E-companion of "Is information transparency a blessing or a curse under trade credit in competing supply chains?"

In this E-companion, we present the technical proofs in Appendix A and the numerical results for the extensions in Appendix B. For convenience, we initially summarize the key mathematical notations used in the proofs in Table EC.1.

Table EC.1 Key notations and mathematical expressions

Notation	Mathematical expression
$\bar{a}_1$	$\frac{1+2\rho}{1+6\rho}$
$\bar{a}_2$	$\frac{1}{2}$
a_1^t	$\frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}}$
a_2^t	$\frac{1}{4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho}}$
a_1^n	$\frac{3}{12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho}}$
a_2^n	$\frac{3}{10+15\sqrt{1-\rho}+2\rho-3\rho\sqrt{1-\rho}}$
a_3^n	$\frac{1}{4(1+\sqrt{1-\rho})}$
a_1	$\frac{3(1-\rho)}{12-6\rho+4\rho^2}$
a_2	$\frac{5(5-\rho)(2\rho+3)\sqrt{\rho(\rho(\rho(\rho(16\rho(\rho+12)-1349)+2141)-2082)+1244)-288}-675)}{-\rho(\rho(2\rho(2\rho(64\rho^2-962\rho+2163)-11327)+18209)-17040)+9225} + \frac{3(3\rho(\rho(\rho(2\rho(4\rho-5)+731)-1144)+640)}{-\rho(\rho(2\rho(2\rho(64\rho^2-962\rho+2163)-11327)+18209)-17040)+9225}$
a_3	$\frac{12\rho^3-81\rho^2-174\rho+243}{64\rho^4+736\rho^3+1428\rho^2-120\rho-108}$

Appendix A: Technical Proofs

We employ the backward induction approach to derive the equilibrium results under the transparency and non-transparency regimes when both retailers are well-capitalized.

(i) Under the transparency regime, if the retailers observe the high demand signal, retailer R_i's profit function is given by $\pi^{ct}_{R_i|S_h} = (\lambda_h - q^{ct}_{ih} - q^{ct}_{jh} - w^{ct}_{ih})q^{ct}_{ih}$. Taking the first-order condition of $\pi^{ct}_{R_i|S_h}$ regarding q^{ct}_{ih}, we obtain retailer R_i's best response function: $q^{ct}_{ih} = \frac{\lambda_h - q^{ct}_{jh} - w^{ct}_{ih}}{2}$. Substituting q^{ct}_{ih} into manufacturer M_i's profit function, we have $\pi^{ct}_{M_i|S_h} = w^{ct}_{ih}(\frac{\lambda_h - q^{ct}_{jh} - w^{ct}_{ih}}{2})$. Solving the manufacturer's profit maximization problem, we deduce that $w^{ct}_{ih} = \frac{\lambda_h - q^{ct}_{jh}}{2}$. Substituting back, we have $q^{ct}_{ih} = \frac{\lambda_h - q^{ct}_{jh}}{4}$. By simultaneously solving $q^{ct}_{ih} = \frac{\lambda_h - q^{ct}_{jh}}{4}$ and $q^{ct}_{jh} = \frac{\lambda_h - q^{ct}_{ih}}{4}$, we obtain that $q^{ct}_{ih} = \frac{\lambda_h}{5}$. Then, the remaining equilibrium results can be derived: $w^{ct}_{ih} = \frac{2\lambda_h}{5}$, $\pi^{ct}_{R_i|S_h} = \frac{\lambda_h^2}{25}$, $\pi^{ct}_{M_i|S_h} = \frac{2\lambda_h^2}{25}$. Similarly, if the retailers observe the low demand signal, the equilibrium results can be easily obtained: $w_{il}^{ct} = \frac{2\lambda_l}{5}$, $q_{il}^{ct} = \frac{\lambda_l}{5}$, $\pi_{R_i|S_l}^{ct} = \frac{\lambda_l^2}{25}$, $\pi_{M_i|S_l}^{ct} = \frac{2\lambda_l^2}{25}$. Thus, the expected profits of manufacturer M_i and the retailer R_i are given by $\pi_{R_i}^{ct} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{ct} = \frac{\lambda_h^2+\lambda_l^2}{25}$.

(ii) Under the non-transparency regime, we first obtain retailer R_i's response functions under the high and low demand signals: $q_{ih}^{cn} = \frac{\lambda_h - q_{jh}^{cn} - w_i^{cn}}{2}$ and $q_{il}^{cn} = \frac{\lambda_l - q_{jl}^{cn} - w_i^{cn}}{2}$. Substituting these expressions into manufacturer M_i's profit function, we have $\pi_{M_i}^{cn} = \frac{w_i^{cn}(\frac{\lambda_h - q_{jh}^{cn} - w_i^{cn}}{2} + \frac{\lambda_l - q_{jl}^{cn} - w_i^{cn}}{2})}{2}$. Maximizing this profit with respect to w_i^{cn} yields $w_i^{cn} = \frac{2 - q_{jh}^{cn} - q_{jl}^{cn}}{4}$. Substituting back, we have $q_{ih}^{cn} = \frac{2+4a(2\rho-1)-3q_{jh}^{cn}+q_{jl}^{cn}}{8}$ and $q_{il}^{cn} = \frac{2+4a(1-2\rho)-3q_{jl}^{cn}+q_{jh}^{cn}}{8}$. By simultaneously solving $q_{ih}^{cn} = \frac{2+4a(2\rho-1)-3q_{jh}^{cn}+q_{jl}^{cn}}{8}$, $q_{il}^{cn} = \frac{2+4a(1-2\rho)-3q_{jl}^{cn}+q_{jh}^{cn}}{8}$,$q_{jh}^{cn} = \frac{2+4a(2\rho-1)-3q_{ih}^{cn}+q_{il}^{cn}}{8}$, and $q_{jl}^{cn} = \frac{2+4a(1-2\rho)-3q_{il}^{cn}+q_{ih}^{cn}}{8}$, we derive that $q_{ih}^{cn} = \frac{3+5a(2\rho-1)}{15}$ and $q_{il}^{cn} = \frac{3+5a(1-2\rho)}{15}$. Then, the remaining equilibrium results can be deduced: $w_i^{cn} = \frac{2}{5}$, $\pi_{R_i|S_h}^{cn} = \frac{(3+5a(2\rho-1))^2}{225}$, $\pi_{R_i|S_l}^{cn} = \frac{(3+5a(1-2\rho))^2}{225}$, $\pi_{R_i}^{cn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, and $\pi_{M_i}^{cn} = \frac{2}{25}$.

To ensure that the equilibrium results under both information regimes are positive, we assume that $0 < a < \frac{9}{5(1+4\rho)}$.

Proof of Proposition 1. By comparing the firms' expected profits between the transparency and non-transparency regimes, we have: $\pi_{R_i}^{ct} - \pi_{R_i}^{cn} = -\frac{16a^2(2\rho-1)^2}{225} < 0$, and $\pi_{M_i}^{ct} - \pi_{M_i}^{cn} = \frac{2a^2(2\rho-1)^2}{25} > 0$.

Proof of Lemma 1. We employ backward induction to obtain the equilibrium outcomes under the transparency regime when both retailers are financially distressed. Since the manufacturers can observe the demand signal information from the retailers, the equilibrium results are divided into the following two cases:

(1) We begin by examining the case where the retailers observe the high demand signal. The optimal order quantities and retailers' profits are given by $q_{ih}^{dt} = \begin{cases} \frac{\lambda_h - q_{jh}^{dt} - w_{ih}^{dt}}{2}, & w_{ih}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}, \\ \frac{1+a-q_{jh}^{dt} - w_{ih}^{dt}}{2}, & w_{ih}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}. \end{cases}$, and $\pi_{R_i|S_h}^{dt} = \begin{cases} \frac{(\lambda_h - q_{jh}^{dt} - w_{ih}^{dt})^2}{4}, & w_{ih}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}, \\ \frac{\rho(1+a-q_{jh}^{dt} - w_{ih}^{dt})^2}{4}, & w_{ih}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}. \end{cases}$.

Based on the equilibrium order quantities of the two retailers, we consider the following four scenarios:

Scenario (1.1): If $w_{ih}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}$, this scenario implies that both retailers operate normally under the low realized demand state. Thus, the manufacturers can recover the full payment from the retailers: $\pi_{M_i|S_h}^{dt} = w_{ih}^{dt} q_{ih}^{dt}$. Solving the manufacturers' problems, we obtain that $w_{ih}^{dt} =$

$\frac{\lambda_h - q_{jh}^{dt}}{2}$. With this value substituted into q_{ih}^{dt}, we derive that $q_{ih}^{dt} = \frac{\lambda_h - q_{jh}^{dt}}{4}$. Similarly, we yield that $q_{jh}^{dt} = \frac{\lambda_h - q_{ih}^{dt}}{4}$. Solving the two equations together, we have $q_{ih}^{dt} = q_{jh}^{dt} = \frac{\lambda_h}{5}$. Next, the remaining optimal results can be derived: $w_{ih}^{dt} = \frac{2\lambda_h}{5}$, $\pi_{R_i|S_h}^{dt} = \frac{\lambda_h^2}{25}$, $\pi_{M_i|S_h}^{dt} = \frac{2\lambda_h^2}{25}$. Substituting these values into $w_{ih}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}$, we yield that $0 < a < \frac{1}{1+5\sqrt{\rho}+3\rho}$.

Scenario (1.2): If $w_{1h}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dt}$ and $w_{2h}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dt}$, this scenario indicates that retailer 1 operates normally but retailer 2 files for bankruptcy under the low realized demand state. Thus, manufacturer 2 receives a portion of repayment from retailer 2: $\pi_{M_1|S_h}^{dt} = w_{1h}^{dt} q_{1h}^{dt}, \pi_{M_2|S_h}^{dt} = \rho w_{2h}^{dt} q_{2h}^{dt} + (1-\rho)(1 - a - q_{2h}^{dt} - q_{1h}^{dt}) q_{2h}^{dt}$. Solving the two manufacturers' problems, we yield that $w_{1h}^{dt} = \frac{\lambda_h - q_{2h}^{dt}}{2}$ and $w_{2h}^{dt} = \frac{2a+\rho-a\rho-\rho q_{1h}^{dt}}{1+\rho}$. With these values substituted into q_{1h}^{dt} and q_{2h}^{dt}, respectively, we have $q_{1h}^{dt} = \frac{\lambda_h - q_{2h}^{dt}}{4}$ and $q_{2h}^{dt} = \frac{\lambda_h - q_{1h}^{dt}}{2(1+\rho)}$. Solving the two equations simultaneously, we deduce that $q_{1h}^{dt} = \frac{1-a+2\rho+4a\rho^2}{7+8\rho}$ and $q_{2h}^{dt} = \frac{3\lambda_h}{7+8\rho}$. Then, it can be derived that $w_{1h}^{dt} = \frac{2(1-a+2\rho+4a\rho^2)}{7+8\rho}$ and $w_{2h}^{dt} = \frac{6\rho - 2a(-7+2\rho+2\rho^2)}{7+8\rho}$. Substituting these values into $w_{1h}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dt}$ and $w_{2h}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dt}$, we find that no feasible range exists. This result indicates that this scenario is not viable.

Scenario (1.3): If $w_{1h}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dt}$ and $w_{2h}^{dt} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dt}$, this scenario indicates that retailer 2 operates normally but retailer 1 goes bankrupt under the low realized demand state. Apparently, Scenario (1.2) and Scenario (1.3) describe the identical situation in which only one of the two retailers goes bankrupt under the low realized demand state. Hence, this scenario is also infeasible.

Scenario (1.4): If $w_{ih}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}$, this scenario suggests that both retailers go bankrupt under the low realized demand state. Thus, the manufacturers only receive a portion of repayment from the retailers:

$$\pi_{M_i|S_h}^{dt} = \rho w_{ih}^{dt} q_{ih}^{dt} + (1-\rho)(1 - a - q_{ih}^{dt} - q_{jh}^{dt}) q_{ih}^{dt}.$$

Solving the manufacturers' problems, we deduce that $w_{ih}^{dt} = \frac{2a+\rho-a\rho-\rho q_{jh}^{dt}}{1+\rho}$. With this value substituted into q_{ih}^{dt}, we yield that $q_{ih}^{dt} = \frac{\lambda_h - q_{jh}^{dt}}{2(1+\rho)}$. Similarly, we yield that $q_{jh}^{dt} = \frac{\lambda_h - q_{ih}^{dt}}{2(1+\rho)}$. Solving the two equations together, we have $q_{ih}^{dt} = q_{jh}^{dt} = \frac{\lambda_h}{3+2\rho}$. Then, the remaining optimal results can be derived: $w_{ih}^{dt} = \frac{2(3+2\rho-2a\rho)}{3+2\rho}$, $\pi_{R_i|S_h}^{dt} = \frac{\rho\lambda_h^2}{(3+2\rho)^2}$, $\pi_{M_i|S_h}^{dt} = \frac{(1+\rho)\lambda_h^2}{(3+2\rho)^2}$. Substituting these values into $w_{ih}^{dt} \geq 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dt}$, we yield that $a \geq \frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}}$. In addition, to guarantee that the retail prices under the low realized demand state are positive, we substitute the values of q_{ih}^{dt} and q_{jh}^{dt} into $1 - a - q_{ih}^{dt} - q_{jh}^{dt} > 0$. It follows that $0 < a < \frac{1+2\rho}{1+6\rho}$. Hence, we yield $\frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}} \leq a < \frac{1+2\rho}{1+6\rho}$ to ensure the feasibility of this scenario.

Organizing the above scenarios and noticing that $\frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}} < \frac{1}{1+5\sqrt{\rho}+3\rho} < \frac{1+2\rho}{1+6\rho}$, we can summarize the equilibrium outcomes under the high demand signal according to the profit maximization of the manufacturer:

(a) When $0 < a < \frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}}$, $w_{ih}^{dt} = \frac{2\lambda_h}{5}$, $q_{ih}^{dt} = \frac{\lambda_h}{5}$, $\pi_{R_i|S_h}^{dt} = \frac{\lambda_h^2}{25}$, $\pi_{M_i|S_h}^{dt} = \frac{2\lambda_h^2}{25}$.

(b) When $\frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}} \leq a < \frac{1+2\rho}{1+6\rho}$, $w_{ih}^{dt} = \frac{2(3+2\rho-2a\rho)}{3+2\rho}$, $q_{ih}^{dt} = \frac{\lambda_h}{3+2\rho}$, $\pi_{R_i|S_h}^{dt} = \frac{\rho\lambda_h^2}{(3+2\rho)^2}$, $\pi_{M_i|S_h}^{dt} = \frac{(1+\rho)\lambda_h^2}{(3+2\rho)^2}$.

(2) Subsequently, we discuss the case where the retailers observe the low demand signal. Similar to the proof under the high demand signal, we can deduce the equilibrium results under the low demand signal as follows:

(a) When $0 < a < \frac{1}{4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho}}$, $w_{il}^{dt} = \frac{2\lambda_l}{5}$, $q_{il}^{dt} = \frac{\lambda_l}{5}$, $\pi_{R_i|S_l}^{dt} = \frac{\lambda_l^2}{25}$, $\pi_{M_i|S_l}^{dt} = \frac{2\lambda_l^2}{25}$.

(b) When $\frac{1}{4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho}} \leq a < \frac{3-2\rho}{7-6\rho}$, $w_{il}^{dt} = \frac{2(1+a-\rho+2a\rho)}{5-2\rho}$, $q_{il}^{dt} = \frac{\lambda_l}{5-2\rho}$, $\pi_{R_i|S_l}^{dt} = \frac{(1-\rho)\lambda_l^2}{(5-2\rho)^2}$, $\pi_{M_i|S_l}^{dt} = \frac{(2-\rho)\lambda_l^2}{(5-2\rho)^2}$.

Here, to ensure that the retail prices under the low realized demand state are positive, $a < \frac{3-2\rho}{7-6\rho}$ must hold. Given that $\frac{1+2\rho}{1+6\rho} < \frac{3-2\rho}{7-6\rho}$, we assume $a < \bar{a}_1 \equiv \frac{1+2\rho}{1+6\rho}$ to ensure the positivity of retail prices under both demand signals.

Combining the equilibrium results under the high and low demand signals and noting that $\frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}} < \frac{1}{4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho}}$, we conclude the optimal results under the transparency regime in the competing supply chains as follows:

(i) When $0 < a < a_1^t$, $w_{ih}^{dt} = \frac{2\lambda_h}{5}$, $w_{il}^{dt} = \frac{2\lambda_l}{5}$, $q_{ih}^{dt} = \frac{\lambda_h}{5}$, $q_{il}^{dt} = \frac{\lambda_l}{5}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$, $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$;

(ii) When $a_1^t \leq a < a_2^t$, $w_{ih}^{dt} = \frac{2(3a+\rho-2a\rho)}{3+2\rho}$, $w_{il}^{dt} = \frac{2\lambda_l}{5}$, $q_{ih}^{dt} = \frac{\lambda_h}{3+2\rho}$, $q_{il}^{dt} = \frac{\lambda_l}{5}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$, $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$;

(iii) When $a_2^t \leq a < \bar{a}_1$, $w_{ih}^{dt} = \frac{2(3a+\rho-2a\rho)}{3+2\rho}$, $w_{il}^{dt} = \frac{2(1+a-\rho+2a\rho)}{5-2\rho}$, $q_{ih}^{dt} = \frac{\lambda_h}{3+2\rho}$, $q_{il}^{dt} = \frac{\lambda_l}{5-2\rho}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2}$, $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2}$.

Here, we set $a_1^t \equiv \frac{1}{4+3\sqrt{\rho}+2\rho\sqrt{\rho}}$ and $a_2^t \equiv \frac{1}{4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho}}$ for the coherence of the main text.

Proof of Corollary 1. Based on the bankruptcy thresholds obtained in Lemma 1, we deduce their derivatives regarding signal accuracy ρ as follows: $\frac{\mathrm{d}a_1^t}{\mathrm{d}\rho} = -\frac{3(2\rho+1)}{2\sqrt{\rho}(4+3\sqrt{\rho}+2\rho\sqrt{\rho})^2} < 0$ and $\frac{\mathrm{d}a_2^t}{\mathrm{d}\rho} = \frac{9-6\rho}{2\sqrt{1-\rho}(4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho})^2} > 0$.

Proof of Lemma 2. We adopt backward induction to deduce the equilibrium outcomes under the non-transparency regime in the competing supply chains. We first obtain the retailers' response functions under both demand signals: $q_{ih}^{dn} = \begin{cases} \frac{\lambda_h - q_{jh}^{dn} - w_i^{dn}}{2}, & w_i^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dn}, \\ \frac{1+a-q_{jh}^{dn}-w_i^{dn}}{2}, & w_i^{dn} \geq 1 - a(1+2\sqrt{\rho}) - q_{jh}^{dn}. \end{cases}$, and $q_{il}^{dn} = \begin{cases} \frac{\lambda_l - q_{jl}^{dn} - w_i^{dn}}{2}, & w_i^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{jl}^{dn}, \\ \frac{1+a-q_{jl}^{dn}-w_i^{dn}}{2}, & w_i^{dn} \geq 1 - a(1+2\sqrt{\rho}) - q_{jl}^{dn}. \end{cases}$

Based on the retailers' equilibrium order quantities, we discuss the following scenarios:

Scenario (1): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, this scenario indicates that the two retailers operate normally under both demand signals. The manufacturers' profits are given: $\pi_{M_1}^{dn} = \frac{w_1^{dn} q_{1h}^{dn} + w_1^{dn} q_{1l}^{dn}}{2}$, $\pi_{M_2}^{dn} = \frac{w_2^{dn} q_{2h}^{dn} + w_2^{dn} q_{2l}^{dn}}{2}$. Solving the manufacturers' problems, we yield that $w_1^{dn} = \frac{2 - q_{2h}^{dn} - q_{2l}^{dn}}{4}$ and $w_2^{dn} = \frac{2 - q_{1h}^{dn} - q_{1l}^{dn}}{4}$. With these values substituted into $q_{1h}^{dn}, q_{1l}^{dn}, q_{2h}^{dn}$ and q_{2l}^{dn}, respectively, we obtain that $q_{1h}^{dn} = \frac{2+4a(2\rho-1)-3q_{2h}^{dn}+q_{2l}^{dn}}{8}$, $q_{1l}^{dn} = \frac{2+4a(1-2\rho)-3q_{2l}^{dn}+q_{2h}^{dn}}{8}$, $q_{2h}^{dn} = \frac{2+4a(2\rho-1)-3q_{1h}^{dn}+q_{1l}^{dn}}{8}$ and $q_{2l}^{dn} = \frac{2+4a(1-2\rho)-3q_{1l}^{dn}+q_{1h}^{dn}}{8}$. Solving the four equations simultaneously, we deduce that $q_{1h}^{dn} = q_{2h}^{dn} = \frac{3+5a(2\rho-1)}{15}$ and $q_{1l}^{dn} = q_{2l}^{dn} = \frac{3+5a(1-2\rho)}{15}$. Then, the remaining optimal results can be derived: $w_i^{dn} = \frac{2}{5}$, $\pi_{R_i|S_h}^{dn} = \frac{(3+5a(2\rho-1))^2}{225}$, $\pi_{R_i|S_l}^{dn} = \frac{(3+5a(1-2\rho))^2}{225}$, $\pi_{R_i}^{dn} = \frac{1}{25} + \frac{a^2(1-2\rho)^2}{9}$, $\pi_{M_i}^{dn} = \frac{2}{25}$. Substituting these values into $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, we get $0 < a < \frac{3}{5(1+3\sqrt{\rho}+\rho)}$.

Scenario (2): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} \geq 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, suggesting that this scenario does not exist.

Scenario (3): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} \geq 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, implying that this scenario does not exist.

Scenario (4): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} \geq 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} \geq 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (5): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} \geq 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} < 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (6): When $w_1^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{2h}^{dn}$, $w_1^{dn} \geq 1 - a(1+2\sqrt{1-\rho}) - q_{2l}^{dn}$, $w_2^{dn} < 1 - a(1+2\sqrt{\rho}) - q_{1h}^{dn}$ and $w_2^{dn} \geq 1 - a(1+2\sqrt{1-\rho}) - q_{1l}^{dn}$, this scenario implies that both retailers face bankruptcy risk under the low demand signal but operate normally under the high demand signal. The manufacturers' profits are given: $\pi_{M_1}^{dn} = \frac{w_1^{dn} q_{1h}^{dn} + (1-\rho) w_1^{dn} q_{1l}^{dn} + \rho(1-a-q_{1l}^{dn}-q_{2l}^{dn}) q_{1l}^{dn}}{2}$, and $\pi_{M_2}^{dn} = \frac{w_2^{dn} q_{2h}^{dn} + (1-\rho) w_2^{dn} q_{2l}^{dn} + \rho(1-a-q_{2l}^{dn}-q_{1l}^{dn}) q_{2l}^{dn}}{2}$. Similar to the proof of Scenario (1), we can yield the equilibrium results as follows: $w_i^{dn} = \frac{2+4a\rho-\rho}{5-\rho}$, $q_{ih}^{dn} = \frac{3-a(5-7\rho+2\rho^2)}{3(5-\rho)}$, $q_{il}^{dn} = \frac{3+5a(1-\rho)}{3(5-\rho)}$, $\pi_{R_i|S_h}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2}{9(5-\rho)^2}$, $\pi_{R_i|S_l}^{dn} = \frac{(1-\rho)(3+5a(1-\rho))^2}{9(5-\rho)^2}$, $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, and $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$. Substituting these values into $w_1^{dn} < 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} < 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, we obtain that this scenario exists when $\frac{1}{2} < \rho < \bar{\rho}_1 \approx 0.595$ and $\frac{3}{10+15\sqrt{1-\rho}+2\rho-3\rho\sqrt{1-\rho}} \leq a < \frac{3}{5+15\sqrt{\rho}+8\rho-3\rho\sqrt{\rho}-\rho^2}$. Besides, it can be readily derived that the retail prices are positive under this range, that is, $1-a-q_{il}^{dn}-q_{jl}^{dn} > 0$.

Scenario (7): When $w_1^{dn} < 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (8): When $w_1^{dn} < 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (9): When $w_1^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} < 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (10): When $w_1^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} < 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} \geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (11): When $w_1^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, this scenario implies that both retailers face bankruptcy risk under the high demand signal but operate normally under the low demand signal. The manufacturers' profits are given: $\pi_{M_1}^{dn} = \frac{\rho w_1^{dn} q_{1h}^{dn} + (1-\rho)(1-a-q_{1h}^{dn}-q_{2h}^{dn}) q_{1h}^{dn} + w_1^{dn} q_{1l}^{dn}}{2}$, $\pi_{M_2}^{dn} = \frac{\rho w_2^{dn} q_{2h}^{dn} + (1-\rho)(1-a-q_{2h}^{dn}-q_{1h}^{dn}) q_{2h}^{dn} + w_2^{dn} q_{2l}^{dn}}{2}$. Similar to the proof of Scenario (1), we can yield the equilibrium results as follows: $w_i^{dn} = \frac{1+\rho+4a(1-\rho)}{4+\rho}$, $q_{ih}^{dn} = \frac{3+5a\rho}{3(4+\rho)}$, $q_{il}^{dn} = \frac{3-a\rho(3+2\rho)}{3(4+\rho)}$, $\pi_{R_i|S_h}^{dn} = \frac{\rho(3+5a\rho)^2}{9(4+\rho)^2}$, $\pi_{R_i|S_l}^{dn} = \frac{(3-a\rho(3+2\rho))^2}{9(4+\rho)^2}$, $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, and $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$. Substituting these values into $w_1^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn} \geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn} < 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, we obtain that this scenario exists when $\frac{3}{12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho}} \leq a < \frac{3}{12(1+\sqrt{1-\rho})+3\rho(\sqrt{1-\rho}-2)-\rho^2}$. Besides, to ensure the positivity of retail prices, we substitute the values of q_{ih}^{dn} and q_{jh}^{dn} into $1-a-q_{ih}^{dn}-q_{jh}^{dn}>0$. It follows that $0<a<\frac{6+3\rho}{12+13\rho}$. It can be readily deduced that when $\frac{1}{2}<\rho\leq\bar{\rho}_2\approx 0.961$, $\frac{6+3\rho}{12+13\rho}>\frac{3}{12(1+\sqrt{1-\rho})+3\rho(\sqrt{1-\rho}-2)-\rho^2}$; otherwise, $\frac{6+3\rho}{12+13\rho}\leq\frac{3}{12(1+\sqrt{1-\rho})+3\rho(\sqrt{1-\rho}-2)-\rho^2}$. Therefore, we obtain that when $\frac{1}{2}<\rho\leq\bar{\rho}_2$ and $\frac{3}{12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho}}\leq a<\frac{3}{12(1+\sqrt{1-\rho})+3\rho(\sqrt{1-\rho}-2)-\rho^2}$, or when $\bar{\rho}_2<\rho\leq 1$ and $\frac{3}{12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho}}\leq a<\frac{6+3\rho}{12+13\rho}$, this scenario exists.

Scenario (12): When $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}<1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (13): When $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}<1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}<1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (14): When $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}<1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (15): When $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}<1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, similar to the proof of Scenario (1), we find that no feasible range exists, indicating that this scenario does not exist.

Scenario (16): When $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, this scenario indicates that both retailers face bankruptcy risk under both demand signals. The manufacturers' profits are given: $\pi_{M_1}^{dn}=\frac{\rho w_1^{dn}q_{1h}^{dn}+(1-\rho)(1-a-q_{1h}^{dn}-q_{2h}^{dn})q_{1h}^{dn}+(1-\rho)w_1^{dn}q_{1l}^{dn}+\rho(1-a-q_{1l}^{dn}-q_{2l}^{dn})q_{1l}^{dn}}{2}$, and $\pi_{M_2}^{dn}=\frac{\rho w_2^{dn}q_{2h}^{dn}+(1-\rho)(1-a-q_{2h}^{dn}-q_{1h}^{dn})q_{2h}^{dn}+(1-\rho)w_2^{dn}q_{2l}^{dn}+\rho(1-a-q_{2l}^{dn}-q_{1l}^{dn})q_{2l}^{dn}}{2}$. Similar to the proof of Scenario (1), we can yield the equilibrium results as follows: $w_i^{dn}=\frac{1}{4}+a$, $q_{ih}^{dn}=q_{il}^{dn}=\frac{1}{4}$, $\pi_{R_i|S_h}^{dn}=\frac{\rho}{16}$, $\pi_{R_i|S_l}^{dn}=\frac{1-\rho}{16}$, $\pi_{R_i}^{dn}=\frac{1}{32}$, and $\pi_{M_i}^{dn}=\frac{3}{32}$. Substituting these values into $w_1^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{2h}^{dn}$, $w_1^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{2l}^{dn}$, $w_2^{dn}\geq 1-a(1+2\sqrt{\rho})-q_{1h}^{dn}$ and $w_2^{dn}\geq 1-a(1+2\sqrt{1-\rho})-q_{1l}^{dn}$, we obtain that this scenario exists when $a\geq\frac{1}{4(1+\sqrt{1-\rho})}$. Besides, to ensure the positivity of retail prices, we substitute the values of q_{1h}^{dn}, q_{1l}^{dn}, q_{2h}^{dn} and q_{2l}^{dn} into $1-a-q_{1h}^{dn}-q_{2h}^{dn}>0$ and $1-a-q_{1l}^{dn}-q_{2l}^{dn}>0$. It follows that $a<\frac{1}{2}$. Thus, we yield $\frac{1}{4(1+\sqrt{1-\rho})}\leq a<\frac{1}{2}$ to guarantee the feasibility of this scenario.

In summary, we find that only Scenarios (1), (6), (11) and (16) exist under certain conditions. Next, we proceed to combine and organize these four scenarios. According to the manufacturers' profit maximization principle, we summarize the optimal results under the non-transparency regime in the competing supply chains as follows:

(i) When $\frac{1}{2} < \rho \le \frac{5}{9}$,

(a) If $0 < a < a_1^n$, $w_i^{dn} = \frac{2}{5}$, $q_{ih}^{dn} = \frac{3+5a(2\rho-1)}{15}$, $q_{il}^{dn} = \frac{3+5a(1-2\rho)}{15}$, $\pi_{R_i|S_h}^{dn} = \frac{(3+5a(2\rho-1))^2}{225}$, $\pi_{R_i|S_l}^{dn} = \frac{(3+5a(1-2\rho))^2}{225}$, $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$;

(b) If $a_1^n \le a < a_2^n$, $w_i^{dn} = \frac{1+\rho+4a(1-\rho)}{4+\rho}$, $q_{ih}^{dn} = \frac{3+5a\rho}{3(4+\rho)}$, $q_{il}^{dn} = \frac{3-a\rho(3+2\rho)}{3(4+\rho)}$, $\pi_{R_i|S_h}^{dn} = \frac{\rho(3+5a\rho)^2}{9(4+\rho)^2}$, $\pi_{R_i|S_l}^{dn} = \frac{(3-a\rho(3+2\rho))^2}{9(4+\rho)^2}$, $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$;

(c) If $a_2^n \le a < a_3^n$, $w_i^{dn} = \frac{2-\rho+4a\rho}{5-\rho}$, $q_{ih}^{dn} = \frac{3-a(5-7\rho+2\rho^2)}{3(5-\rho)}$, $q_{il}^{dn} = \frac{3+5a(1-\rho)}{3(5-\rho)}$, $\pi_{R_i|S_h}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2}{9(5-\rho)^2}$, $\pi_{R_i|S_l}^{dn} = \frac{(1-\rho)(3+5a(1-\rho))^2}{9(5-\rho)^2}$, $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$;

(d) If $a_3^n \le a < \bar{a}_2$, $w_i^{dn} = \frac{1}{4} + a$, $q_{ih}^{dn} = q_{il}^{dn} = \frac{1}{4}$, $\pi_{R_i|S_h}^{dn} = \frac{\rho}{16}$, $\pi_{R_i|S_l}^{dn} = \frac{1-\rho}{16}$, $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$.

(ii) When $\frac{5}{9} < \rho \le 1$,

(a) If $0 < a < a_1^n$, $w_i^{dn} = \frac{2}{5}$, $q_{ih}^{dn} = \frac{3+5a(2\rho-1)}{15}$, $q_{il}^{dn} = \frac{3+5a(1-2\rho)}{15}$, $\pi_{R_i|S_h}^{dn} = \frac{(3+5a(2\rho-1))^2}{225}$, $\pi_{R_i|S_l}^{dn} = \frac{(3+5a(1-2\rho))^2}{225}$, $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$;

(b) If $a_1^n \le a < a_3^n$, $w_i^{dn} = \frac{1+\rho+4a(1-\rho)}{4+\rho}$, $q_{ih}^{dn} = \frac{3+5a\rho}{3(4+\rho)}$, $q_{il}^{dn} = \frac{3-a\rho(3+2\rho)}{3(4+\rho)}$, $\pi_{R_i|S_h}^{dn} = \frac{\rho(3+5a\rho)^2}{9(4+\rho)^2}$, $\pi_{R_i|S_l}^{dn} = \frac{(3-a\rho(3+2\rho))^2}{9(4+\rho)^2}$, $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$;

(c) If $a_3^n \le a < \bar{a}_2$, $w_i^{dn} = \frac{1}{4} + a$, $q_{ih}^{dn} = q_{il}^{dn} = \frac{1}{4}$, $\pi_{R_i|S_h}^{dn} = \frac{\rho}{16}$, $\pi_{R_i|S_l}^{dn} = \frac{1-\rho}{16}$, $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$.

Here, $a_1^n \equiv \frac{3}{12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho}}$, $a_2^n \equiv \frac{3}{10+15\sqrt{1-\rho}+2\rho-3\rho\sqrt{1-\rho}}$, $a_3^n \equiv \frac{1}{4(1+\sqrt{1-\rho})}$, and $\bar{a}_2 \equiv \frac{1}{2}$.

Proof of Corollary 2. Given the bankruptcy thresholds derived in Lemma 2, we deduce their derivatives regarding signal accuracy ρ as follows: $\frac{\mathrm{d}a_1^n}{\mathrm{d}\rho} = -\frac{3(12-4\sqrt{\rho}+9\rho)}{2\sqrt{\rho}(12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho})^2} < 0$, and $\frac{\mathrm{d}a_3^n}{\mathrm{d}\rho} = \frac{1}{8\sqrt{1-\rho}(1+\sqrt{1-\rho})^2} > 0$.

Proof of Proposition 2. We compare the bankruptcy thresholds between the transparency and non-transparency regimes in the competing supply chains.

(1) By examining the difference between a_1^n and a_1^t, we have $a_1^n - a_1^t = \frac{3\rho\sqrt{\rho}+2\rho-3\sqrt{\rho}}{(4+3\sqrt{\rho}+2\rho\sqrt{\rho})(12+12\sqrt{\rho}-2\rho+3\rho\sqrt{\rho})}$. Since the denominator of $a_1^n - a_1^t$ is positive, the sign of $a_1^n - a_1^t$ depends on $3\rho\sqrt{\rho} + 2\rho - 3\sqrt{\rho}$. It can be readily derived that when $\frac{1}{2} < \rho \le \frac{11-2\sqrt{10}}{9}$, $3\rho\sqrt{\rho} + 2\rho - 3\sqrt{\rho} < 0$; whereas when $\frac{11-2\sqrt{10}}{9} < \rho \le 1$, $3\rho\sqrt{\rho} + 2\rho - 3\sqrt{\rho} \ge 0$. Thus, when $\frac{1}{2} < \rho \le \frac{11-2\sqrt{10}}{9}$, $a_1^n < a_1^t$; when $\frac{11-2\sqrt{10}}{9} < \rho \le 1$, $a_1^n \ge a_1^t$.

(2) By examining the difference between a_3^n and a_2^t, we have $a_3^n - a_2^t = -\frac{(2\rho-1)\sqrt{1-\rho}}{4(1+\sqrt{1-\rho})(4+5\sqrt{1-\rho}-2\rho\sqrt{1-\rho})} < 0$, implying that $a_3^n < a_2^t$.

Proof of Propositions 3-4. By referencing Lemmas 1-2, we compare the expected profits of the manufacturers and retailers between different information structures. Due to $\bar{a}_1 < \bar{a}_2$, we assume $0 < a < \bar{a}_1$ to ensure the positivity of retail prices under both regimes. The comparison results can be categorized into four cases depending on signal accuracy.

Case (1): When $\frac{1}{2} < \rho \leq \rho_1 \approx 0.503$, it can be readily derived that $a_1^n < a_2^n < a_3^t < a_3^n < a_2^t < \bar{a}_1$. The comparison of firms' profits can be further divided into six scenarios as follows:

(1.1) When $0 < a < a_1^n$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. By comparing the retailers' profits, we yield that $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{a^2(1-2\rho)^2}{225} > 0$, i.e., $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$. Then, we contrast the manufacturers' profits, we obtain that $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{-2a^2(1-2\rho)^2}{25} < 0$, that is, $\pi_{M_i}^{dn} < \pi_{M_i}^{dt}$.

(1.2) When $a_1^n \leq a < a_2^n$, we have $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison of the retailers' profits is obtained: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{a^2(28\rho^4+421\rho^3-369\rho^2+1008\rho-288)+450a(\rho-1)\rho-9(2\rho^2-9\rho+7)}{450(\rho+4)^2}$. It can be deduced that $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} < 0$, suggesting $\pi_{R_i}^{dn} < \pi_{R_i}^{dt}$. Moving forward, the comparison of the manufacturer's profits is derived: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{a^2(-144\rho^4-283\rho^3+487\rho^2-384\rho-576)+150a\rho(-\rho^2-2\rho+3)+9(-4\rho^2-7\rho+11)}{450(\rho+4)^2} > 0$, indicating $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$.

(1.3) When $a_3^n \leq a < a_1^t$, we have $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. We first contrast the retailers' profits: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{a^2(28\rho^4-533\rho^3+1062\rho^2-1645\rho+800)+450a(\rho-1)\rho-9\rho(2\rho+5)}{450(\rho-5)^2} < 0$, implying $\pi_{R_i}^{dn} < \pi_{R_i}^{dt}$. Then, we compare the manufacturers' profits: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{a^2(-144\rho^4+859\rho^3-1226\rho^2+835\rho-900)+150a\rho(\rho^2-5\rho+4)+9\rho(15-4\rho)}{450(\rho-5)^2} > 0$, suggesting $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$.

(1.4) When $a_1^t \leq a < a_3^n$, we have $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. By comparing the retailers' profits, we obtain that $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{S_1(a)}{450(\rho-5)^2(2\rho+3)^2}$, where $S_1(a) = a^2(\rho(\rho(2\rho(2\rho(64\rho^2-962\rho+2163)-11327)+18209)-17040)+9225)+18a(\rho-1)(\rho(\rho(8\rho^2-2\rho+729)-415)+225)-9\rho(\rho(\rho(4\rho+97)-161)+460)+2025$. Since the denominator of $\pi_{R_i}^{dn}-\pi_{R_i}^{dt}$ is positive, the sign of $\pi_{R_i}^{dn}-\pi_{R_i}^{dt}$ depends on the numerator $S_1(a)$. It can be deduced that when $\frac{1}{2} < \rho \leq \rho_2 \approx 0.501$ and $a_1^t \leq a < \min\{a_3^n, a_2\}$, we have $S_1(a) > 0$; otherwise, we have $S_1(a) \leq 0$. Here, $a_2 \equiv -\frac{3(3\rho(\rho(\rho(2\rho(4\rho-5)+731)-1144)+640)+5(5-\rho)(2\rho+3)\sqrt{\rho(\rho(\rho(\rho(16\rho(\rho+12)-1349)+2141)-2082)+1244)-288}-675)}{\rho(\rho(2\rho(2\rho(64\rho^2-962\rho+2163)-11327)+18209)-17040)+9225}$. Hence, we can summarize that when $\frac{1}{2} < \rho \leq \rho_2 \approx 0.501$ and $a_1^t \leq a < \min\{a_3^n, a_2\}$, $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$; otherwise, $\pi_{R_i}^{dn} \leq \pi_{R_i}^{dt}$. Subsequently, we contrast the manufacturers' profits as follows: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{S_2(a)}{450(\rho-5)^2(2\rho+3)^2}$, where $S_2(a) = -(a^2(\rho(\rho(2\rho(2\rho(72\rho^2+374\rho-3601)+5029)+10707)-3420)+9675))+6a(\rho-1)(2\rho(\rho(\rho(74\rho-281)+12)-120)-525)-9(\rho(\rho(\rho(8\rho+69)-347)-180)+175)$. Similarly, the sign of $\pi_{M_i}^{dn}-\pi_{M_i}^{dt}$ depends on the numerator $S_2(a)$. It can be deduced that $S_2(a) > 0$, suggesting $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$.

(1.5) When $a_3^n \le a < a_2^t$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. We begin by comparing the retailers' profits as follows: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{-16a^2(1-2\rho)^2(4\rho^2+37\rho+9)+32a(8\rho^3-30\rho^2+31\rho-9)+36\rho^2-292\rho+81}{800(2\rho+3)^2} < 0$, i.e., $\pi_{R_i}^{dn} < \pi_{R_i}^{dt}$. Next, the comparison result of the manufacturers' profits is obtained: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{-16a^2(1-2\rho)^2(8\rho^2+49\rho+43)+32a(16\rho^3-10\rho^2-13\rho+7)+172\rho^2+116\rho-13}{800(2\rho+3)^2} > 0$, suggesting $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$.

(1.6) When $a_2^t \le a < \bar{a}_1$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2}$. We first compare the retailers' profits: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{(4a-1)(1-2\rho)^2(4a(28(\rho-1)\rho-9)-4(\rho-1)\rho-81)}{32(15-4(\rho-1)\rho)^2}$. It can be deduced that when $a_2^t \le a < \frac{1}{4}$, $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} > 0$, i.e., $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$; whereas when $\frac{1}{4} \le a < \bar{a}_1$, $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} \le 0$, i.e., $\pi_{R_i}^{dn} \le \pi_{R_i}^{dt}$. Then, the manufacturers' profit comparison result is obtained: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{(4a-1)(1-2\rho)^2(4a(20(\rho-1)\rho-43)-12(\rho-1)\rho+13)}{32(15-4(\rho-1)\rho)^2}$. Similarly, we can yield that when $a_2^t \le a < \frac{1}{4}$, $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} > 0$, i.e., $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$; whereas when $\frac{1}{4} \le a < \bar{a}_1$, $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} \le 0$, i.e., $\pi_{M_i}^{dn} \le \pi_{M_i}^{dt}$.

Case (2): When $\rho_1 < \rho \le \frac{11-2\sqrt{10}}{9}$, it can be deduced that $a_1^n < a_1^t \le a_2^n < a_3^n < a_2^t < \bar{a}_1$. The comparison outcomes can be categorized into the following scenarios:

(2.1) When $0 < a < a_1^n$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison results of the retailers and the manufacturers are consistent with those in (1.1).

(2.2) When $a_1^n \le a < a_1^d$, we have $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison outcomes of the retailers and the manufacturers align with those in (1.2).

(2.3) When $a_1^t \le a < a_2^n$, we have $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. By contrasting the retailers' profits, we obtain that $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{S_3(a)}{450(\rho+4)^2(2\rho+3)^2}$, where $S_3(a) = 8a^2(32\rho^6 + 320\rho^5 + 258\rho^4 - 27\rho^3 + 2178\rho^2 - 99\rho - 162) + 18a(8\rho^5 + 134\rho^4 + 119\rho^3 - 316\rho^2 + 199\rho - 144) - 9(\rho - 1)^2(4\rho^2 - 23\rho - 81)$. Since the denominator of $\pi_{R_i}^{dn} - \pi_{R_i}^{dt}$ is positive, the sign of $\pi_{R_i}^{dn} - \pi_{R_i}^{dt}$ depends on the numerator $S_3(a)$. It can be derived that when $a_1^t \le a < \min\{a_2^n, a_1\}$, we have $S_3(a) > 0$; otherwise, we have $S_3(a) \le 0$. Here, $a_1 \equiv \frac{3(1-\rho)}{12-6\rho+4\rho^2}$. Hence, we can summarize that when $a_1^t \le a < \min\{a_2^n, a_1\}$, $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$; otherwise, $\pi_{R_i}^{dn} \le \pi_{R_i}^{dt}$. Next, we compare the manufacturers' profits as follows: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = -\frac{(2a(\rho(2\rho-3)+6)+3(\rho-1))(a(2\rho(\rho(4\rho(9\rho+41)+247)+420)+516)+3(\rho-1)(\rho(8\rho+29)+13))}{450(\rho+4)^2(2\rho+3)^2}$. Similarly, we can yield that when $a_1^t \le a < \min\{a_2^n, a_1\}$, $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} > 0$, i.e., $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$; otherwise, $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} \le 0$, that is, $\pi_{M_i}^{dn} \le \pi_{M_i}^{dt}$.

(2.4) When $a_2^n \le a < a_3^n$, we have $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. Different from the comparison results in (1.4), we find that $\pi_{R_i}^{dn} < \pi_{R_i}^{dt}$ and $\pi_{M_i}^{dn} > \pi_{M_i}^{dt}$ for $\rho_1 < \rho \le \frac{11-2\sqrt{10}}{9}$ and $a_2^n \le a < a_3^n$.

(2.5) When $a_3^n \leq a < a_2^t$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$, and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. The comparison results of the retailers and the manufacturers are consistent with those in (1.5).

(2.6) When $a_2^t \leq a < \bar{a}_1$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2}$. The comparison results of the retailers and the manufacturers are consistent with those in (1.6).

Case (3): When $\frac{11-2\sqrt{10}}{9} < \rho \leq \frac{5}{9}$, it can be deduced that $a_1^t \leq a_1^n < a_2^n < a_3^n < a_2^t < \bar{a}_1$. The comparison results can be further divided into the following six scenarios:

(3.1) When $0 < a < a_1^t$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison results of the retailers and the manufacturers are consistent with those in (1.1).

(3.2) When $a_1^t \leq a < a_1^n$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. We begin by comparing the retailers' profits as follows: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{(\rho(164\rho+267)+369)(a-2a\rho)^2+18a(\rho-1)(2\rho-1)(4\rho-9)+9(\rho-1)(4\rho-9)}{450(2\rho+3)^2} > 0$, implying $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$. Then, we contrast the manufacturers' profits: $\pi_{M_i}^{dn} - \pi_{M_i}^{dt} = \frac{-((\rho(8\rho+49)+43)(a-2a\rho)^2)+2a(\rho-1)(2\rho-1)(8\rho+7)+(\rho-1)(8\rho+7)}{50(2\rho+3)^2} < 0$, indicating $\pi_{M_i}^{dn} < \pi_{M_i}^{dt}$.

(3.3) When $a_1^n \leq a < a_2^n$, we have $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. Different from the comparison results in (2.3), we find that $\pi_{R_i}^{dn} < \pi_{R_i}^{dt}$ and $\pi_{M_i}^{dn} < \pi_{M_i}^{dt}$ for $\frac{11-2\sqrt{10}}{9} < \rho \leq \frac{5}{9}$ and $a_1^n \leq a < a_2^n$.

(3.4) When $a_2^n \leq a < a_3^n$, we have $\pi_{R_i}^{dn} = \frac{(3-a(5-7\rho+2\rho^2))^2+(1-\rho)(3+5a(1-\rho))^2}{18(5-\rho)^2}$, $\pi_{M_i}^{dn} = \frac{36+(-9+24a-125a^2)\rho+2a(77a-15)\rho^2+(6-29a)a\rho^3}{18(5-\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison results of the retailers and the manufacturers are consistent with those in (2.4).

(3.5) When $a_3^n \leq a < a_2^t$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$, and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. The comparison outcomes of the retailers and the manufacturers are consistent with those in (1.5).

(3.6) When $a_2^t \leq a < \bar{a}_1$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2}$. The comparison results of the retailers and the manufacturers align with those in (1.6).

Case (4): When $\frac{5}{9} < \rho \leq 1$, it can be obtained that $a_1^t < a_1^n < a_3^n < a_2^t < \bar{a}_1$. The comparison results can be further divided into the following five scenarios:

(4.1) When $0 < a < a_1^t$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{\lambda_h^2+\lambda_l^2}{25}$. The comparison results of the retailers and the manufacturers are consistent with those in (1.1).

(4.2) When $a_1^t \le a < a_1^n$, we have $\pi_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\pi_{M_i}^{dn} = \frac{2}{25}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. The comparison outcomes of the retailers and the manufacturers are consistent with those in (3.2).

(4.3) When $a_1^n \le a < a_3^n$, we have $\pi_{R_i}^{dn} = \frac{\rho(3+5a\rho)^2+(3-a\rho(3+2\rho))^2}{18(4+\rho)^2}$, $\pi_{M_i}^{dn} = \frac{27+\rho(9+a(\rho-1)(-6(\rho+3)+a(29\rho+96)))}{18(4+\rho)^2}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. The comparison result of the retailers' profits is obtained as follows: $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} = \frac{S_4(a)}{450(\rho+4)^2(2\rho+3)^2}$. It can be deduced that when $\rho_3 < \rho \le 1$ and $\max\{a_1^n, a_3\} \le a < a_3^n$, $S_4(a) > 0$; when $\frac{5}{9} < \rho \le \rho_4$ and $a_1^n \le a < \min\{a_3, a_3^n\}$, $S_4(a) \le 0$. Here, $\rho_3 \approx 0.65$, $\rho_4 \approx 0.671$, and $a_3 \equiv \frac{12\rho^3-81\rho^2-174\rho+243}{64\rho^4+736\rho^3+1428\rho^2-120\rho-108}$. Thus, we summarize that when $\rho_3 < \rho \le 1$ and $\max\{a_1^n, a_3\} \le a < a_3^n$, $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} > 0$, i.e., $\pi_{R_i}^{dn} > \pi_{R_i}^{dt}$; when $\frac{5}{9} < \rho \le \rho_4$ and $a_1^n \le a < \min\{a_3, a_3^n\}$, $\pi_{R_i}^{dn} - \pi_{R_i}^{dt} \le 0$, i.e., $\pi_{R_i}^{dn} \le \pi_{R_i}^{dt}$. Next, the comparison result of the manufacturers' profits is consistent with that in (3.3).

(4.4) When $a_3^n \le a < a_2^t$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{50}$, and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{\lambda_l^2}{25}$. The comparison outcomes of the retailers and the manufacturers are consistent with those in (1.5).

(4.5) When $a_2^t \le a < \bar{a}_1$, we have $\pi_{R_i}^{dn} = \frac{1}{32}$, $\pi_{M_i}^{dn} = \frac{3}{32}$, $\pi_{R_i}^{dt} = \frac{\rho\lambda_h^2}{2(3+2\rho)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2}$ and $\pi_{M_i}^{dt} = \frac{(1+\rho)\lambda_h^2}{2(3+2\rho)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2}$. The comparison outcomes of the retailers and the manufacturers align with those in (1.6).

In summary, we conclude that: (i) When $\frac{1}{2} < \rho \le \frac{11-2\sqrt{10}}{9}$ and $a_1^n \le a < \min\{a_2^n, a_1\}$, or when $\frac{1}{2} < \rho \le \frac{5}{9}$ and $a_2^n \le a < a_3^n$, or when $a_3^n \le a < \frac{1}{4}$, $\pi_{Mi}^{dn} > \pi_{Mi}^{dt}$; otherwise, $\pi_{Mi}^{dn} \le \pi_{Mi}^{dt}$. (ii) When $\frac{1}{2} < \rho \le \rho_2$ and $a_1^t \le a < \min\{a_3^n, a_2\}$, or when $\rho_1 < \rho \le \frac{11-2\sqrt{10}}{9}$ and $a_1^t \le a < \min\{a_2^n, a_1\}$, or when $\rho_3 < \rho \le 1$ and $\max\{a_1^n, a_3\} \le a < a_3^n$, or when $0 < a < a_1^n$, or when $a_2^t \le a < \frac{1}{4}$, $\pi_{Ri}^{dn} > \pi_{Ri}^{dt}$; otherwise, $\pi_{Ri}^{dn} \le \pi_{Ri}^{dt}$.

Proof of Corollary 3. We can obtain this corollary based on the proofs of Proposition 3-4.

Proof of Proposition 5. We compare the manufacturers' and retailers' expected profits in the scenarios with and without trade credit. Under the transparency regime, there are three cases: (i) When $0 < a < a_1^t$, we obtain $\pi_{R_i}^{dt} - \pi_{R_i}^{ct} = 0$ and $\pi_{M_i}^{dt} - \pi_{M_i}^{ct} = 0$; (ii) When $a_1^t \le a < a_2^t$, we have $\pi_{R_i}^{dt} - \pi_{R_i}^{ct} = -\frac{(4\rho^2-13\rho+9)(a(2\rho-1)+1)^2}{50(2\rho+3)^2} < 0$ and $\pi_{M_i}^{dt} - \pi_{M_i}^{ct} = \frac{(-8\rho^2+\rho+7)(a(2\rho-1)+1)^2}{50(2\rho+3)^2} > 0$; (iii) When $a_2^t \le a < \bar{a}_1$, we have $\pi_{R_i}^{dt} - \pi_{R_i}^{ct} = \frac{\rho\lambda_h^2}{2(2\rho+3)^2} + \frac{(1-\rho)\lambda_l^2}{2(5-2\rho)^2} - \frac{\lambda_h^2+\lambda_l^2}{50} < 0$ and $\pi_{M_i}^{dt} - \pi_{M_i}^{ct} = \frac{(\rho+1)\lambda_h^2}{2(2\rho+3)^2} + \frac{(2-\rho)\lambda_l^2}{2(5-2\rho)^2} - \frac{\lambda_h^2+\lambda_l^2}{25} > 0$. Similarly, under the non-transparency regime, it can be deduced that $\pi_{R_i}^{dn} \le \pi_{R_i}^{cn}$ and $\pi_{M_i}^{dn} \ge \pi_{M_i}^{cn}$.

Proof of Proposition 6. Similar to the proofs in Lemmas 1-2, we can obtain the equilibrium results when considering competition intensity.

Under the transparency regime, we have: (i) When $0 < a < \tilde{a}_1^t$, $\tilde{\pi}_{R_i}^{dt} = \frac{(a^2(1-2\rho)^2+1)}{(\lambda+4)^2}$, $\tilde{\pi}_{M_i}^{dt} = \frac{2(a^2(1-2\rho)^2+1)}{(\lambda+4)^2}$; (ii) When $\tilde{a}_1^t \le a < \tilde{a}_2^t$, $\tilde{\pi}_{R_i}^{dt} = \frac{(a(2\rho-1)-1)^2}{2(\lambda+4)^2} + \frac{\rho(a(2\rho-1)+1)^2}{2(\lambda+2\rho+2)^2}$, $\tilde{\pi}_{M_i}^{dt} = \frac{(a(2\rho-1)-1)^2}{(\lambda+4)^2} + \frac{(\rho+1)(a(2\rho-1)+1)^2}{2(\lambda+2\rho+2)^2}$; (iii) When $\tilde{a}_2^t \le a < \bar{a}_3$, $\tilde{\pi}_{R_i}^{dt} = \frac{\rho(a(2\rho-1)+1)^2}{2(\lambda+2\rho+2)^2} + \frac{(1-\rho)(a(2\rho-1)-1)^2}{2(\lambda-2\rho+4)^2}$, $\tilde{\pi}_{M_i}^{dt} = \frac{(\rho+1)(a(2\rho-1)+1)^2}{2(\lambda+2\rho+2)^2} + \frac{(2-\rho)(a(2\rho-1)-1)^2}{2(\lambda-2\rho+4)^2}$.

Here, we set $\tilde{a}_1^t \equiv \frac{1}{\lambda\sqrt{\rho}+\lambda+2\rho^{3/2}+2\sqrt{\rho}+3}$, $\tilde{a}_2^t \equiv \frac{1}{\lambda\sqrt{1-\rho}+\lambda-2\rho\sqrt{1-\rho}+4\sqrt{1-\rho}+3}$, and $\bar{a}_3 = \frac{2\rho+1}{2\lambda\rho+4\rho+1}$.

Under the non-transparency regime, we have:

(i) When $2(\sqrt{2}-1) < \lambda \le 1$ and $\frac{1}{2} < \rho \le \frac{4\lambda+\lambda^2}{4+4\lambda+\lambda^2}$,

(a) If $0 < a < \tilde{a}_1^n$, $\tilde{\pi}_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{9} + \frac{1}{25}$, $\tilde{\pi}_{M_i}^{dn} = \frac{2}{(\lambda+4)^2}$; (b) If $\tilde{a}_1^n \le a < \tilde{a}_2^n$, $\tilde{\pi}_{R_i}^{dn} = \frac{\rho(a(\lambda+4)\rho+\lambda+2)^2+(\lambda(a\rho-1)+2a\rho(\rho+1)-2)^2}{2(\lambda+2)^2(\lambda+\rho+3)^2}$, $\tilde{\pi}_{M_i}^{dn} = \frac{2a^2\lambda^3(\rho-1)\rho+\lambda^2(3a^2\rho^3+13a^2\rho^2-16a^2\rho+\rho+3)+2\lambda(\rho+3)(7a^2(\rho-1)\rho-a(\rho-1)\rho+2)+4(\rho+3)(3a^2(\rho-1)\rho-a(\rho-1)\rho+1)}{2(\lambda+2)^2(\lambda+\rho+3)^2}$;
(c) If $\tilde{a}_2^n \le a < \tilde{a}_3^n$, $\tilde{\pi}_{R_i}^{dn} = \frac{(a(\rho-1)(\lambda-2\rho+4)+\lambda+2)^2-(\rho-1)(-a(\lambda+4)(\rho-1)+\lambda+2)^2}{2(\lambda+2)^2(\lambda-\rho+4)^2}$, $\tilde{\pi}_{M_i}^{dn} = -\frac{-2a^2\lambda^3(\rho-1)\rho+\lambda^2\left(3a^2\rho^3-22a^2\rho^2+19a^2\rho+\rho-4\right)+2\lambda(\rho-4)\left(7a^2(\rho-1)\rho-a(\rho-1)\rho+2\right)+4(\rho-4)\left(3a^2(\rho-1)\rho-a(\rho-1)\rho+1\right)}{2(\lambda+2)^2(\lambda-\rho+4)^2}$;
(d) If $\tilde{a}_3^n \le a < \bar{a}_4$, $\tilde{\pi}_{R_i}^{dn} = \frac{1}{2(\lambda+3)^2}$, $\tilde{\pi}_{M_i}^{dn} = \frac{3}{2(\lambda+3)^2}$.

(ii) When $0 < \lambda \le 2(\sqrt{2}-1)$ and $\frac{1}{2} < \rho < \frac{4\lambda+\lambda^2}{4+4\lambda+\lambda^2}$, or when $2(\sqrt{2}-1) < \lambda \le 1$ and $\frac{4\lambda+\lambda^2}{4+4\lambda+\lambda^2} < \rho \le 1$,

(a) If $0 < a < \tilde{a}_1^n$, $\tilde{\pi}_{R_i}^{dn} = \frac{a^2(1-2\rho)^2}{(\lambda+2)^2} + \frac{1}{(\lambda+4)^2}$, $\tilde{\pi}_{M_i}^{dn} = \frac{2}{(\lambda+4)^2}$; (b) If $\tilde{a}_1^n \le a < \tilde{a}_3^n$, $\tilde{\pi}_{R_i}^{dn} = \frac{\rho(a(\lambda+4)\rho+\lambda+2)^2+(\lambda(a\rho-1)+2a\rho(\rho+1)-2)^2}{2(\lambda+2)^2(\lambda+\rho+3)^2}$, $\tilde{\pi}_{M_i}^{dn} = \frac{2a^2\lambda^3(\rho-1)\rho+\lambda^2(3a^2\rho^3+13a^2\rho^2-16a^2\rho+\rho+3)+2\lambda(\rho+3)(7a^2(\rho-1)\rho-a(\rho-1)\rho+2)+4(\rho+3)(3a^2(\rho-1)\rho-a(\rho-1)\rho+1)}{2(\lambda+2)^2(\lambda+\rho+3)^2}$; (c) If $\tilde{a}_3^n \le a < \bar{a}_4$, $\tilde{\pi}_{R_i}^{dn} = \frac{1}{2(\lambda+3)^2}$, $\tilde{\pi}_{M_i}^{dn} = \frac{3}{2(\lambda+3)^2}$.

Here, $\tilde{a}_1^n \equiv \frac{\lambda+2}{\lambda^2(\sqrt{\rho}+1)+\lambda(\rho^{3/2}+5\sqrt{\rho}+5)+2(\rho^{3/2}-\rho+3\sqrt{\rho}+3)}$, $\tilde{a}_3^n \equiv \frac{1}{(\lambda+3)\rho} - \sqrt{\frac{1-\rho}{(\lambda+3)^2\rho^2}}$, $\bar{a}_4 \equiv \frac{\lambda+1}{\lambda+3}$, and $\tilde{a}_2^n \equiv \frac{\lambda+2}{\lambda^2(\sqrt{1-\rho}+1)+\lambda(-\rho\sqrt{1-\rho}+6\sqrt{1-\rho}+5)-2\sqrt{1-\rho}-1)\rho+8\sqrt{1-\rho}+4}$.

We then explore how the competition intensity λ affects the demand uncertainty thresholds under the transparency and non-transparency regimes. Under the transparency regime, we obtain that $\frac{\partial\bar{a}_1^t}{\partial\lambda} = -\frac{\sqrt{\rho}+1}{(\lambda\sqrt{\rho}+\lambda+2\rho^{3/2}+2\sqrt{\rho}+3)^2} < 0$, $\frac{\partial\bar{a}_2^t}{\partial\lambda} = -\frac{\sqrt{1-\rho}+1}{(\lambda\sqrt{1-\rho}+\lambda-2\sqrt{1-\rho}\rho+4\sqrt{1-\rho}+3)^2} < 0$, and $\frac{\partial(\bar{a}_2^t-\bar{a}_1^t)}{\partial\lambda} < 0$. Under the non-transparency regime, we have $\frac{\partial\bar{a}_1^n}{\partial\lambda} = -\frac{\lambda^2(\sqrt{\rho}+1)+4\lambda(\sqrt{\rho}+1)+2(\rho+2\sqrt{\rho}+2)}{(\lambda^2(\sqrt{\rho}+1)+\lambda(\rho^{3/2}+5\sqrt{\rho}+5)+2(\rho^{3/2}-\rho+3\sqrt{\rho}+3))^2} < 0$, $\frac{\partial\bar{a}_3^n}{\partial\lambda} = \frac{\sqrt{1-\rho}-1}{(\lambda+3)^2\rho} < 0$, and $\frac{\partial(\bar{a}_3^n-\bar{a}_1^n)}{\partial\lambda} = \frac{\sqrt{1-\rho}-1}{(\lambda+3)^2\rho} - \left(-\frac{\lambda^2(\sqrt{\rho}+1)+4\lambda(\sqrt{\rho}+1)+2(\rho+2\sqrt{\rho}+2)}{(\lambda^2(\sqrt{\rho}+1)+\lambda(\rho^{3/2}+5\sqrt{\rho}+5)+2(\rho^{3/2}-\rho+3\sqrt{\rho}+3))^2}\right)$. It follows that $\frac{\partial(\bar{a}_3^n-\bar{a}_1^n)}{\partial\lambda} > 0$ when $\rho < \rho_5$, whereas $\frac{\partial(\bar{a}_3^n-\bar{a}_1^n)}{\partial\lambda} \le 0$ when $\rho \ge \rho_5$. The expression of ρ_5 is so complicated that we omit it.

Appendix B: Numerical Results for Extensions

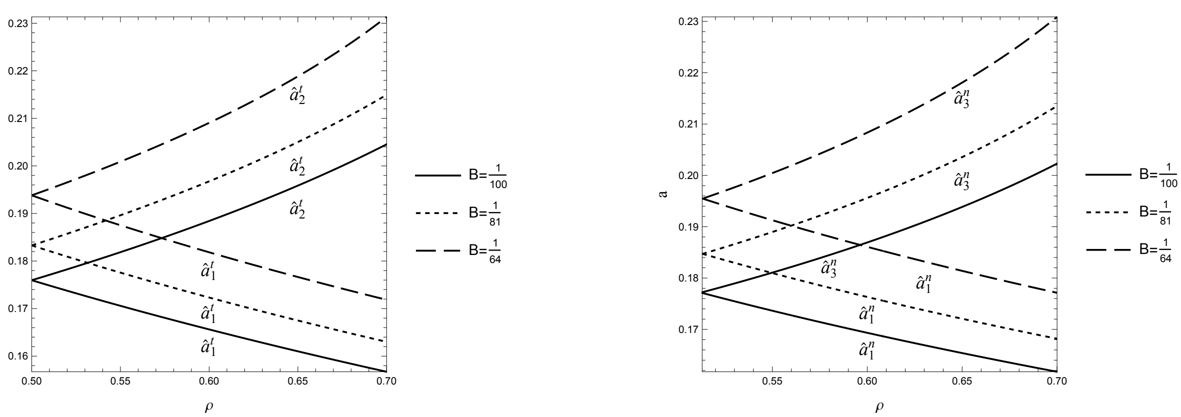
B1: Initial Capital

By incorporating positive initial capital, we first define the firms' profit functions as follows:

Under the transparency regime, the retailers' and the manufacturers' problems are given: $\hat{\pi}^{dt}_{R_i|S_h} = \rho(1+a-\hat{q}^{dt}_{ih}-\hat{q}^{dt}_{jh}-\hat{w}^{dt}_{ih})\hat{q}^{dt}_{ih} + (1-\rho)([(1-a-\hat{q}^{dt}_{ih}-\hat{q}^{dt}_{jh})\hat{q}^{dt}_{ih}+B-\hat{w}^{dt}_{ih}\hat{q}^{dt}_{ih}]^+ - B)$, $\hat{\pi}^{dt}_{M_i|S_h} = \rho\hat{w}^{dt}_{ih}\hat{q}^{dt}_{ih} + (1-\rho)\min\{\hat{w}^{dt}_{ih}\hat{q}^{dt}_{ih}, (1-a-\hat{q}^{dt}_{ih}-\hat{q}^{dt}_{jh})\hat{q}^{dt}_{ih}+B\}$, $\hat{\pi}^{dt}_{R_i|S_l} = (1-\rho)(1+a-\hat{q}^{dt}_{il}-\hat{q}^{dt}_{jl}-\hat{w}^{dt}_{il})\hat{q}^{dt}_{il} + \rho([(1-a-\hat{q}^{dt}_{il}-\hat{q}^{dt}_{jl})\hat{q}^{dt}_{il}+B-\hat{w}^{dt}_{il}\hat{q}^{dt}_{il}]^+ - B)$ and $\hat{\pi}^{dt}_{M_i|S_l} = (1-\rho)\hat{w}^{dt}_{il}\hat{q}^{dt}_{il} + \rho\min\{\hat{w}^{dt}_{il}\hat{q}^{dt}_{il}, (1-a-\hat{q}^{dt}_{il}-\hat{q}^{dt}_{jl})\hat{q}^{dt}_{il}+B\}$.

Under the non-transparency regime, the retailers' and the manufacturers' profit functions are defined as follows: $\hat{\pi}^{dn}_{R_i|S_h} = \rho(1+a-\hat{q}^{dn}_{ih}-\hat{q}^{dn}_{jh}-\hat{w}^{dn}_{i})\hat{q}^{dn}_{ih} + (1-\rho)([(1-a-\hat{q}^{dn}_{ih}-\hat{q}^{dn}_{jh})\hat{q}^{dn}_{ih}+B-\hat{w}^{dn}_{i}\hat{q}^{dn}_{ih}]^+ - B)$, $\hat{\pi}^{dn}_{R_i|S_l} = (1-\rho)(1+a-\hat{q}^{dn}_{il}-\hat{q}^{dn}_{jl}-\hat{w}^{dn}_{i})\hat{q}^{dn}_{il} + \rho([(1-a-\hat{q}^{dn}_{il}-\hat{q}^{dn}_{jl})\hat{q}^{dn}_{il}+B-\hat{w}^{dn}_{i}\hat{q}^{dn}_{il}]^+ - B)$, and $\hat{\pi}^{dn}_{M_i} = \frac{\rho\hat{w}^{dn}_{i}\hat{q}^{dn}_{ih}+(1-\rho)\min\{\hat{w}^{dn}_{i}\hat{q}^{dn}_{ih},(1-a-\hat{q}^{dn}_{ih}-\hat{q}^{dn}_{jh})\hat{q}^{dn}_{ih}+B\}}{2} + \frac{(1-\rho)\hat{w}^{dn}_{i}\hat{q}^{dn}_{il}+\rho\min\{\hat{w}^{dn}_{i}\hat{q}^{dn}_{il},(1-a-\hat{q}^{dn}_{il}-\hat{q}^{dn}_{jl})\hat{q}^{dn}_{il}+B\}}{2}$.

Introducing positive initial capital renders the analysis analytically intractable. We therefore conduct numerical studies to verify the robustness of the main results. Figure EC.1 shows the demand uncertainty thresholds under the transparency and non-transparency regimes when $B = \frac{1}{100}$, $B = \frac{1}{81}$, and $B = \frac{1}{64}$. We can observe that with the increase in B, the full solvency range expands but the full bankruptcy range contracts. This is intuitive that a higher B relaxes the retailers' financial constraints, thereby contracting the regions in which bankruptcy occurs.

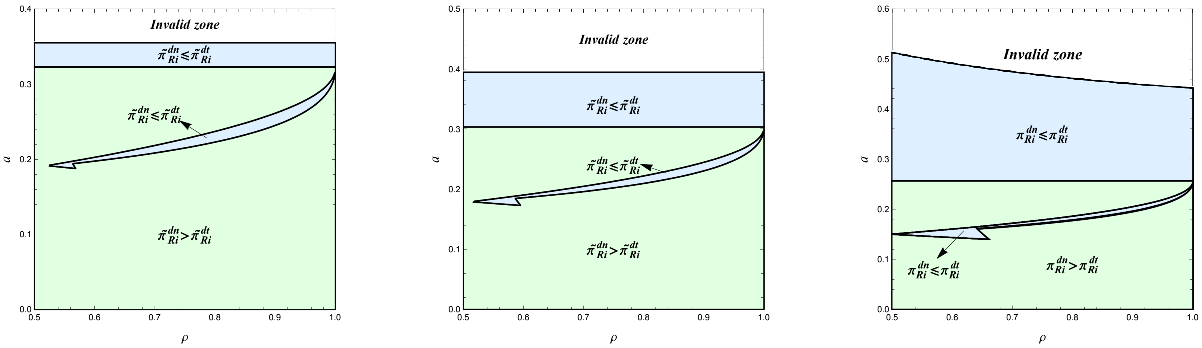


(a) Under the transparency regime (b) Under the non-transparency regime

Figure EC.1 The demand uncertainty thresholds when the retailers have positive initial capital.

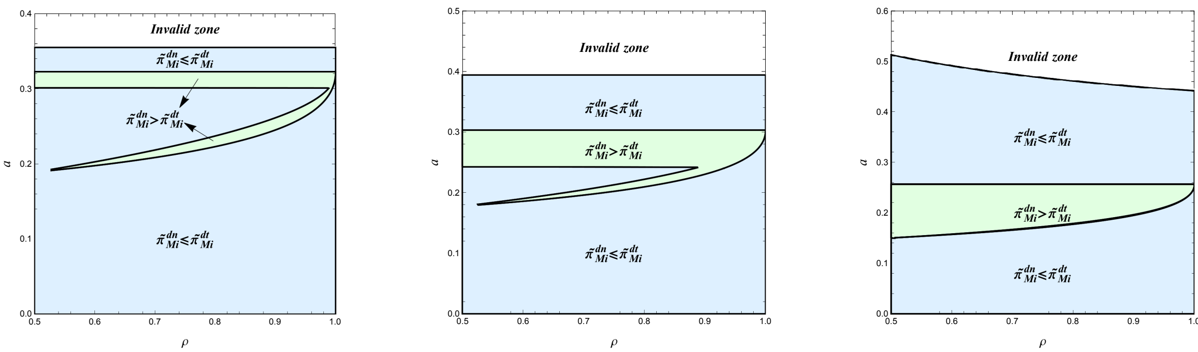
B2: Competition Intensity

We adopt numerical analysis to present the effects of information transparency on the retailers' and the manufacturers' expected profits under varying levels of competition intensity, as illustrated in Figures EC.2-EC.3.



(a) $\lambda = \frac{1}{10}$ (b) $\lambda = \frac{3}{10}$ (c) $\lambda = \frac{9}{10}$

Figure EC.2 The retailers' expected profit comparisons considering competition intensity.

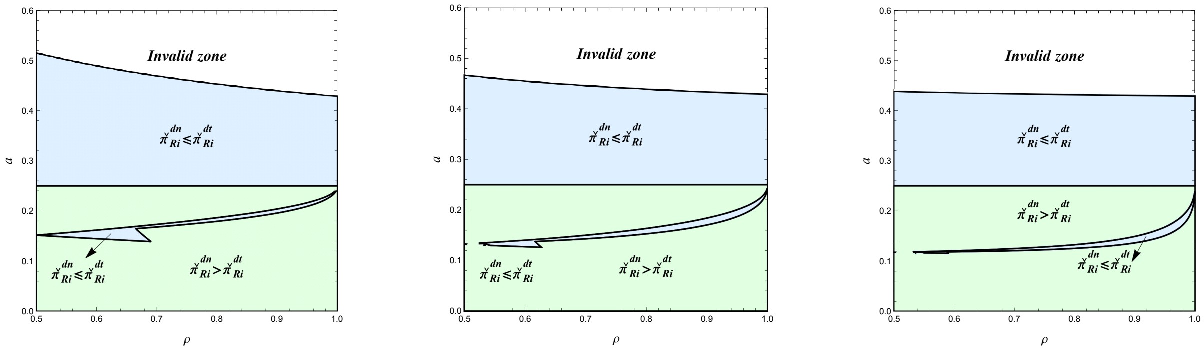


(a) $\lambda = \frac{1}{10}$ (b) $\lambda = \frac{3}{10}$ (c) $\lambda = \frac{9}{10}$

Figure EC.3 The manufacturers' expected profit comparisons considering competition intensity.

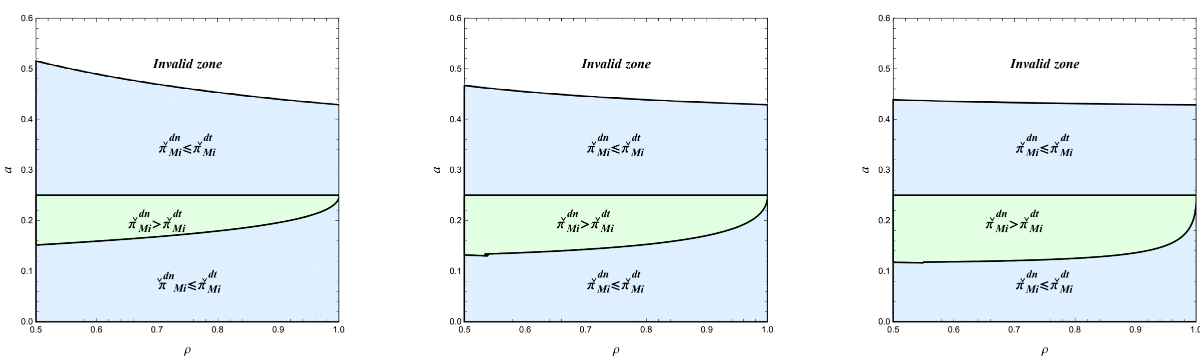
B3: Unequal Probability of Demand States

Figures EC.4-EC.5 show how information transparency affects the manufacturers' and the retailers' expected profits when considering unequal probability of demand states.



(a) $\beta = \frac{4}{9}$ (b) $\beta = \frac{2}{3}$ (c) $\beta = \frac{8}{9}$

Figure EC.4 The retailers' expected profit comparisons considering unequal probability of demand states.



(a) $\beta = \frac{4}{9}$ (b) $\beta = \frac{2}{3}$ (c) $\beta = \frac{8}{9}$

Figure EC.5 The manufacturers' expected profit comparisons considering unequal probability of demand states.