1 Appendix A: Proofs

We begin with a definition.

Investment plan. A (fully specified) investment plan is a vector $(k_1, l_1, k_2, l_2, k_1'^L, l_1'^L, k_2'^L, l_2'^L, k_1'^H, l_1'^H, k_2'^H, l_2'^H)$. The plan specifies an initial investment plan (k_1, l_1, k_2, l_2) in period 0, an updated investment plan $\left(k_1'^L, l_1'^L, k_2'^L, l_2'^L\right)$ in period 1 if demand is low, and an updated investment plan $\left(k_1'^H, l_1'^H, k_2'^H, l_2'^H\right)$ in period 1 if demand is high.

Lemma A1 provides a partial characterization of an optimal investment plan that holds across all our scenarios.

Lemma A1. Suppose entry occurs. Then the optimal choice of inputs satisfies:
(i) $k_1 = l_1$.
(ii) Output $m - \Delta/2$ is produced with T_1 in both periods: $k_1 \geq m - \Delta/2$, $l_1 \geq m - \Delta/2$, $k_1'^L \geq m - \Delta/2$, $l_1'^L \geq m - \Delta/2$, $k_1'^H \geq m - \Delta/2$, $l_1'^H \geq m - \Delta/2$.
(iii) $\min\{k_1, l_1\} + \frac{\alpha}{2}(k_2 + l_2) \leq m + \Delta/2$.

Proof. (i). The proof proceeds by contradiction. Suppose there is an optimal investment plan $P = (k_1, l_1, k_2, l_2, k_1'^L, l_1'^L, k_2'^L, l_2'^L, k_1'^H, l_1'^H, k_2'^H, l_2'^H)$ such that $k_1 < l_1$. Consider an alternative investment plan $\widehat{P} = (\widehat{k_1}, \widehat{l_1}, \widehat{k_2}, \widehat{l_2}, \widehat{k_1'^L}, \widehat{l_1'^L}, \widehat{k_2'^L}, \widehat{l_2'^L}, \widehat{k_1'^H}, \widehat{l_1'^H}, \widehat{k_2'^H}, \widehat{l_2'^H})$ where all input choices are the same as in the original plan P except that $\widehat{l_1} = k_1$. By construction $\widehat{P}$ guarantees the same level of output as P but costs strictly less because $\widehat{l_1} < l_1$ (note that $\widehat{l_1} < l_1$ also implies that adjustment costs are lower with $\widehat{P}$ than with P). Thus any plan such that $k_1 < l_1$ cannot be optimal. A completely analogous argument shows that any plan such that $k_1 > l_1$ cannot be optimal. Thus we must have $k_1 = l_1$ in the optimum.

(ii). Because $w = r$ and $\alpha < 1$, the cheapest way to produce the $m - \Delta/2$ units of output demanded for sure in both periods is to use T_1 with $k_1 = l_1$. This implies $k_1 \geq m - \Delta/2$, $l_1 \geq m - \Delta/2$, $k_1'^L \geq m - \Delta/2$, $l_1'^L \geq m - \Delta/2$, $k_1'^H \geq m - \Delta/2$, $l_1'^H \geq m - \Delta/2$.

(iii). In period 1 it cannot be optimal to produce more than the maximum quantity demanded, $m + \Delta/2$. Any plan P such as $\min\{k_1, l_1\} + \frac{\alpha}{2}(k_2 + l_2) > m + \Delta/2$ can easily be improved upon by reducing inputs. ■

Proof of Proposition 1. Suppose the firm enters in period 0. The proof proceeds in two steps.

Step 1. First, we show that technology T_2 is never used in the optimum**.** Remember that, by Lemma A1, the output $m - \Delta/2$ demanded for sure is produced with T_1 in both periods. Thus, we only need to establish whether T_2 is used to serve the uncertain demand.

Consider the profits that the firm can make by serving the uncertain demand. Suppose that, in period 1, the firm starts by producing $x \in [0, \Delta]$ units of this uncertain demand. We compare the total profits (in period 1 and period 2) associated with initially producing x with T_1 or T_2.

If the firm starts by producing x units of uncertain output with T_1, it must select $k_1 = l_1 = x$. Period-1 expected profits are therefore $(p/2 - 2)x$. In period 2, if demand is low and $c < 1$, the firm eliminates these inputs and adjustment costs are $2cx$. (If $c \geq 1$, the firm retains these inputs and costs are $2x = (w + r)x$.) If demand is high, the firm not only retains the inputs, but increases them until $k_1'^H = l_1'^H = \Delta$ and demand Δ is fully served. Period-2 profits in this case are therefore $(p - 2)\Delta$. Thus, the firm's total expected profits associated with this strategy are:

$$\left(\frac{1}{2}p - 2\right)x - \frac{1}{2}\beta(2cx) + \frac{1}{2}\beta(p - 2)\Delta \tag{1}$$

where $c \in [0,1]$ and β is the discount factor.

If instead the firm starts by producing x units of uncertain output with T_2, it must select k_2 and l_2 such that $k_2 + l_2 = \frac{2}{\alpha}x$. Period-1 expected profits are $(p/2 - 2/\alpha)x$. Note that, if $x > 0$, $(p/2 - 2/\alpha)x$ is strictly less than $(p/2 - 2)x$ that can be obtained with T_1. In period 2, if demand is low and $c < 1$, the firm eliminates the inputs and adjustment costs are $(2/\alpha)cx \geq 2cx$. (If $c \geq 1$, the firm retains the inputs and costs are $(2/\alpha)x \geq 2x$.) Finally, if demand is high, no matter what the firm does, its period-2 profits cannot be higher than $(p-2)\Delta$, because this is the maximum profit achievable in a single period.

The above discussion demonstrates that producing $x \in [0, \Delta]$ units of uncertain output in period 1 with T_1 always yields higher profits than producing x units of uncertain output in period 1 with T_2 (strictly so if $x > 0$). Hence technology T_2 is never used in the optimum: $k_2 = l_2 = 0$.

Step 2. Step 1 demonstrates that, if $x \in [0, \Delta]$ units of uncertain output are produced, they are produced with T_1. Now we determine the optimal x. To do so, we maximize (1) with respect to x, subject to $x \in [0, \Delta]$. Clearly, in the optimum, $x = 0$ if $\frac{1}{2}p - 2 - \beta c < 0$, and $x = \Delta$ if $\frac{1}{2}p - 2 - \beta c \geq 0$ (we assume that, when indifferent between output levels, the firm breaks the tie in favor of higher output). Thus, the firm selects the starting small strategy if $p < 4 + 2\beta c$, and the starting large and efficient strategy if $p \geq 4 + 2\beta c$. Proposition 1 easily follows. ■

Proof of Proposition 2. This proof also proceeds in two steps.

Step 1. First, we show that when labor is flexible ($c_L = 0$) and capital is rigid ($c_K = c \in (0,1]$), then $k_2 = 0$. The argument is similar to the one in Step 1 of Proposition 1. Suppose by contradiction that there is an optimal investment plan such that $k_2 = a > 0$. Then we can construct an alternative investment plan that strictly improves upon this posited optimal plan.

In the new plan, k_2 is set equal to zero and k_1 and l_1 are increased by $\frac{\alpha}{2}a$ each. By construction, this new plan yields the same output in period 1 than the posited optimal plan, but costs strictly less. In period 2, if demand turns out to be low, adjustment costs are also lower with the new plan than with the posited optimal plan. (Eliminating $\frac{\alpha}{2}a$ units of k_1 and $\frac{\alpha}{2}a$ units of l_1 costs $\frac{\alpha}{2}ac$ in total, because $c_K = c$ and $c_L = 0$. Eliminating a units of k_2 costs ac.) Finally, if demand is high, the most efficient way to produce output in period 2 is to use T_1 with $k_1 = l_1$. The new plan does not require adjustments to do that.

We conclude that because any plan such that $k_2 = a > 0$ can be strictly improved upon, $k_2 = 0$ in the optimum.

Step 2. Next, we characterize when T_1 or T_2 are used to serve the uncertain demand.

Suppose the firm starts in period 1 by producing $x \in [0, \Delta]$ units of uncertain output with T_1. Following the proof of Proposition 1, but noting that now $c_L = 0$, the firm's total expected profits associated with this strategy are

$$\left(\frac{1}{2}p - 2\right)x - \frac{1}{2}\beta c x + \frac{1}{2}\beta(p-2)\Delta.$$

Thus, the firm prefers starting small ($x = 0$) if $p < 4 + \beta c$, and starting large and efficient ($x = \Delta$) if $p \geq 4 + \beta c$. The payoff associated with serving the uncertain demand Δ with the starting small strategy is

$$\frac{1}{2}\beta(p-2)\Delta \tag{2}$$

and payoff associated with serving the uncertain demand Δ with the starting large and efficient strategy is

$$\left(\frac{1}{2}p - 2\right)\Delta + \frac{1}{2}\beta\left[-c + (p-2)\right]\Delta. \tag{3}$$

Suppose instead the firm starts by producing $x \in [0, \Delta]$ units of uncertain output in period 1 with T_2. Because $k_2 = 0$ in the optimum, it must be that $l_2 = \frac{2}{\alpha}x$. Period-1 expected profits are therefore $(p/2 - 2/\alpha)x$. In period 2, if demand is low and $c < 1$, the firm eliminates the excess labor without incurring adjustment costs because $c_L = 0$. If demand is high, the firm optimally selects $k_1^H = l_1^H = \Delta$ (if necessary, by firing some labor) and demand Δ is fully served. Period-2 profits in this case are therefore $(p-2)\Delta$. Thus, the firm's total expected profits associated with this strategy are

$$\left(\frac{1}{2}p - \frac{2}{\alpha}\right)x + \frac{1}{2}\beta(p-2)\Delta.$$

Note that selecting $x = 0$ is equivalent to choosing the starting small strategy. The other possible profit maximizing choice is to select $x = \Delta$, which corresponds to the exploratory labor-intensive strategy discussed in the text. This strategy yields payoff

$$\left(\frac{1}{2}p - \frac{2}{\alpha}\right)\Delta + \frac{1}{2}\beta(p-2)\Delta. \tag{4}$$

By comparing profits (2), (3) and (4), Proposition 2 easily follows. ■