CORPORATE DIVERSIFICATION AND RISK: PORTFOLIO EFFECTS AND RESOURCE REDEPLOYABILITY

ONLINE APPENDIX A: Two-period numerical example

This example considers a firm that has a unit of resources which can be used in two discrete periods, $t = 0$ and $t = 1$, and in two (product or geographic) markets i and j. It is denoted that, at time t, proportion $0 \le m_{it} \le 1$ of resources is used in market i and another proportion $(1 - m_{it})$ is used in market j. The focus on two markets is needed to capture how the firm's diversification affects risk, whereas two periods allow the minimal consideration of inter-temporal economies of scope, or resource redeployability. For inter-temporal discounting, the interest rate is set $r = 0.08$. To enable an exogenous source for risk, future margins in the two markets, C_{i1} and C_{j1}, are made uncertain as the firm sees them at the initial time $t = 0$. Specifically, C_{it} and C_{jt} follow independent binomial distributions such that $C_{i0} = C_{j0} = 0.080$, $C_{i1}^u = 0.217$ with probability $q_i^u = 0.304$, $C_{i1}^d = 0.029$ with probability $q_i^d = 0.696$, $C_{j1}^u = 0.217$ with probability $q_j^u = 0.304$, and $C_{j1}^d = 0.029$ with probability $q_j^d = 0.696$. Here, if C_{it} goes up (denoted as superscript *u*) from its original value C_{i0}, then $C_{i1}^u = 0.217$. Alternatively, if C_{it} goes down (denoted as superscript *d*) from its original value C_{i0}, then $C_{i1}^d = 0.029$. Similarly, if C_{jt} goes up from C_{j0}, then $C_{j1}^u = 0.217$; if C_{jt} goes down from C_{j0}, then $C_{j1}^d = 0.029$.[1] These dynamics for the two margins are summarized in Panel A of Figure A1. To identify the mechanisms with which corporate diversification affect risk in this setting, the firm is placed in the following three alternative contexts.

[1] This specification is not arbitrary. To make sure that the two states for each variable completely represent the nature, it should be that $q_i^u + q_i^d = 1$ and $q_j^u + q_j^d = 1$. Moreover, this specification is designed to be comparable to the model in the manuscript. To make this example generalizable to more longitudinal contexts, the example represents the setting with $\sigma = \sigma_i = \sigma_j = 1$ and $\rho = 0$, in which $C_{i1}^u = u_i C_{i0}$, $C_{i1}^d = d_i C_{i0}$, $u_i = e^{\sigma_i \Delta t}$, $d_i = 1/u_i$, $q_i^u = \left(e^{r\Delta t} - d_i\right)/\left(u_i - d_i\right)$, $C_{j1}^u = u_j C_{j0}$, $C_{j1}^d = d_j C_{j0}$, $u_j = e^{\sigma_j \Delta t}$, $d_j = 1/u_j$, and $q_j^u = \left(e^{r\Delta t} - d_j\right)/\left(u_j - d_j\right)$. Finally, this setting implies the equilibrium market because the used binomial probability distributions are Martingales such that $C_{i0} = e^{-r\Delta t}\mathrm{E}\left[C_{i1}\right]$ and $C_{j0} = e^{-r\Delta t}\mathrm{E}\left[C_{j1}\right]$.

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Insert Figure A1 about here

Context 1: Diversified firm with resource redeployability

In the first context, the firm enters the model while having been already diversified from its original market i to another market j: the firm had proportion $m_{i(-1)} = 0.5$ of its resources deployed in market i and the remaining proportion $(1 - m_{i(-1)}) = 0.5$ deployed in market j.[2] This context lets the firm redeploy resources between i and j at $t = 0$ and at $t = 1$. Panel B of Figure A1 shows resource allocation choices that the firm can make in the first context—in a single scenario for the margins possible at time $t = 0$, and in four scenarios for the two margins possible at time $t = 1$.[3] If the firm redeploys proportion $\left(m_{it} - m_{it-1}\right)$ from j to i or proportion $\left(m_{it-1} - m_{it}\right)$ from i to j, it pays a redeployment cost $S\left[\max\left(0, m_{it} - m_{it-1}\right)C_{it} + \max\left(0, m_{it-1} - m_{it}\right)C_{jt}\right]$. Here, $S = 0.05$ represents the marginal redeployment cost that reflects the five-percent loss in efficiency in redeploying a unit of resources from another market relative to their permanent use in the focal destination; that loss is mitigated by relatedness between i and j (Montgomery & Wernerfelt 1988). At both times $t = 0$ and $t = 1$ considered in the model, the firm is assumed to maximize the net cash flow V_t it expects to accumulate over the remaining lifecycle of its resources. Formally, in each period, the firm's resource (re-)deployment choices must comply with the following Bellman equation (Bellman 1957):

$$\left(m_{it}^{*} \middle| m_{it-1}\right) = \arg\max_{m_{it}} \left\{V_t\left(m_{it} \middle| m_{it-1}\right)\right\} = \arg\max_{m_{it}} \left\{F_t\left(m_{it} \middle| m_{it-1}\right) + \left[1/\left(1+r\right)\right]\mathrm{E}\left[V_{t+1}\left(m_{it+1}^{*} \middle| m_{it}\right)\right]\right\}. \quad \text{(A.1)}$$

In Equation A.1, dependence of the current net cash flow $F_t\left(m_{it} \middle| m_{it-1}\right)$ and, respectively, of the optimal choice $\left(m_{it}^{*} \middle| m_{it-1}\right)$ on the previous choice reflects 'conditioning on the past' (Feldman &

[2] With this notation, resource deployment choice $m_{i(-1)}$, with which the firm enters time $t = 0$, corresponds to time $t = -1$.

[3] The names of the scenarios list the realization for C_{it} (*i.e.*, *u* or *d*) first and the realization for C_{jt} (*i.e.*, *u* or *d*) second.

Sakhartov 2021). This conditioning captures path-dependence and takes place because, in the general case of non-trivial redeployment cost $S > 0$, the current net cash flow $F_t\left(m_{it} \middle| m_{it-1}\right)$ generated by the considered current choice depends on where resources were deployed in the previous time and, thus, on whether a redeployment cost needs to be paid if the resources need to be withdrawn from their previous use. In turn, conditional expectation $\mathrm{E}\left[V_{t+1}\left(m^*_{it+1} \middle| m_{it}\right)\right]$ for the net cash flow the firm expects to accumulate starting at the immediate next time $t+1$ reflects 'projecting into the future' (Feldman & Sakhartov 2021). Such projecting is necessary to assess the expected impact of the current resource deployment choice on future uncertain net cash flow given that the selected current choice m_{it} will be impossible to undo in the future.[4] The use of m^*_{it+1}, rather than m_{it+1}, highlights that the same maximization procedure specified in Equation A.1 must be applied to $V_{t+1}\left(m_{it+1} \mid m_{it}\right)$.

The first step in calculating risk is the estimation of the optimal conditional choices $\left(m^*_{it} \middle| m_{it-1}\right)$ in both periods. The need for the firm to project its choices into the future, by estimating $\mathrm{E}\left[V_{t+1}\left(m^*_{it+1} \middle| m_{it}\right)\right]$, necessitates the use of the backward induction—the estimation should start at time $t = 1$ when no future implications of the considered choices are expected (*i.e.*, $\mathrm{E}\left[V_{t>1}\right] = 0$ because resources fully depreciate over the two periods). In this case, Equation A.1 degenerates to the following:

$$\left(m^*_{i1} \middle| m_{i0}\right) = \arg\max_{m_{i1}} \left\{m_{i1}C_{i1} + (1-m_{i1})C_{j1} - S\left[\max\left(0, m_{i1} - m_{i0}\right)C_{i1} + \max\left(0, m_{i0} - m_{i1}\right)C_{j1}\right]\right\}. \quad \text{(A.2)}$$

In terms of the value maximized at time $t = 1$, Equation A.2 can be restated as follows:

$$V_1\left(m^*_{i1} \middle| m_{i0}\right) = \max_{m_{i1}} \left\{m_{i1}C_{i1} + (1-m_{i1})C_{j1} - S\left[\max\left(0, m_{i1} - m_{i0}\right)C_{i1} + \max\left(0, m_{i0} - m_{i1}\right)C_{j1}\right]\right\}. \quad \text{(A.3)}$$

[4] It is still possible for the firm to select m_{it+1} that is the same as m_{it-1} even if m_{it} is different from m_{it-1}, but this will have to be done through the use of the costly reverse redeployment because m_{it} will have already been committed by time $t+1$.

To prepare this maximization, each of the four tables of Panel B where $t=1$ evaluates $V_1\left(m_{i1}\middle|m_{i0}\right)$ for all nine combinations of m_{i0} and m_{i1} allowed in this example using the following formula:

$$V_1\left(m_{i1}\middle|m_{i0}\right)=m_{i1}C_{i1}+(1-m_{i1})C_{j1}-S\left[\max\left(0,m_{i1}-m_{i0}\right)C_{i1}+\max\left(0,m_{i0}-m_{i1}\right)C_{j1}\right]. \quad \text{(A.4)}$$

For each m_{i0}, the maximum value $V_1\left(m_{i1}^*\middle|m_{i0}\right)$ is in bold font to highlight optimality, and it maps onto the optimal conditional choice $\left(m_{i1}^*\middle|m_{i0}\right)$.

The values $V_1\left(m_{i1}^*\middle|m_{i0}\right)$ estimated for $t=1$ can now be plugged in the estimation of the optimal conditional choices $\left(m_{i0}^*\middle|m_{i(-1)}\right)$ in time $t=0$ that are assessed for the only feasible value of $m_{i(-1)}=0.5$:

$$\begin{aligned}\left(m_{i0}^*\middle|m_{i(-1)}\right)&=\arg\max_{m_{i0}}\left\{\begin{array}{l}m_{i0}C_{i0}+(1-m_{i0})C_{j0}\\-S\left[\max\left(0,m_{i0}-m_{i(-1)}\right)C_{i0}+\max\left(0,m_{i(-1)}-m_{i0}\right)C_{j0}\right]\\+\left[1/(1+r)\right]\mathrm{E}\left[V_1\left(m_{i1}^*\middle|m_{i0}\right)\right]\end{array}\right\}\\&=\arg\max_{m_{i0}}\left\{\begin{array}{l}m_{i0}C_{i0}+(1-m_{i0})C_{j0}\\-S\left[\max\left(0,m_{i0}-m_{i(-1)}\right)C_{i0}+\max\left(0,m_{i(-1)}-m_{i0}\right)C_{j0}\right]\\+\left[1/(1+r)\right]\left(\begin{array}{l}q_i^uq_j^uV_1^{uu}\left(m_{i1}^*\middle|m_{i0}\right)+q_i^uq_j^dV_1^{ud}\left(m_{i1}^*\middle|m_{i0}\right)\\+q_i^dq_j^uV_1^{du}\left(m_{i1}^*\middle|m_{i0}\right)+q_i^dq_j^dV_1^{dd}\left(m_{i1}^*\middle|m_{i0}\right)\end{array}\right)\end{array}\right\}.\end{aligned}[5] \quad \text{(A.5)}$$

In terms of the maximized function, Equation A.5 can be restated as follows:

[5] Superscripts for $V_1\left(m_{i1}^*\middle|m_{i0}\right)$ list the realization for C_{it} (*i.e.*, *u* or *d*) first and the realization for C_{jt} (*i.e.*, *u* or *d*) second.

$$V_0\left(m_{i0}^*\left|m_{i(-1)}\right.\right)=\max_{m_{i0}}\left\{\begin{array}{l} m_{i0}C_{i0}+(1-m_{i0})C_{j0} \\ -S\left[\max\left(0,m_{i0}-m_{i(-1)}\right)C_{i0}+\max\left(0,m_{i(-1)}-m_{i0}\right)C_{j0}\right] \\ +\left[1/(1+r)\right]\left(\begin{array}{l} q_i^u q_j^u V_1^{uu}\left(m_{i1}^*\left|m_{i0}\right.\right)+q_i^u q_j^d V_1^{ud}\left(m_{i1}^*\left|m_{i0}\right.\right) \\ +q_i^d q_j^u V_1^{du}\left(m_{i1}^*\left|m_{i0}\right.\right)+q_i^d q_j^d V_1^{dd}\left(m_{i1}^*\left|m_{i0}\right.\right) \end{array}\right) \end{array}\right\}. \qquad \text{(A.6)}$$

To run the maximization in Equation A.6, three values of $V_0\left(m_{i0}\left|m_{i(-1)}\right.\right)$ that are possible with $m_{i(-1)}=0.5$ and with three values of m_{i0} are estimated as follows and are put in the table for $t=0$ in Panel B:

$$V_0\left(m_{i0}\left|m_{i(-1)}\right.\right)=m_{i0}C_{i0}+(1-m_{i0})C_{j0}-S\left[\max\left(0,m_{i0}-m_{i(-1)}\right)C_{i0}+\max\left(0,m_{i(-1)}-m_{i0}\right)C_{j0}\right]$$
$$+\left[1/(1+r)\right]\left(\begin{array}{l} q_i^u q_j^u V_1^{uu}\left(m_{i1}^*\left|m_{i0}\right.\right)+q_i^u q_j^d V_1^{ud}\left(m_{i1}^*\left|m_{i0}\right.\right) \\ +q_i^d q_j^u V_1^{du}\left(m_{i1}^*\left|m_{i0}\right.\right)+q_i^d q_j^d V_1^{dd}\left(m_{i1}^*\left|m_{i0}\right.\right) \end{array}\right). \qquad \text{(A.7)}$$

The maximum of these values is marked in bold font to highlight optimality, and it maps on the optimal unconditional choice $m_{i0}^*=0.5$ realized at time $t=0$, which is also marked in red font to note the lack of conditionality.

The second step in calculating risk is the estimation of all possible two-period paths (*i.e.*, realizations) for the discounted net cash flow accumulated by the firm over the lifecycle of its resources (W) in the presence of uncertainty in the two markets. By going recursively forward in time, from the optimal unconditional choice m_{i0}^* identified in the first step through the conditional optimal choices $\left(m_{i1}^*\left|m_{i0}\right.\right)$ also found in the first step, Panel B of Figure A1 enables the identification of the unconditional optimal choices m_{i1}^*. These four unconditional optimal choices that are now marked in red font in Panel B of Figure A1 (*i.e.*, only some cells with numbers highlighted in bold font turn into highlighted in red font based on this recursive conditioning on the past) form the respective four optimal paths. The discounted net cash flow accumulated by the firm over the two periods on those four paths is estimated as follows:

$$\begin{aligned} W^{uu} &= m_{i0}^{*}C_{i0} + (1-m_{i0}^{*})C_{j0} - S\begin{bmatrix} \max\left(0, m_{i0}^{*} - m_{i(-1)}\right)C_{i0} \\ +\max\left(0, m_{i(-1)} - m_{i0}^{*}\right)C_{j0} \end{bmatrix} + \left[1/\left(1+r\right)\right]V_1^{uu}\left(m_{i1}^{*}\middle|m_{i0}\right), \quad \text{(A.8)} \\ &= 0.5*0.080 + (1-0.5)*0.080 - 0.05*0 + 1/(1+0.08)*0.217 = 0.281 \end{aligned}$$

$$\begin{aligned} W^{ud} &= m_{i0}^{*}C_{i0} + (1-m_{i0}^{*})C_{j0} - S\begin{bmatrix} \max\left(0, m_{i0}^{*} - m_{i(-1)}\right)C_{i0} \\ +\max\left(0, m_{i(-1)} - m_{i0}^{*}\right)C_{j0} \end{bmatrix} + \left[1/\left(1+r\right)\right]V_1^{ud}\left(m_{i1}^{*}\middle|m_{i0}\right), \quad \text{(A.9)} \\ &= 0.5*0.080 + (1-0.5)*0.080 - 0.05*0 + 1/(1+0.08)*0.212 = 0.276 \end{aligned}$$

$$\begin{aligned} W^{du} &= m_{i0}^{*}C_{i0} + (1-m_{i0}^{*})C_{j0} - S\begin{bmatrix} \max\left(0, m_{i0}^{*} - m_{i(-1)}\right)C_{i0} \\ +\max\left(0, m_{i(-1)} - m_{i0}^{*}\right)C_{j0} \end{bmatrix} + \left[1/\left(1+r\right)\right]V_1^{du}\left(m_{i1}^{*}\middle|m_{i0}\right), \quad \text{(A.10)} \\ &= 0.5*0.080 + (1-0.5)*0.080 - 0.05*0 + 1/(1+0.08)*0.212 = 0.276 \end{aligned}$$

$$\begin{aligned} W^{dd} &= m_{i0}^{*}C_{i0} + (1-m_{i0}^{*})C_{j0} - S\begin{bmatrix} \max\left(0, m_{i0}^{*} - m_{i(-1)}\right)C_{i0} \\ +\max\left(0, m_{i(-1)} - m_{i0}^{*}\right)C_{j0} \end{bmatrix} + \left[1/\left(1+r\right)\right]V_1^{dd}\left(m_{i1}^{*}\middle|m_{i0}\right). \quad \text{(A.11)} \\ &= 0.5*0.080 + (1-0.5)*0.080 - 0.05*0 + 1/(1+0.08)*0.029 = 0.107 \end{aligned}$$

The third step in calculating risk is the estimation of the expectation for the discounted net cash flow accumulated by the firm by weighting the four realizations estimated with Equations A.8-A.11 by their respective probabilities:

$$\begin{aligned} \mathrm{E}\left[W\right] &= q_i^u q_j^u W^{uu} + q_i^u q_j^d W^{ud} + q_i^d q_j^u W^{du} + q_i^d q_j^d W^{dd} \\ &= 0.304*0.304*0.281 + 0.304*0.696*0.276 \qquad . \quad \text{(A.12)} \\ &+ 0.696*0.304*0.276 + 0.696*0.696*0.107 = 0.195 \end{aligned}$$

Naturally, the estimated expectation for the accumulated net cash flow $\mathrm{E}\left[W\right] = 0.195$ is the same as $V_0\left(m_{i0}^{*}\right) = 0.195$ in Panel B of Figure A1.

The final step is the estimation of risk as the variance of the discounted net cash flow W accumulated by the diversified firm with redeployability over the two-period lifecycle of its resources:

$$\begin{aligned} R &= \mathrm{Var}\left[W\right] = q_i^u q_j^u \left(W^{uu} - \mathrm{E}\left[W\right]\right)^2 + q_i^u q_j^d \left(W^{ud} - \mathrm{E}\left[W\right]\right)^2 \\ &+ q_i^d q_j^u \left(W^{du} - \mathrm{E}\left[W\right]\right)^2 + q_i^d q_j^d \left(W^{dd} - \mathrm{E}\left[W\right]\right)^2 \\ &= 0.304 * 0.304 * \left(0.281 - 0.195\right)^2 + 0.304 * 0.696 * \left(0.276 - 0.195\right)^2 \\ &+ 0.696 * 0.304 * \left(0.276 - 0.195\right)^2 + 0.696 * 0.696 * \left(0.107 - 0.195\right)^2 = 0.007 \end{aligned} \quad . \quad \text{(A.13)}$$

Context 2: Diversified firm without resource redeployability

In the second context, the firm enters the model while having been already diversified from its original market i to another market j: the firm had proportion $m_{i(-1)} = 0.5$ of its resources deployed in market i and the remaining proportion $(1 - m_{i(-1)}) = 0.5$ deployed in market j. In contrast to the first context, the firm is now forced to stay diversified in both periods: $m_{i0} = m_{i1} = 0.5$. By disallowing the use of inter-temporal economies of scope from resource redeployment, this scenario eliminates the primary difference between corporate diversification and portfolio diversification.

Panel C of Figure A1 depicts resource allocation that is committed by the diversified firm in the absence of resource redeployability for the single possible realization of the margins at time $t = 0$, and for the four scenarios for the two margins possible at time $t = 1$. In this second context, the estimation of the discounted net cash flow accumulated by the firm degenerates from the more general setting in Equations A.8-A.11 to the following:

$$\begin{aligned} W^{uu} &= m_{i0} C_{i0} + (1 - m_{i0}) C_{j0} + \left[1/(1+r)\right] V_1^{uu}\left(m_{i1}\right) \\ &= 0.5 * 0.080 + (1 - 0.5) * 0.080 + 1/(1 + 0.08) * 0.217 = 0.281 \end{aligned} \quad , \quad \text{(A.14)}$$

$$\begin{aligned} W^{ud} &= m_{i0} C_{i0} + (1 - m_{i0}) C_{j0} + \left[1/(1+r)\right] V_1^{ud}\left(m_{i1}\right) \\ &= 0.5 * 0.080 + (1 - 0.5) * 0.080 + 1/(1 + 0.08) * 0.123 = 0.194 \end{aligned} \quad , \quad \text{(A.15)}$$

$$\begin{aligned} W^{du} &= m_{i0} C_{i0} + (1 - m_{i0}) C_{j0} + \left[1/(1+r)\right] V_1^{du}\left(m_{i1}\right) \\ &= 0.5 * 0.080 + (1 - 0.5) * 0.080 + 1/(1 + 0.08) * 0.123 = 0.194 \end{aligned} \quad , \quad \text{(A.16)}$$

$$\begin{aligned} W^{dd} &= m_{i0} C_{i0} + (1 - m_{i0}) C_{j0} + \left[1/(1+r)\right] V_1^{dd}\left(m_{i1}\right) \\ &= 0.5 * 0.080 + (1 - 0.5) * 0.080 + 1/(1 + 0.08) * 0.029 = 0.107 \end{aligned} \quad . \quad \text{(A.17)}$$

Then, the expectation for the discounted net cash flow accumulated by the diversified firm without resource redeployability is calculated by weighting the four realizations estimated with Equations A.14-A.17 by their respective probabilities:

$$\begin{aligned}\mathrm{E}\left[W\right] &= q_i^u q_j^u W^{uu} + q_i^u q_j^d W^{ud} + q_i^d q_j^u W^{du} + q_i^d q_j^d W^{dd} \\ &= 0.304 * 0.304 * 0.281 + 0.304 * 0.696 * 0.194 \\ &+ 0.696 * 0.304 * 0.194 + 0.696 * 0.696 * 0.107 = 0.160\end{aligned} \quad . \qquad \text{(A.18)}$$

The final step is the estimation of risk as the variance of the discounted net cash flow W accumulated by the diversified firm without redeployability over the two-period lifecycle of its resources:

$$\begin{aligned}R = \mathrm{Var}\left[W\right] &= q_i^u q_j^u \left(W^{uu} - \mathrm{E}\left[W\right]\right)^2 + q_i^u q_j^d \left(W^{ud} - \mathrm{E}\left[W\right]\right)^2 \\ &+ q_i^d q_j^u \left(W^{du} - \mathrm{E}\left[W\right]\right)^2 + q_i^d q_j^d \left(W^{dd} - \mathrm{E}\left[W\right]\right)^2 \\ &= 0.304 * 0.304 * \left(0.281 - 0.160\right)^2 + 0.304 * 0.696 * \left(0.194 - 0.160\right)^2 \\ &+ 0.696 * 0.304 * \left(0.194 - 0.160\right)^2 + 0.696 * 0.696 * \left(0.107 - 0.160\right)^2 = 0.003\end{aligned} \quad . \qquad \text{(A.19)}$$

Context 3: Undiversified firm

In the third context, the firm enters the model while having been already focused on market i: the firm had proportion $m_{i(-1)} = 1$ of its resources deployed in market i and proportion $(1 - m_{i(-1)}) = 0$ deployed in another market j. In contrast to the first context and like in the second context, the firm is forced to stay in its original scope in both periods: $m_{i0} = m_{i1} = 1$. By keeping the distributions for C_{it} and C_{jt} symmetric, this context is made identical to the second context in terms of the value the firm is expected to accumulate over its resources' lifecycle, but the risk profiles can vary between the two contexts.

Panel D of Figure A1 depicts resource allocation that is committed by the undiversified firm for the single possible realization of the margins at time $t = 0$, and for the four scenarios possible at time $t = 1$. In this context, the calculation of the discounted net cash flow accumulated by the firm reduces from the more general setting in Equations A.8-A.11 to the following:

$$\begin{aligned} W^{uu} &= m_{i0}C_{i0} + (1-m_{i0})C_{j0} + \left[1/(1+r)\right]V_1^{uu}\left(m_{i1}\right) \\ &= 0.5*0.080 + (1-0.5)*0.080 + 1/(1+0.08)*0.217 = 0.281 \end{aligned}, \qquad \text{(A.20)}$$

$$\begin{aligned} W^{ud} &= m_{i0}C_{i0} + (1-m_{i0})C_{j0} + \left[1/(1+r)\right]V_1^{ud}\left(m_{i1}\right) \\ &= 0.5*0.080 + (1-0.5)*0.080 + 1/(1+0.08)*0.217 = 0.281 \end{aligned}, \qquad \text{(A.21)}$$

$$\begin{aligned} W^{du} &= m_{i0}C_{i0} + (1-m_{i0})C_{j0} + \left[1/(1+r)\right]V_1^{du}\left(m_{i1}\right) \\ &= 0.5*0.080 + (1-0.5)*0.080 + 1/(1+0.08)*0.029 = 0.107 \end{aligned}, \qquad \text{(A.22)}$$

$$\begin{aligned} W^{dd} &= m_{i0}C_{i0} + (1-m_{i0})C_{j0} + \left[1/(1+r)\right]V_1^{dd}\left(m_{i1}\right) \\ &= 0.5*0.080 + (1-0.5)*0.080 + 1/(1+0.08)*0.029 = 0.107 \end{aligned}. \qquad \text{(A.23)}$$

Then, the expectation for the discounted net cash flow accumulated by the undiversified firm is assessed by weighting the four realizations estimated with Equations A.20-A.23 by their respective probabilities:

$$\begin{aligned} \mathrm{E}\left[W\right] &= q_i^u q_j^u W^{uu} + q_i^u q_j^d W^{ud} + q_i^d q_j^u W^{du} + q_i^d q_j^d W^{dd} \\ &= 0.304*0.304*0.281 + 0.304*0.696*0.281 \\ &+ 0.696*0.304*0.107 + 0.696*0.696*0.107 = 0.160 \end{aligned}. \qquad \text{(A.24)}$$

The final step is the evaluation of risk as the variance of the discounted net cash flow W accumulated by the undiversified firm over the two-period lifecycle of its resources:

$$\begin{aligned} R = \mathrm{Var}\left[W\right] &= q_i^u q_j^u \left(W^{uu} - \mathrm{E}\left[W\right]\right)^2 + q_i^u q_j^d \left(W^{ud} - \mathrm{E}\left[W\right]\right)^2 \\ &+ q_i^d q_j^u \left(W^{du} - \mathrm{E}\left[W\right]\right)^2 + q_i^d q_j^d \left(W^{dd} - \mathrm{E}\left[W\right]\right)^2 \\ &= 0.304*0.304*\left(0.281-0.160\right)^2 + 0.304*0.696*\left(0.281-0.160\right)^2 \\ &+ 0.696*0.304*\left(0.107-0.160\right)^2 + 0.696*0.696*\left(0.107-0.160\right)^2 = 0.006 \end{aligned}. \qquad \text{(A.25)}$$

Summary of the example

The comparison of the third context to the second context confirms the portfolio effect of corporate diversification. In particular, the pooling of risk between less-than-perfectly correlated markets i and j reduces corporate risk from 0.006 to 0.003, which is by 0.003. However, the comparison of the third context to the first context reveals that, despite the portfolio effect that is still present in the first scenario, the presence of inter-temporal economies of scope from resource redeployment makes the risk in the diversified firm estimated as 0.007 higher than risk in the comparable undiversified firm estimated as 0.006, which is by 0.001. The risk is added specifically due to the presence of resource redeployability, as the comparison of the first context to the second context reveals. The only difference between the first context where risk is 0.007 and the second context where risk is 0.003 is the presence of resource redeployability in the former and the absence of resource redeployability in the latter. The result is the enhancement of risk by redeployability by 0.004. Thus, it can be summarized that the reduction of risk by 0.003 due to the portfolio effect is exceeded by the enhancement of risk due to redeployability by 0.004, which in the considered example leads to the eventual enhancement of corporate risk by 0.001.

Although the simple example considered in this appendix demonstrates the possibility that risk in the diversified firm can exceed risk in the comparable undiversified firm, this result is not expected to be general. Whether risk in the diversified firm exceeds risk in the comparable undiversified firm depends on the efficiency of the risk-pooling and on how much resource redeployability increases risk. The efficiency of risk-pooling depends on the initial margins in the two businesses (*i.e.*, C_{i0} and C_{j0} in this example), volatilities of these margins (*i.e.*, σ_i and σ_j involved implicitly in this example), and correlation between these margins (*i.e.*, ρ assumed zero in this example). In turn, the increase of risk by resource redeployability is rather complex. Not only the presence of resource redeployability raises the expectation for the accumulated net cash flow that in turn factors in the estimation of risk as variance of that cash flow, but also the contingent use of resource redeployment in some states of the nature increases the positive deviation of that cash flow from its expected value. This dual impact of resource redeployability (*i.e.*, through changing the mean and through increasing the positive deviation from the mean) is compounded with the possibility that, in more longitudinal settings, the diversified firm very carefully selects the time for resource redeployment from many opportunities available over the lifecycle of the firm's resources (as was demonstrated in Feldman & Sakhartov 2021), instead of exercising such redeployment at the final time (as happened in this two-period numerical example). This is why this example, while helpful in providing transparency in how corporate diversification can increase risk above the level in undiversified firms, is generalized in the main model to a more longitudinal setting that, moreover, captures the essential determinants of the portfolio effect and of resource redeployability.

CORPORATE DIVERSIFICATION AND RISK: PORTFOLIO EFFECTS AND RESOURCE REDEPLOYABILITY

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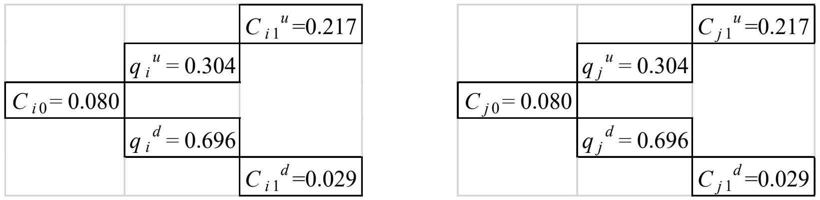

A. Dynamics of margins

		m_{i0}		
		0	0.5	1
$m_{i(-1)}$	0			
	0.5	0.193	0.195	0.193
	1			

uu

		m_{i1}		
		0	0.5	1
m_{i0}	0	0.217	0.212	0.207
	0.5	0.212	0.217	0.212
	1	0.207	0.212	0.217

ud

		m_{i1}		
		0	0.5	1
m_{i0}	0	0.029	0.118	0.207
	0.5	0.029	0.123	0.212
	1	0.028	0.123	0.217

du

		m_{i1}		
		0	0.5	1
m_{i0}	0	0.217	0.123	0.028
	0.5	0.212	0.123	0.029
	1	0.207	0.118	0.029

dd

		m_{i1}		
		0	0.5	1
m_{i0}	0	0.029	0.029	0.029
	0.5	0.029	0.029	0.029
	1	0.028	0.029	0.029

B. Context 1: Diversified firm with redeployability

		m_{i0}		
		0	1	2
$m_{i(-1)}$	0			
	1		0.160	
	2			

uu

		m_{i1}		
		0	1	2
m_{i0}	0			
	1		0.160	
	2			

ud

		m_{i1}		
		0	1	2
m_{i0}	0			
	1		0.123	
	2			

du

		m_{i1}		
		0	1	2
m_{i0}	0			
	1		0.123	
	2			

dd

		m_{i1}		
		0	1	2
m_{i0}	0			
	1		0.029	
	2			

C. Context 2: Diversified firm without redeployability

		m_{i0}		
		0	1	2
$m_{i(-1)}$	0			
	1			
	2			0.160

uu

		m_{i1}		
		0	1	2
m_{i0}	0			
	1			
	2			0.217

ud

		m_{i1}		
		0	1	2
m_{i0}	0			
	1			
	2			0.217

du

		m_{i1}		
		0	1	2
m_{i0}	0			
	1			
	2			0.029

dd

		m_{i1}		
		0	1	2
m_{i0}	0			
	1			
	2			0.029

D. Context 3: Undiversified firm

Figure A1. Estimation of the net cash flow accumulated by the firm in three alternative contexts

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ONLINE APPENDIX B: Estimation of corporate risk

In the focal context where the diversified firm counts on the optimal use economies of scope, at any time $t \in [0,T)$ the firm maximizes the net cash flow V_t it expects to accumulate over the remaining lifecycle of its resources. Formally, the firm's resource deployment choice m_{it} must comply with the following Bellman equation (Bellman 1957):

$$\left(m_{it}^{*} \middle| m_{it-\partial t}\right) = \arg\max_{m_{it}} \left\{ F_t \; + e^{-r\partial t} \mathrm{E}\left[V_{t+\partial t}\left(m_{it+\partial t}^{*} \mid m_{it}\right)\right]\right\}, \qquad \text{(B.1)}$$

where $r = 0.08$ is the risk-free interest rate used for temporal discounting. This equation can be restated for corporate value that the firm that previously made choice $m_{it-\partial t}$ and is currently making choice m_{it}^{*} will accumulate between the current time t and the end of the useful life of the firm's resources $t = T$:

$$V_t\left(m_{it}^{*} \mid m_{it-\partial t}\right) = \max_{m_{it}} \left\{ F_t \; + e^{-r\partial t} \mathrm{E}\left[V_{t+\partial t}\left(m_{it+\partial t}^{*} \mid m_{it}\right)\right]\right\}. \qquad \text{(B.2)}$$

In Equations B.1 and B.2, $\mathrm{E}\left[V_{t+\partial t}\left(m_{it+\partial t}^{*} \mid m_{it}\right)\right]$ is the corporate value that is expected to be accumulated between the immediate next time $t + \partial t$ and the end of the useful life of the firm's resources $t = T$, based on the information available to the firm at time t and conditioned on the considered current choice m_{it}. The use of $m_{it+\partial t}^{*}$, rather than $m_{it+\partial t}$, highlights that the same maximization procedure specified in Equation B.2 must be used to assess $V_{t+\partial t}\left(m_{it+\partial t}^{*} \mid m_{it}\right)$. The set over which $V_t\left(m_{it}^{*} \mid m_{it-\partial t}\right)$ is maximized is formed by estimating $V_t\left(m_{it} \mid m_{it-\partial t}\right)$ for all possible values of the current choice m_t and of the immediate previous choice $m_{it-\partial t}$:

$$V_t\left(m_{it} \mid m_{it-\partial t}\right) = F_t \; + e^{-r\partial t} \mathrm{E}\left[V_{t+\partial t}\left(m_{it+\partial t}^{*} \mid m_{it}\right)\right], \qquad \text{(B.3)}$$

where the current net cash flow F_t is defined with Equation 6.

Equation B.1 splits the analytically intractable estimation of resource allocation choices into a sequence of simpler problems that are amenable to a semi-analytical solution. Namely, corporate diversification is stated in a recursive form that uses backward induction to find an optimal resource allocation choice $\left(m_{it}^{*}\middle|m_{it-\partial t}\right)$ in each time that is conditioned on the immediate previous choice. The solution involves discretization of the continuous-time distribution for C_{it} and C_{jt}. Like Sakhartov & Folta (2014; 2015), this model follows Boyle, Evnine & Gibbs (1989) to discretize Equations 1–3 with a binomial lattice, which preserves the mean and the variance of the distribution if the number of time discretization steps N is sufficiently large and, thus, each time step $\partial t = T / N$ on the lattice is sufficiently short.[1] On this lattice, the next-period margins $C_{it+\partial t}$ and $C_{jt+\partial t}$ have four states: $C_{it+\partial t}^{u}$ and $C_{jt+\partial t}^{u}$ with probability q^{uu}; $C_{it+\partial t}^{u}$ and $C_{jt+\partial t}^{d}$ with probability q^{ud}; $C_{it+\partial t}^{d}$ and $C_{jt+\partial t}^{u}$ with probability q^{du}; or $C_{it+\partial t}^{d}$ and $C_{jt+\partial t}^{d}$ with probability q^{dd}. The formulas for the involved discretization parameters that are based on Boyle et al. (1989) are provided below:

$$C_{it+\partial t}^{u} = u_i C_{it} \qquad \text{(B.4)}$$

$$C_{it+\partial t}^{d} = d_i C_{it} \qquad \text{(B.5)}$$

$$u_i = e^{\sigma_i \partial t} \qquad \text{(B.6)}$$

$$d_i = 1/u_i \qquad \text{(B.7)}$$

$$C_{jt+\partial t}^{u} = u_j C_{jt} \qquad \text{(B.8)}$$

$$C_{jt+\partial t}^{d} = d_j C_{jt} \qquad \text{(B.9)}$$

$$u_j = e^{\sigma_j \partial t} \qquad \text{(B.10)}$$

$$d_j = 1/u_j \qquad \text{(B.11)}$$

$$q^{uu} = 0.25\left\{1 + \rho + \sqrt{\partial t}\left[\left(r - 0.5\sigma_i^2\right)/\sigma_i + \left(r - 0.5\sigma_j^2\right)/\sigma_j\right]\right\} \qquad \text{(B.12)}$$

[1] This study uses $N = 100$.

$$q^{ud} = 0.25\left\{1-\rho+\sqrt{\partial t}\left[\left(r-0.5\sigma_i^2\right)/\sigma_i-\left(r-0.5\sigma_j^2\right)/\sigma_j\right]\right\} \qquad \text{(B.13)}$$

$$q^{du} = 0.25\left\{1-\rho+\sqrt{\partial t}\left[-\left(r-0.5\sigma_i^2\right)/\sigma_i+\left(r-0.5\sigma_j^2\right)/\sigma_j\right]\right\} \qquad \text{(B.14)}$$

$$q^{dd} = 0.25\left\{1+\rho+\sqrt{\partial t}\left[-\left(r-0.5\sigma_i^2\right)/\sigma_i-\left(r-0.5\sigma_j^2\right)/\sigma_j\right]\right\}. \qquad \text{(B.15)}$$ [2]

With this discretization,

$$\begin{aligned}\mathrm{E}\left[V_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)\right] &= q^{uu}V^{uu}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)+q^{ud}V^{ud}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)\\ &\quad +q^{du}V^{du}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)+q^{dd}V^{dd}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)\end{aligned}, \qquad \text{(B.16)}$$

where, conditioned on the firm's current choice m_{it}, $V^{uu}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)$ is calculated using $C^u_{it+\partial t}$ and $C^u_{jt+\partial t}$; $V^{ud}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)$ is estimated using $C^u_{it+\partial t}$ and $C^d_{jt+\partial t}$; $V^{du}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)$ is assessed using $C^d_{it+\partial t}$ and $C^u_{jt+\partial t}$; and $V^{dd}_{t+\partial t}\left(m^*_{it+\partial t}\mid m_{it}\right)$ is computed using $C^d_{it+\partial t}$ and $C^d_{jt+\partial t}$.

The backward induction starts at the penultimate time $t = T-\partial t$ with the terminal condition $V_T = 0$ (*i.e.*, the firm's resources will have exhausted ability to generate cash flows by time $t = T$). The use of Equations B.1 and B.2 proceeds recursively backward in time with a step ∂t until time $t = 0$. In each step and for each possible realization of the margins, the two equations return respectively the optimal corporate value $V_t\left(m^*_{it}\middle|m_{it-\partial t}\right)$ and the optimal use of the firm's resources $\left(m^*_{it}\middle|m_{it-\partial t}\right)$ that is conditioned on their previous use. Although the backward induction retrieves the choices $\left(m^*_{it}\middle|m_{it-\partial t}\right)$ at each possible realization of the margins and over the whole lifecycle of the resources, the ensuing number of possible realizations for the random variable of the accumulated cash flow W (*i.e.*, the number of paths through which the margins can evolve) on the lattice with N steps is 4^N. With $N = 100$, that number is 4^{100}=$1.6069*10^{60}$. The estimation of variance using the whole sample of such realizations is obviously

[2] Just like the two-period example in ONLINE APPENDIX A, the full continuous-time model implies the equilibrium market because Equations 1 and 2 can be converted to Martingales by substituting r for μ_i and μ_j in those equations.

impossible. Meanwhile, the following Monte-Carlo simulation procedure is used to efficiently estimate the variance of W.

Using the transitional probabilities q^{uu}, q^{ud}, q^{du}, and q^{dd}, one million paths are simulated for margins C_{it} and C_{jt} over time $t \in [0, T)$. The use of the transition probabilities for simulating the sample of paths preserves the properties of the probability distribution for C_{it} and C_{jt} specified with Equations 1–3. Because in the baseline context $m_{i0-\partial t} = 0.5$ (*i.e.*, the firm is initially diversified), starting with the initial time $t = 0$ and using the derived optimal conditional use of resources $\left(m_{it}^{*} \middle| m_{it-\partial t}\right)$, the net cash flow is estimated in each increment of a path based on Equation 6 going recursively forward in time on that path. With this approach, each of the 1,000,000 sampled realizations W^x for W can be expressed as follows (the superscript x indexes paths and thus realizations of F_t and of W):

$$W^x = \sum_{t=0}^{t=T} e^{-rt} F_t^x \,. \qquad \text{(B.17)}$$

The simulated sample of W^x's can then be used to assess corporate risk:

$$R = \text{Var}\left[W^x\right]. \qquad \text{(B.18)}$$

Finally, by imposing the condition $m_{it}^{*} = 1$ for all $t \in [0, T)$, the same procedure can be used to estimate corporate risk in the undiversified firm; with another condition $m_{it}^{*} = 0.5$ for all $t \in [0, T)$, the procedure returns corporate risk in the diversified firm when redeployability is absent.

Summary of the step-by-step procedure for estimating corporate risk

1. Select a sufficiently large number of the time discretization steps N. This study uses $N = 100$. Because the binomial approximation of the geometric Brownian motion is known to make the option value estimated with that approximation converge from below to the true option value with the increase of the time discretization steps, the common heuristic is to increase N until the

respective changes in the estimated option value get insignificant. An additional notorious constraint on N is that too low values of N can turn the transition probabilities negative with Equations B.12-B.15.

2. Using Equations B.4-B.11, create the bivariate binomial lattice. This lattice should be filled in with all values of C_{it} and C_{jt} over time $t \in [0, T)$.
3. Using Equations B.12-B.15, estimate the transitional probabilities q^{uu}, q^{ud}, q^{du}, and q^{dd}.
4. Set terminal value $V_T = 0$.
5. Start at the penultimate time $t = T - \partial t$ and, using Equation B.3, estimate $V_{T-\partial t}\left(m_{iT-\partial t} \mid m_{iT-2\partial t}\right)$ for all feasible combinations of $m_{iT-\partial t}$ (*i.e.*, the current choice) and $m_{iT-2\partial t}$ (*i.e.*, the immediate previous choice) and on all nodes of the lattice feasible at time $t = T - \partial t$ (*i.e.*, $C_{iT-\partial t}$ and $C_{jT-\partial t}$).
6. For each feasible value of $m_{iT-2\partial t}$ (*i.e.*, the immediate previous choice) considered in Step 5, solve Equations B.1 and B.2 with respect to $m_{iT-\partial t}$ (*i.e.*, the current choice) considered in Step 5. Store all the conditional solutions $\left(m^*_{iT-\partial t} \middle| m_{iT-2\partial t}\right)$ and $V_{T-\partial t}\left(m^*_{iT-\partial t} \middle| m_{iT-2\partial t}\right)$.
7. Proceed to the immediate previous time $t = T - 2\partial t$. Use $V_{T-\partial t}\left(m^*_{iT-\partial t} \middle| m_{iT-2\partial t}\right)$'s estimated in Step 6 and, using Equation B.3, estimate $V_{T-\partial t}\left(m_{iT-2\partial t} \mid m_{iT-3\partial t}\right)$ for all feasible combinations of $m_{iT-2\partial t}$ (*i.e.*, the new current choice) and $m_{iT-3\partial t}$ (*i.e.*, the new immediate previous choice) and on all nodes of the lattice feasible at time $t = T - 2\partial t$ (*i.e.*, $C_{iT-2\partial t}$ and $C_{jT-2\partial t}$).

8. For each feasible value of $m_{iT-3\partial t}$ (*i.e.*, the immediate previous choice) considered in Step 7, solve Equations B.1 and B.2 with respect to $m_{iT-2\partial t}$ (*i.e.*, the current choice) considered in Step 7. Store all the conditional solutions $\left(m^*_{iT-2\partial t} \middle| m_{iT-3\partial t}\right)$ and $V_{T-\partial t}\left(m^*_{iT-2\partial t} \middle| m_{iT-3\partial t}\right)$.
9. Procced with Steps 7 and 8 recursively backward in time down to time $t = 0$. When time $t = 0$ is reached as the current time, the backward induction is completed. The stored values of $\left(m^*_{it} \middle| m_{it-\partial t}\right)$ for $t \in \left[0, T - \partial t\right]$ represent the main outcome of Steps 5-9 to be used in next steps.
10. Using the initial margins C_{i0} and C_{j0} and the transitional probabilities q^{uu}, q^{ud}, q^{du}, and q^{dd}, simulate 1,000,000 paths for margins C_{it} and C_{jt} over time $t \in \left[0, T\right)$.
11. On each path x simulated in Step 10, start with the condition $m_{i0-\partial t} = 0.5$ (*i.e.*, the firm is initially diversified) and using the stored values of $\left(m^*_{it} \middle| m_{it-\partial t}\right)$, sequentially retrieve the unconditional resource deployment choices m^*_{it} by going from time $t = 0$ to time $t = T - \partial t$.
12. Use Equation 6 and unconditional choices m^*_{it} retrieved in Step 11 to estimate the current cash flow F^x_t on path x going sequentially from time $t = 0$ to time $t = T - \partial t$.
13. Repeat Steps 11 and 12 for the remaining 999,999 paths.
14. Use Equation B.17 to calculate realizations W^x on 1,000,000 paths.
15. Use Equation B.18 to assess corporate risk in the diversified firm with economies of scope.
16. To assess risk in the diversified firm without redeployability, fix $m^*_{it} = 0.5$ for all $t \in \left[0, T\right)$ and redo Steps 12-15.
17. To assess risk in the undiversified firm, fix $m^*_{it} = 1$ for all $t \in \left[0, T\right)$ and redo Steps 12-15.

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ONLINE APPENDIX C: Additional results

Redeployment cost and return correlation

A few additional observations, not discussed in the results section of the paper, can be made based on Figure 1. For example, when return correlation is not strongly positive in Panel A, the left end of the solid green line converts from convex to concave and the left end of the solid blue line even bends down. This tendency derives from the dependence of the optimal timing of resource redeployment on correlation and on the redeployment cost. With less-positive correlation and with low redeployment costs, redeployment of resources to a currently outperforming market (*i.e.*, i or j) occurs earlier (those additional results on the odds of redeployment are available from the author upon request). In turn, earlier redeployment, before the range of future scenarios expands too broad, entails lower variation of cash flows, thus explaining why the left ends of the solid green and blue lines bend down. In Panel C, the red area at the bottom that depicts the strongest enhancement of risk is isolated from the left margin by a thin stripe where the enhancement of risk is lower. This pattern generalizes the observation in Panel A that, with low redeployment costs and less-positive correlation, the relationship between risk and the redeployment cost becomes concave.

Redeployment cost and current return advantage

Panel A of Figure C1 shows how corporate risk (*i.e.*, the vertical axis) depends on the redeployment cost (*i.e.*, the horizontal axis) when the current return advantage in market j takes three values: (a) negative advantage $\left(C_{j0} - C_{i0}\right)/C_{i0} = -87.5\%$ when $C_{i0} = 0.08$ and $C_{j0} = 0.01$ (*i.e.*, the blue lines); (b) zero advantage $\left(C_{j0} - C_{i0}\right)/C_{i0} = 0.0\%$ when $C_{i0} = C_{j0} = 0.08$ (*i.e.*, the green lines), and (c) positive advantage $\left(C_{j0} - C_{i0}\right)/C_{i0} = 100.0\%$ when $C_{i0} = 0.08$ and $C_{j0} = 0.16$ (*i.e.*, the red lines).[1] The solid lines depict risk in the diversified firm that has redeployabilty. The broken lines illustrate corporate risk

[1] The current return advantage is a continuous variable. None of its three discrete values in Panel A, including zero, has a special theoretical interpretation, which is demonstrated in other panels of Figure C1 where the continuity of the variable is restored.

that emulates risk in portfolio diversification because redeployability is absent. The dash-dot line indicates risk in the undiversified firm that is void of either risk pooling or redeployability because it always focusses on its single business. The following six results in Panel A are noteworthy. First, even without redeployability, risk can be higher in the diversified firm than in the undiversified firm: the broken red line is above the dash-dot line. Second, absent redeployability, the current return advantage raises risk in the diversified firm. This second result is seen in the pattern with which the broken red line (*i.e*., the positive current return advantage) is above the broken green line (*i.e*., zero current advantage) that is, in turn, above the broken blue line (*i.e*., the negative current advantage). Third, that the broken lines are horizontal shows that, when redeployability is absent, risk is insensitive to the redeployment cost. Fourth, redeployability cannot reduce risk in the diversified firm because the solid lines never get below the respective broken lines. Fifth, risk in the diversified firm that has redeployability can be considerably higher than risk the same firm would face if it did not diversify. This idea is illustrated in the substantial elevation of the solid red line over the dash-dot line. Finally, the downward trends in the solid lines (except for the left end of the solid red line) indicate that, with redeployability, the redeployment cost tends to reduce risk.

Insert Figure C1 about here

The main result above is that *risk faced in corporate diversification can remarkably exceed risk in the undiversified firm*. This result contrasts with the popular idea that corporate diversification, just like portfolio diversification, cuts risk. Panel A of Figure C1 traces possible risk enhancement to (a) the risk-pooling effect that is shared by portfolio and corporate diversification, and (b) the effect of economies of scope that is unique to corporate diversification. The risk-pooling effect (*i.e*., ‘a’ above) can be separated from the effect of economies of scope (*i.e*., ‘b’ above) by comparing positions of the broken lines with each other and with the dash-dot line because, on all these lines, the firm is void of economies of scope. The result that the broken red line is above the broken green line that is, in turn, above the broken blue line indicates that the current return advantage reduces the efficacy of risk pooling (*i.e*., increases risk). Also, that the broken red line is above the dash-dot line shows that, even without economies of scope, lower efficacy of risk pooling can eventually grow into the rise of risk over the level in the undiversified firm. Although the current return advantage in the business to which a firm diversifies did not feature directly in previous analogies between portfolio and corporate diversification, this effect can be explained with the following formal logic. If C_{it} *and* C_{jt} are two identically distributed uncorrelated variables such that $C_{i0} = C_{j0}$, $\mu_i = \mu_j = \mu$, $\sigma_i = \sigma_j = \sigma$, and $\rho = 0$, variance of the net cash flow of the firm that is evenly diversified between the two markets and that is void of economies of scope is $\mathrm{Var}\left[F_t\right] = (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) + (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) = 0.5 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. The net cash flow of the firm that is focused on market i is $\mathrm{Var}\left[F_t\right] = C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. If the initial return C_{jt} is scaled up by a positive factor $a > 1$ thus introducing the current return advantage $(a-1)$, variance of the net cash flow of the firm that is evenly diversified between the two markets and void of economies of scope is $\mathrm{Var}\left[F_t\right] = (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) + (0.5)^2 a^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) = 0.25(1 + a^2) C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. This follows that any $a > \sqrt{3}$ makes variance in the net cash flow higher in the diversified firm without economies of scope than in the undiversified firm. This rise of variance happens every period, thus determining the monotonic positive effect of the current return advantage on corporate risk. The effect of economies of scope on risk (*i.e.*, ‘b’ above) can be assessed by comparing positions of lines of the same color that have the same risk-pooling characteristics and, thus, their comparison is free of the risk-pooling effect (*i.e.*, ‘a’ above). An important finding in that respect is that *redeployability cannot reduce risk.* Although this result contrasts with the speculations that redeployability reduces risk, it complies with the idea that variance of value of a project that contains a to-be-exercised real option is higher than variance of value of the same project without that option (Carlson et al. 2006). In other words, future use of redeployment increases variance of the net cash flow. The strongest increase of risk by redeployability with the positive current return advantage and with low redeployment costs compounds the reduction of the efficacy of risk pooling with the positive current return advantage and explains why the left end of the solid red line stands far above the dash-dot line.[2]

Panel B of Figure C1 generalizes the risk-pooling effect that is shared by portfolio and corporate diversification and was revealed in the elevation of the dash-dot line over the broken lines in Panel A to a continuum of the current return advantage that is now shown on the vertical axis. The horizontal axis still captures the redeployment cost. The variation of color in Panel B depicts the efficacy of risk pooling—the difference between risk in the undiversified firm and risk in the diversified firm without redeployability; the scale for this variation is indicated in a vertical sidebar next to the filled contour map. Because the undiversified firm is void of redeployability, the efficacy of risk pooling is free of the effect of economies of scope on risk. The following three patterns in Panel B reconfirm results in Panel A. First, the filled contour map changes color from red at the bottom to dark blue at the top, thus indicating that the current return advantage monotonically increases risk and therefore reduces the efficacy of risk pooling (*i.e*., the second result diagnosed in Panel A). Second, at the top of Panel B where the current return advantage is highly-positive, the efficacy of risk pooling turns negative—corporate diversification adds extra risk (*i.e*., the first result reported with Panel A). Third, absent redeployability, the redeployment cost expectedly has no effect on the efficacy of risk pooling (*i.e*., the third result mentioned with Panel A).

[2] Although Panel A does not show that risk due to redeployability increases monotonically in the current return advantage, the strongest rise of risk occurs when that advantage is strongly positive (*i.e*., the two red lines) because such an advantage intensifies the use of redeployment (Sakhartov & Reuer 2021). The redeployment cost reduces risk (*i.e*., relatedness increases risk) because that cost suppresses redeployment (Feldman & Sakhartov, 2021). A horizontal increment in the right-hand side of each solid line coincides with the respective broken line because high redeployment costs preclude redeployment. In turn, a horizontal increment in the left-hand side of the solid blue line coincides with the dash-dot line because the negative current return advantage and low redeployment costs together make the firm instantly (*i.e*., at time $t = 0$) redeploy all resources to market i and permanently stay in that market, thus making the risk profile of the firm indistinguishable from the profile of the undiversified firm. Finally, the difference in shape of the left end between the solid green and the solid red lines can be explained as follows. Without strong current return advantage (*i.e*., on the solid green line), the increase of the redeployment cost from $S = 0$ to $S = 15$ reduces the probability of redeployment from 100% to 92%. This decrease in the odds of redeployment cuts the variation of the accumulated cash flow, thus explaining the downward slope in the green line between $S = 0$ and $S = 15$. By contrast, with positive current return advantage (*i.e*., on the solid red line), the increase of the redeployment cost from $S = 0$ to $S = 15$ does not change the odds of redeployment—in both cases the firm instantly (*i.e*., at time $t = 0$) redeploys all resources to one market with the probability of 100% (those additional results on the odds of resource redeployment are available from the author upon request). The change in risk occurs only because the nontrivial redeployment cost of $S = 15$ raises the variance in the outcome of such redeployment over the case $S = 0$, thus corresponding to the upward slope in the respective increment of the solid red line.

Panel C of Figure C1 extends the risk-enhancement effect of redeployability that is unique to corporate diversification and was shown in the divergence between lines of the same color in Panel A to a continuum of the current return advantage that is now reflected on the vertical axis. The horizontal axis continues to display the redeployment cost. The variation of color in Panel C depicts the enhancement of risk by redeployability—the difference between risk in the diversified firm with redeployability and without redeployability. Because both parts in that difference include the same risk-pooling effect in the diversified firm, the response variable in Panel C is net of that effect. The following two patterns corroborate results from Panel A. First, the filled contour map does not have negative values, thus revealing that redeployability cannot cut risk (*i.e*., the fourth result reported with Panel A). Second, risk unique to redeployability declines in the redeployment cost (*i.e*., the last result observed in Panel A).

Although the main insight from Panel C is that redeployability can only raise rather than cut risk; the result that redeployment costs decrease the enhancement of risk by redeployability can be compared to the previous speculations that relatedness raises risk. Because relatedness reduces redeployment costs, this second result in Panel C indeed suggests that corporate risk is positively associated with relatedness. However, this positive association is derived in Panel C without assuming that relatedness is synonymous with the correlation of returns and thus reduces the efficacy of risk pooling. In addition to revealing that such an assumption is not necessary, Panel C attributes the positive effect of relatedness on risk to the use of redeployability, a type of economies of scope, rather than to the hampering of the pooling of risk.

The variation of color in Panel D of Figure C1 shows the full effect of corporate diversification on risk—the difference between risk in the diversified firm with redeployability and risk in the undiversified firm. The vertical axis in the panel displays the current return advantage, whereas the horizontal axis captures the redeployment cost. The dash-dot white line separates the area where the response variable is positive from the area where it is negative. The existence of the area above the dash-dot line demonstrates that at least some combinations of the current return advantage and the redeployment cost make risk in the diversified firm with redeployability exceed risk in the undiversified firm. Furthermore, the red-colored part of that area indicates that risk in the diversified firm with redeployability can be much higher than risk the same firm would face without diversification, thus generalizing the main result in Panel A.

Although Figure C1 delivers multiple insights, the most revealing finding is the presence of the sizeable area above the dash-dot white line in Panel D of the figure. This result means that there are copious conditions, with which the firm that diversifies across geographic or product markets ends up with risk above the level without such diversification. This finding contrasts with the popular idea that corporate diversification reduces risk. Furthermore, conditions with which such diversification increases risk can be reformulated in terms of relatedness, which reduces the redeployment cost and was mentioned occasionally in research on risk, and of the current return advantage, which was not previously used to explain risk. Notably, with weak relatedness, corporate diversification adds risk only when the firm diversifies into a business with a highly-positive current return advantages; with strong relatedness, diversification adds risk whenever the firm diversifies into a business that does not underperform the firm's original business by a lot at the time when such diversification starts. Conversely, only when a strong risk-pooling effect farther from the top margin of Panel D (*cf*. Panel B) is not offset by a small rise in risk due to redeployability outside of the top left corner of Panel D (*cf*. Panel C), can corporate diversification reduce risk as was expected in previous research.

Redeployment cost and return volatility

Panel A of Figure C2 shows how risk (*i.e*., the vertical axis) derives from the redeployment cost (*i.e*., the horizontal axis) when return volatility in market j takes three values: (a) low volatility $\sigma_j = 0.05$ (*i.e*., the blue lines); (b) medium volatility $\sigma_j = 1.00$ (*i.e*., the green lines), and (c) high volatility $\sigma_j = 1.95$ (*i.e*., the red lines).[3] The broken lines display risk when economies of scope are absent. The dash-dot line reveals risk in the undiversified firm that is void of either risk pooling or economies of scope. The following three results in Panel A are unique to Figure C2. First, absent economies of scope, return

[3] To compare the diversified firm to the undiversified firm that focuses on market i, volatility in i is held to its default value.

volatility increases risk in the diversified firm (*i.e.*, the broken red line is above the broken green line that is, in turn, above the broken blue line). Second, absent economies of scope, risk in the diversified firm with high return volatility is higher than risk in the undiversified firm (*i.e.*, the broken red line is above the dash-dot line). Finally, the enhancement of risk due to redeployability increases in return volatility (*i.e.*, the divergence between the red lines is greater than the divergence between the green lines that is, in turn, greater than the divergence between the blue lines).[4]

Insert Figure C2 about here

Like in Figure C1, the effect of corporate diversification on risk in Panel A of Figure C2 can be split into the risk-pooling effect and the effect of economies of scope. The risk-pooling effect is diagnosed by comparing positions of the broken lines and of the dash-dot line, on which the firm is void of economies of scope. Even though volatility of returns in the business to which a firm diversifies did not feature directly in previous analogies between portfolio and corporate diversification, this effect can be explained with the following formal logic. If C_{it} *and* C_{jt} are two identically distributed uncorrelated variables such that $C_{i0} = C_{j0}$, $\mu_i = \mu_j = \mu$, $\sigma_i = \sigma_j = \sigma$, and $\rho = 0$, variance of the net cash flow of the firm that is evenly diversified between the two markets and that is void of economies of scope is $\mathrm{Var}\left[F_t\right] = (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) + (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. Variance of the net cash flow of the firm that is focused on market i is $\mathrm{Var}\left[F_t\right] = C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. If volatility σ_j of the market into which the firm diversifies is scaled up by a positive factor $a > 1$, variance of the net cash flow of the firm that is evenly diversified between the two markets and that is void of economies of scope becomes $\mathrm{Var}\left[F_t\right] = (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{\sigma^2 t} - 1) + (0.5)^2 C_{i0}^2 e^{2\mu t}(e^{a^2\sigma^2 t} - 1)$. This is easy to verify that, with values of a such that $(e^{a^2\sigma^2 t} - 1) > 3(e^{\sigma^2 t} - 1)$, variance of the net cash flow of the firm that is evenly diversified

[4] Besides its unique results, Panel A of Figure C2 reconfirms the following results from Figure C1. First, absent redeployability, risk is indifferent to redeployment costs. Second, redeployability cannot reduce risk. Third, risk can be much higher in the diversified firm with redeployability than in the undiversified firm. Finally, with redeployabilty, risk declines in the redeployment cost.

between the two markets and is void of economies of scope becomes higher than in the undiversified firm. This rise of variance happens every period, aggregating in the variance of the net cash flow accumulated by the firm over the lifecycle of its resources. The effect of economies of scope on risk is revealed in the divergence of solid lines from broken lines of the same color in Panel A of Figure C2. A greater divergence takes place on lines with higher return volatility, leading to the positive association of risk due to redeployability with return volatility (*i.e.*, the third result raised in the previous paragraph).[5]

Panel B of Figure C2 extends the risk-pooling effect to a continuum of values of return volatility $\sigma_j \in [0.05, 1.95]$, that is shown in the vertical axis. The horizontal axis captures the redeployment cost. The variation of color in Panel B displays the efficacy of risk pooling—the difference between risk in the undiversified firm and risk in the diversified firm without redeployability. This response variable is free of the effect of economies of scope on risk. That the filled contour map changes color from red at the bottom to dark blue at the top confirms that return volatility monotonically increases risk thus reducing the efficacy of risk pooling (*i.e.*, the first result considered in Panel A). Moreover, at the top of the panel where return volatility is high, the efficacy of risk pooling is negative, thus suggesting that corporate diversification adds extra risk in that area (*i.e.*, the second result observed in Panel A of Figure C2).

Panel C of Figure C2 extends the risk-enhancement effect of redeployability to a continuum of return volatility, $\sigma_j \in [0.05, 1.95]$, that is captured in the vertical axis. The horizontal axis still shows the redeployment cost. The variation of color in Panel C reveals the difference between risk in the diversified firm with redeployability and risk in the diversified firm without redeployability. This response variable is free of the risk-pooling effect. The color patten in Panel C confirms the results in Panel A that the enhancement of risk by redeployability grows in return volatility and declines in the redeployment cost.

The variation of color in Panel D in Figure C2 displays the total effect of corporate diversification on risk—the difference in risk between the diversified and the undiversified firm. The vertical axis shows

[5] The greatest divergence of the solid red lines from each other corresponds to the finding in Sakhartov & Reuer (2021) that return volatility increases the intensity of resource redeployment, which in turn induces higher variation in cash flows.

return volatility. The horizontal axis indicates the redeployment cost. The dash-dot white line splits the map into the areas where the response variable is positive and negative. Like in Figure 1, the presence of the area above the dash-dot line indicates that, at least with some combinations of return volatility and of the redeployment cost, risk in the diversified firm with redeployability surpasses risk in the undiversified firm. Moreover, the red-colored area in the top left corner reveals that risk in the diversified firm with redeployability can be remarkably higher than risk the same firm would face without diversification.

The fact that Panel D of Figure C2 has a vast area above the dash-dot white line demonstrates that there are abundant conditions, with which the diversified firm faces risk that is higher than in the undiversified firm. Given that relatedness reduces redeployment costs, such conditions can be stated in terms of relatedness and of return volatility. Specifically, with weak relatedness (*i.e*., with high redeployment costs), corporate diversification adds risk when the firm diversifies into a highly-volatile market; with strong relatedness (*i.e*., with low redeployment costs), diversification adds risk when the firm diversifies into a business with at least moderate volatility.

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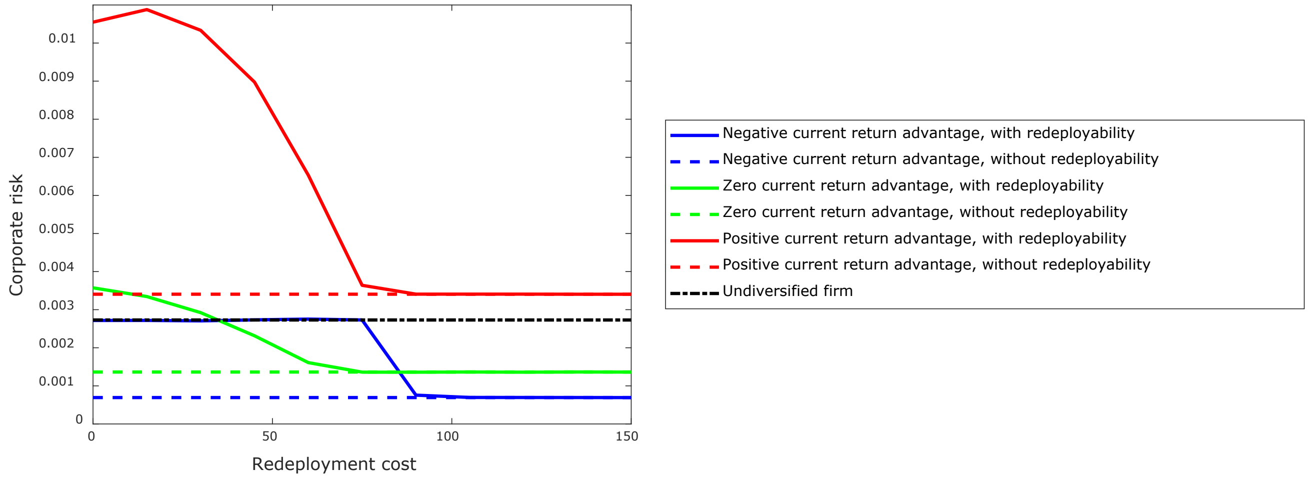
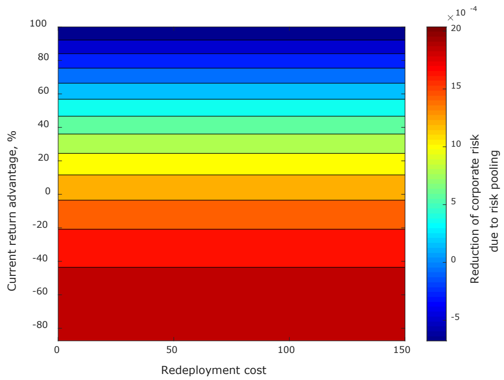
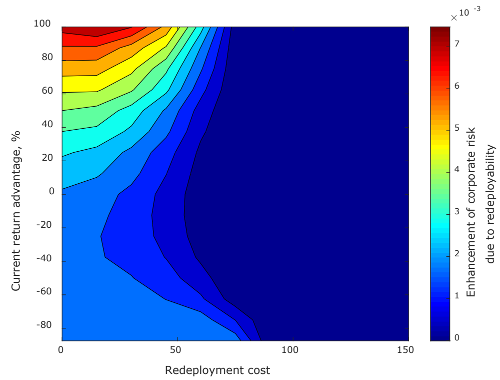
Feldman ER, Sakhartov AV (2021) Resource Redeployment and Divestiture as Strategic Alternatives. *Organ. Sci.* Forthcoming.



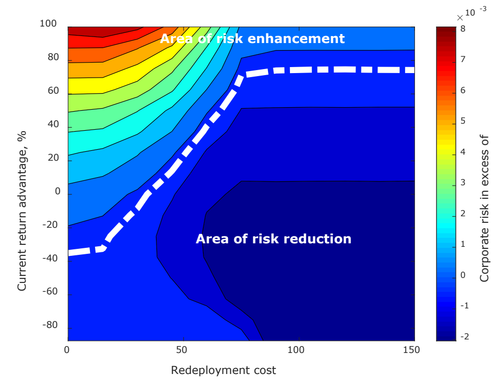
A. Corporate risk for various combinations of the redeployment cost and of the current return advantage



B. Risk reduction due to risk pooling

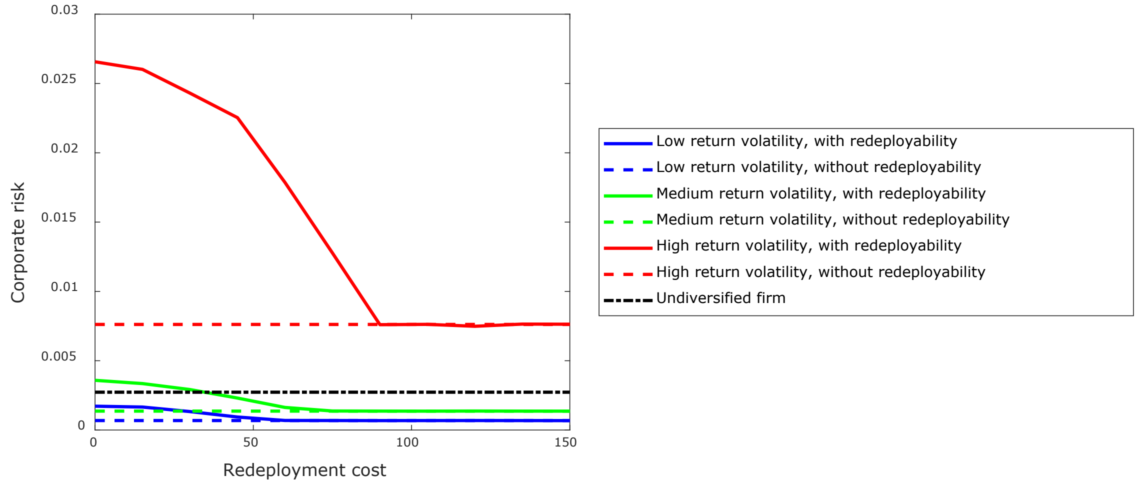


C. Risk enhancement due to redeployability

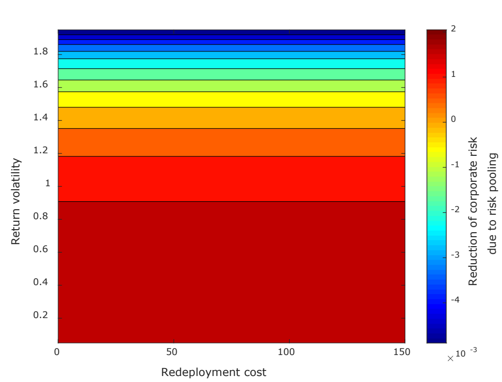


D. Corporate risk in excess of risk in undiversified firm

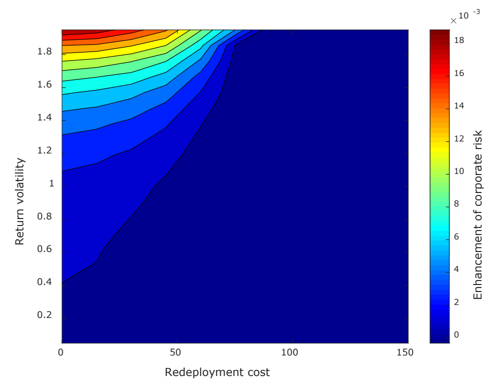
Figure C1. Implications of the redeployment cost and of the current return advantage for corporate risk



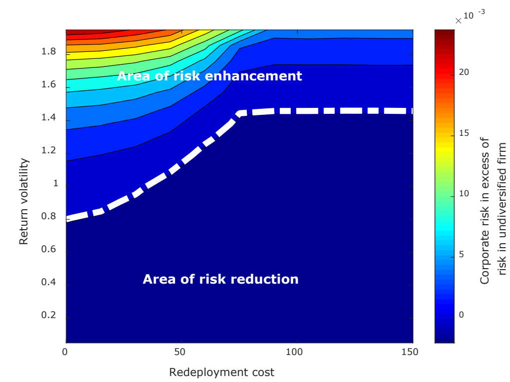
A. Corporate risk for various combinations of the redeployment cost and of return volatility



B. Risk reduction due to risk pooling



C. Risk enhancement due to redeployability



D. Corporate risk in excess of risk in undiversified firm

Figure C2. Implications of the redeployment cost and of return volatility for corporate risk