldsymbol{x},\ell)$ for all $(\boldsymbol{x},\ell)$. Additionally,

$$\mathbb{E}(V) - \mathbb{E}(\tilde{V}) = \boldsymbol{c}\boldsymbol{z}^* - \boldsymbol{c}\boldsymbol{y}^* = \boldsymbol{c}(\boldsymbol{z}^* - \boldsymbol{y}^*) \leq \|\boldsymbol{c}\| \cdot \|\boldsymbol{z}^* - \boldsymbol{y}^*\|. \tag{EC.5}$$

The first equality holds due to strong duality and the inequality is a direct consequence of the Cauchy–Schwarz inequality. Therefore, by (EC.4) and (EC.5), the result is obtained. □

EC.9. Proof of Lemma 3

Proof. We show there exists $y \geq 0$, supported on C_0, that satisfies every DALP equality constraint (10b).

Step 1: Construct y on C_0. Define the aggregate weights

$$\alpha_0(\ell) := \alpha(\boldsymbol{0},\ell) \quad \text{and} \quad \alpha_{\mathrm{nz}}(\ell) := \sum_{\boldsymbol{x}\in\mathcal{X}_0\backslash\{\boldsymbol{0}\}} \alpha(\boldsymbol{x},\ell), \;\; \ell \in \mathcal{L},$$

and set

$$y(\boldsymbol{x},\ell,\mathbf{1}) := \alpha(\boldsymbol{x},\ell) \quad \text{for all } \boldsymbol{x}\in\mathcal{X}_0\setminus\{\mathbf{0}\},\ \ell\in\mathcal{L}, \tag{EC.6}$$
$$y(\mathbf{0},\ell,\mathbf{1}) := v(\ell), \quad \ell\in\mathcal{L},$$

where $v\in\mathbb{R}^{|\mathcal{L}|}$ is the unique solution of

$$(I-\lambda\mathbf{P}^\top)v = \alpha_0 + \lambda\mathbf{P}^\top\alpha_{\mathrm{nz}}. \tag{EC.7}$$

Since $\lambda\in(0,1)$ and $\mathbf{P}$ is IFR, $(I-\lambda\mathbf{P}^\top)$ is invertible and $(I-\lambda\mathbf{P}^\top)^{-1}\geq 0$, hence $v\geq 0$. By construction, all y in (EC.6) are nonnegative; hence $y\geq 0$ on C_0, and $y=0$ off C_0.

Step 2: Verify (10b). Fix (n,x_n,ℓ) and consider the following two cases.

Case 2a: $x_n>0$. Every active column in C_0 has action $\boldsymbol{a}'=\mathbf{1}$, and by the perfect replacement property it follows that any predecessor $(\boldsymbol{x}',\ell',\boldsymbol{a}')\in\mathcal{Q}$ must satisfy $\hat{x}_n(\cdot)=0$ next stage; thus no predecessor in C_0 can lead to a constraint indexed by $x_n>0$. Hence the discounted term in (10b) is zero. Remaining on the left hand side, only the column $y((x_n,\mathbf{0}_{-n}),\ell,\mathbf{1})$ is present in C_0, so

$$\sum_{(\boldsymbol{x}_{-n},\boldsymbol{a})} y\big((x_n,\boldsymbol{x}_{-n}),\ell,\boldsymbol{a}\big) = y\big((x_n,\mathbf{0}_{-n}),\ell,\mathbf{1}\big) = \alpha\big((x_n,\mathbf{0}_{-n}),\ell\big),$$

by (EC.6). Because α is supported on $\mathcal{X}_0$, the right-hand side equals

$$\sum_{\boldsymbol{x}_{-n}} \alpha\big((x_n,\boldsymbol{x}_{-n}),\ell\big) = \alpha\big((x_n,\mathbf{0}_{-n}),\ell\big).$$

Thus (10b) holds for $x_n>0$.

Case 2b: $x_n=0$. Write $\boldsymbol{x}=(0,\boldsymbol{x}_{-n})$. The forward term collects mass from the all-zero state and from one-nonzero states whose nonzero coordinate lies at some $m\neq n$:

$$\sum_{(\boldsymbol{x}_{-n},\boldsymbol{a})} y\big((0,\boldsymbol{x}_{-n}),\ell,\boldsymbol{a}\big) = v(\ell) + \alpha_{\mathrm{nz}}^{(-n)}(\ell),$$

where $\alpha_{\mathrm{nz}}^{(-n)}(\ell)$ sums α over one-nonzero states in $\mathcal{X}_0$ with the nonzero at indices $m\neq n$. For the discounted term, the outer sum in (10b) enforces $\boldsymbol{a}'_{-n}=\boldsymbol{a}_{-n}$; since y is nonzero only at $\boldsymbol{a}'=\mathbf{1}$, we must have $\boldsymbol{a}_{-n}=\mathbf{1}_{-n}$. The perfect replacement implies that *every* state in C_0 can serve as a predecessor to any constraint with $x_n=0$. Summing over $(\boldsymbol{x}_{-n},\boldsymbol{a}_{-n})$ and then over $\mathcal{Q}$, the discounted term reduces to

$$\lambda\sum_{\ell'\in\mathcal{L}} p_{\ell'\ell}\left(v(\ell')+\alpha_{\mathrm{nz}}(\ell')\right) \;=\; \lambda\left(\mathbf{P}^\top\left[\,v+\alpha_{\mathrm{nz}}\,\right]\right)(\ell).$$

Therefore, the left-hand side equals

$$\Big(v(\ell) - \lambda(\mathbf{P}^\top v)(\ell)\Big) \;+\; \Big(\alpha_{\text{nz}}^{(-n)}(\ell) - \lambda(\mathbf{P}^\top \alpha_{\text{nz}})(\ell)\Big) = \;\alpha_0(\ell) \;+\; \alpha_{\text{nz}}^{(-n)}(\ell) \tag{EC.8}$$

Equality (EC.8) holds using (EC.7), i.e., $(I - \lambda\mathbf{P}^\top)v = \alpha_0 + \lambda\mathbf{P}^\top\alpha_{\text{nz}}$. Additionally, the right-hand side equals

$$\sum_{\boldsymbol{x}_{-n}} \alpha\big((0, \boldsymbol{x}_{-n}), \ell\big) \;=\; \alpha_0(\ell) \;+\; \alpha_{\text{nz}}^{(-n)}(\ell).$$

Equality of the left- and right-hand sides proves (10b) for $x_n = 0$. Since both cases hold and $y \geq 0$, the RMP with column set C_0 is feasible. □

EC.10. Proof of Proposition 4

Proof. The upper bound is obtained directly from the definition of $H_i^D(\boldsymbol{\alpha})$. On the other hand, for the lower bound, Lemma 1 provides an upper bound of $1/(1-\lambda)$ for the sum of the optimal values of the variables in the master problem. Consequently, according to the result in Lübbecke and Desrosiers (2005), the lower bound is given by $H_i^C(\boldsymbol{\alpha}) + \frac{R_i^*(\boldsymbol{\alpha})}{1-\lambda}$. □

EC.11. Calculating Bounds via CG and Simulation

We utilize the CG algorithm to compute the lower bound (LB). We set $\epsilon = 0.01$, and the CG algorithm terminates at iteration i_ϵ when

$$\frac{|R_{i_\epsilon}^*(\boldsymbol{\alpha})|}{1-\lambda} = \frac{-R_{i_\epsilon}^*(\boldsymbol{\alpha})}{1-\lambda} < \epsilon = 0.01,$$

i.e., when $R_{i_\epsilon}^*(\boldsymbol{\alpha}) > -0.0005$. This termination criterion ensures that the bounds on $\tilde{V}(\mathbf{0}, 1)$ are sufficiently close to each other. Utilizing the value obtained for $\tilde{V}_{i_\epsilon}(\boldsymbol{x}, \ell)$ via the CG algorithm and employing (20), we compute policy $\tilde{\pi}$.

To obtain the upper bound (UB), we simulate the respective costs of policy $\tilde{\pi}$ using a large number of sample paths, and take their average to be the upper bound. For each sample path, the simulation run length, which is the number of decision epochs, is denoted by T_s ($T_s \in \mathbb{N}$). Since the expected one-step costs are bounded, and the cost function is discounted, the simulation run length can be determined *a priori* to ensure that the total discounted cost is accurate to a fixed constant. The simulation run length T_s is chosen such that

$$T \geq [\ln((1-\lambda)\epsilon_s/C)/\ln(\lambda)] - 1,$$

where C is any valid upper bound on the expected one-step costs and λ is the discount factor. For all numerical examples, T_s is chosen to correspond to $\epsilon_s = 0.01$, $\lambda = 0.95$, and

$$C = c_s + N\left[\max_{n\in\mathcal{N}}\{c_r^n\} + \max_{n\in\mathcal{N}}\{c_d^n(L)\}\right].$$

For example, in the case of $N = 20$ homogeneous turbines, we chose $T_s = 292$. To compute the average simulated total discounted cost under policy $\tilde{\pi}$, we selected the number of replications (R_s) to achieve a desired margin of error, utilizing the well-known relationship

$$R_s = \left(z_{\alpha/2}\,\hat{\sigma}/\Delta\right)^2, \tag{EC.9}$$

where $z_{\alpha/2}$ is the critical z-value above which lies an area of $\alpha/2$ under the standard normal density function, $\hat{\sigma}$ is the estimated standard deviation, and Δ is the desired margin of error. For each of the following models, we chose a 95% confidence level ($\alpha = 0.05$, $z_{\alpha/2} = 1.96$) and $\Delta = 1$. For example, in the case of $N = 20$ homogeneous turbines, we chose $R_s = 3,898,823$.

EC.12. Figures

Figure EC.1 Single period degradation of a turbine with no interventions.

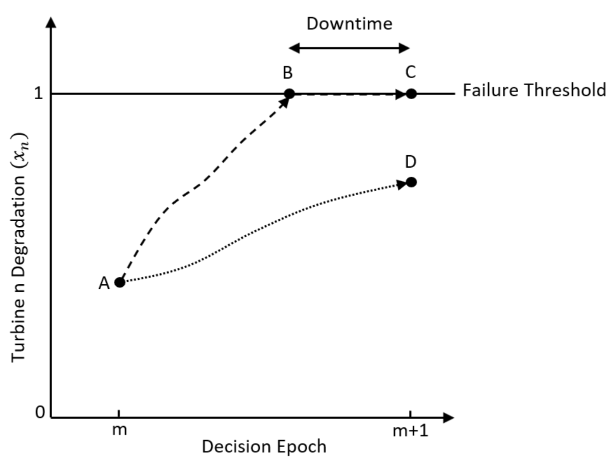


Figure EC.2 Comparison of optimal and approximate policies for different environment states.

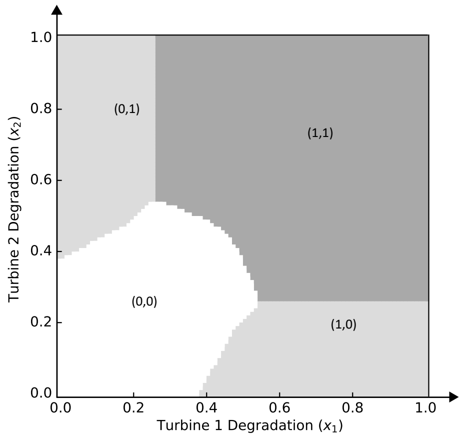
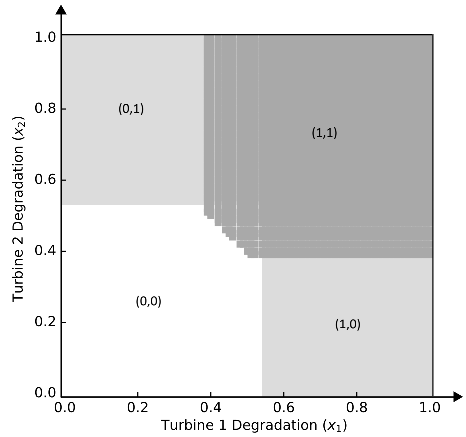
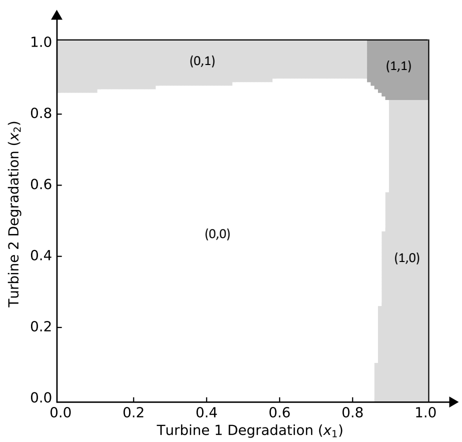
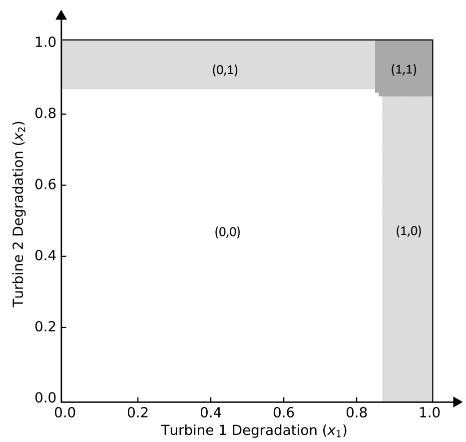
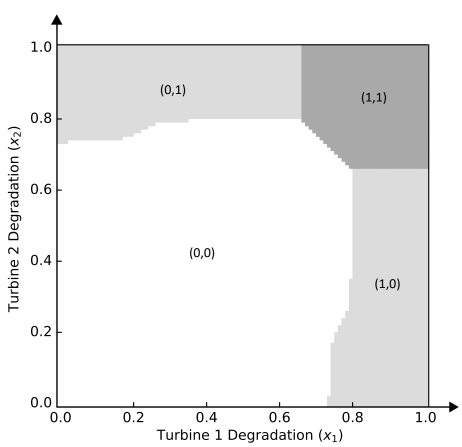
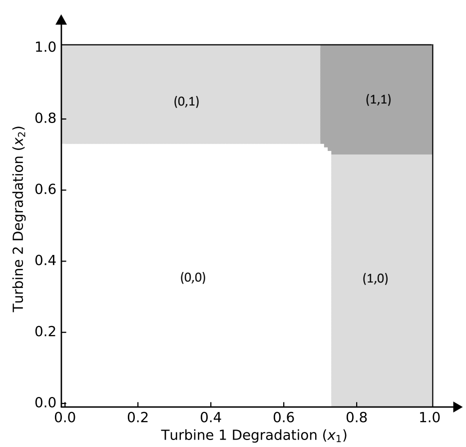
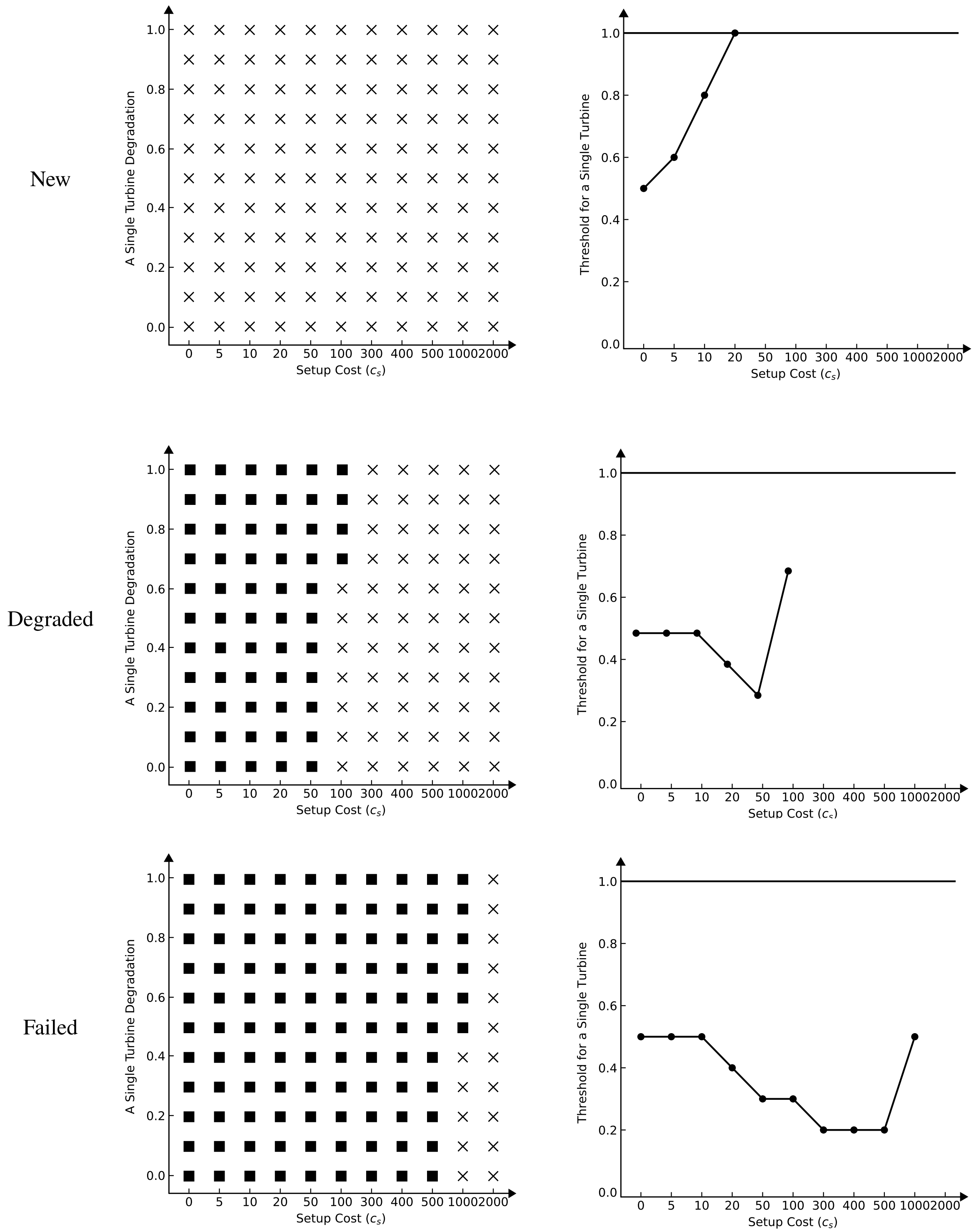
Environment (ℓ)	Exact Policy	Approximate Policy
$\ell = 1$ (Mild)		
$\ell = 3$ (Moderate)		
$\ell = 6$ (Severe)		

Figure EC.3 **Sensitivity of the replacement threshold to c_s for a given turbine when other turbine degradation states are new (state 0), degraded (state 0.5) and failed (state 1).**
(■ Replace other turbines ×Do nothing to other turbines)



EC.13. Parameter Values for Heterogeneous Experiments

Table EC.1 Summary of scenarios for parameter randomization.

Parameter	Possible value or vector
c_r	$\{2, 3, \ldots, 15\}$
$\boldsymbol{c}_d = (c_d(1), \ldots, c_d(6))$	$(10, 27, 41, 55, 68, 80)$
	$(8, 22, 39, 53, 66, 78)$
	$(13, 57, 63, 65, 67, 74)$
	$(9, 23, 38, 54, 71, 90)$
	$(9, 21, 35, 50, 68, 88)$
	$(10, 24, 37, 53, 70, 92)$
	$(10, 30, 48, 60, 70, 75)$
	$(14, 35, 55, 70, 83, 92)$
	$(20, 49, 72, 86, 95, 104)$
	$(9, 22, 36, 52, 69, 88)$
	$(11, 25, 40, 56, 73, 92)$
	$(8, 20, 33, 48, 65, 84)$
	$(10, 23, 38, 55, 72, 90)$
$\boldsymbol{r} = (r(1), \ldots, r(6))$	$(0.05, 0.12, 0.19, 0.27, 0.36, 0.48)$
	$(0.07, 0.13, 0.21, 0.29, 0.38, 0.50)$
	$(0.12, 0.31, 0.43, 0.52, 0.58, 0.64)$
	$(0.08, 0.15, 0.23, 0.31, 0.44, 0.59)$
	$(0.05, 0.11, 0.18, 0.27, 0.38, 0.52)$
	$(0.06, 0.13, 0.21, 0.30, 0.42, 0.58)$
	$(0.06, 0.15, 0.21, 0.30, 0.42, 0.50)$
	$(0.08, 0.22, 0.33, 0.41, 0.47, 0.56)$
	$(0.13, 0.28, 0.41, 0.49, 0.58, 0.66)$
	$(0.06, 0.12, 0.19, 0.28, 0.40, 0.55)$
	$(0.04, 0.10, 0.17, 0.25, 0.36, 0.50)$
	$(0.05, 0.11, 0.18, 0.27, 0.39, 0.54)$
	$(0.07, 0.13, 0.20, 0.29, 0.42, 0.57)$

Table EC.2 Transition probability matrices for parameter randomization.

$$\mathbf{P}_1 = \begin{bmatrix} 0.40 & 0.25 & 0.15 & 0.10 & 0.07 & 0.03 \\ 0.25 & 0.30 & 0.18 & 0.12 & 0.10 & 0.05 \\ 0.15 & 0.25 & 0.25 & 0.15 & 0.13 & 0.07 \\ 0.10 & 0.15 & 0.25 & 0.25 & 0.17 & 0.08 \\ 0.07 & 0.10 & 0.18 & 0.25 & 0.25 & 0.15 \\ 0.03 & 0.07 & 0.12 & 0.18 & 0.25 & 0.35 \end{bmatrix} \quad \mathbf{P}_2 = \begin{bmatrix} 0.45 & 0.22 & 0.13 & 0.10 & 0.07 & 0.03 \\ 0.30 & 0.28 & 0.16 & 0.12 & 0.09 & 0.05 \\ 0.20 & 0.25 & 0.22 & 0.14 & 0.12 & 0.07 \\ 0.12 & 0.18 & 0.24 & 0.22 & 0.16 & 0.08 \\ 0.08 & 0.12 & 0.18 & 0.24 & 0.25 & 0.13 \\ 0.04 & 0.07 & 0.12 & 0.18 & 0.24 & 0.35 \end{bmatrix}$$

$$\mathbf{P}_3 = \begin{bmatrix} 0.38 & 0.26 & 0.16 & 0.10 & 0.07 & 0.03 \\ 0.24 & 0.30 & 0.20 & 0.12 & 0.09 & 0.05 \\ 0.16 & 0.24 & 0.26 & 0.16 & 0.12 & 0.06 \\ 0.10 & 0.16 & 0.24 & 0.26 & 0.16 & 0.08 \\ 0.07 & 0.10 & 0.16 & 0.24 & 0.26 & 0.17 \\ 0.05 & 0.07 & 0.12 & 0.18 & 0.24 & 0.34 \end{bmatrix} \quad \mathbf{P}_4 = \begin{bmatrix} 0.50 & 0.20 & 0.12 & 0.08 & 0.06 & 0.04 \\ 0.32 & 0.28 & 0.16 & 0.10 & 0.08 & 0.06 \\ 0.22 & 0.24 & 0.22 & 0.12 & 0.12 & 0.08 \\ 0.14 & 0.18 & 0.22 & 0.20 & 0.16 & 0.10 \\ 0.09 & 0.12 & 0.18 & 0.22 & 0.22 & 0.17 \\ 0.05 & 0.08 & 0.12 & 0.18 & 0.22 & 0.35 \end{bmatrix}$$

$$\mathbf{P}_5 = \begin{bmatrix} 0.40 & 0.22 & 0.14 & 0.10 & 0.08 & 0.06 \\ 0.32 & 0.20 & 0.14 & 0.10 & 0.12 & 0.12 \\ 0.25 & 0.18 & 0.14 & 0.11 & 0.14 & 0.18 \\ 0.19 & 0.16 & 0.14 & 0.11 & 0.16 & 0.24 \\ 0.15 & 0.14 & 0.14 & 0.11 & 0.18 & 0.28 \\ 0.12 & 0.12 & 0.14 & 0.10 & 0.20 & 0.32 \end{bmatrix} \quad \mathbf{P}_6 = \begin{bmatrix} 0.35 & 0.30 & 0.15 & 0.12 & 0.07 & 0.01 \\ 0.18 & 0.35 & 0.19 & 0.15 & 0.10 & 0.03 \\ 0.10 & 0.20 & 0.30 & 0.20 & 0.15 & 0.05 \\ 0.08 & 0.10 & 0.20 & 0.35 & 0.20 & 0.07 \\ 0.07 & 0.08 & 0.15 & 0.25 & 0.30 & 0.15 \\ 0.05 & 0.05 & 0.10 & 0.20 & 0.35 & 0.25 \end{bmatrix}$$

$$\mathbf{P}_7 = \begin{bmatrix} 0.38 & 0.22 & 0.14 & 0.10 & 0.09 & 0.07 \\ 0.32 & 0.20 & 0.14 & 0.10 & 0.11 & 0.13 \\ 0.26 & 0.18 & 0.14 & 0.10 & 0.13 & 0.19 \\ 0.20 & 0.16 & 0.14 & 0.10 & 0.15 & 0.25 \\ 0.16 & 0.14 & 0.14 & 0.10 & 0.17 & 0.29 \\ 0.12 & 0.12 & 0.14 & 0.10 & 0.19 & 0.33 \end{bmatrix}.$$