The Value of Information in Container Transport - Online Appendix

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Abstract

Planning the transport of maritime containers from the sea port to final inland destinations is challenged by uncertainties regarding the time the container is released for further transport and the transit time from the port to its final destination. This paper assesses the value of information in container transport in terms of efficiency and reliability. The analysis uses a stylized single period model where a decision maker allocates released containers to two transport modes (slow, low price, no flexible departure times versus fast, high price, flexible departure times), where the decision maker plans the departure time of the inflexible mode. We construct Pareto frontiers and the corresponding Pareto optimal decisions under various information scenarios and show that the Pareto frontiers move in a favorable direction when the level of information increases. The mathematical results are explained and illustrated by means of a numerical example involving barge transport. We also perform a sensitivity analysis and study the impact of erroneous information on efficiency and reliability based on a numerical analysis.

Keywords: Intermodal transport; value of information; Pareto optimality; single period model

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A Mathematical Proofs

A.1 Proof of Lemma 1

Given γ and t_0, the expected transit costs are given by

$$\gamma \text{Prob}(t \leq t_0) c_{\text{barge}} + \gamma \text{Prob}(t > t_0) c_{\text{rctruck}} + (1-\gamma) c_{\text{truck}} = \\ \gamma(1-\alpha) c_{\text{barge}} + \gamma \alpha c_{\text{rctruck}} + (1-\gamma) c_{\text{truck}}.$$

By using (1), we obtain the efficiency (3) where θ is defined as in (2). The reliability is given by

$$\rho_2(\alpha, \gamma) = \gamma \text{Prob}(t + \tau_{\text{rctruck}} \leq T \, \wedge \, t > t_0) +$$

$$\gamma \text{Prob}(t_0 + \tau_{\text{barge}} \leq T \, \wedge \, t \leq t_0) +$$

$$(1-\gamma) \text{Prob}(t + \tau_{\text{truck}} \leq T) =$$

$$\gamma \int_{t_0}^{T} G_{\text{rctruck}}(T-t) f(t) \, dt + \gamma(1-\alpha) G_{\text{barge}}(T-t_0) +$$

$$(1-\gamma) \int_{0}^{T} G_{\text{truck}}(T-t) f(t) \, dt. \ \square$$

A.2 Proof of Lemma 2

Observe that

$$\frac{d\rho(\alpha)}{d\alpha} = \frac{d\rho(\alpha)}{dt_0} \frac{dt_0}{d\alpha} =$$

$$\{G_{\text{rctruck}}(T-t_0) f(t_0) + g_{\text{barge}}(T-t_0) F(t_0) - G_{\text{barge}}(T-t_0) f(t_0)\} \frac{-1}{f(t_0)} \leq 0,$$

which implies that $\alpha \mapsto \rho(\alpha)$ is decreasing. Further,

$$\rho(1) = \rho_0 - \int_0^T G_{\mathrm{rctruck}}(T-t) f(t)\, dt \geq 0,$$

and $\rho(\alpha) \geq 0$ follows from

$$\int_{t_0}^T G_{\mathrm{rctruck}}(T-t) f(t)\, dt + \int_0^{t_0} G_{\mathrm{barge}}(T-t_0) f(t)\, dt \leq$$

$$\int_{t_0}^T G_{\mathrm{rctruck}}(T-t) f(t)\, dt + \int_0^{t_0} G_{\mathrm{barge}}(T-t) f(t)\, dt \leq \int_0^T G_{\mathrm{truck}}(T-t) f(t) = \rho_0.$$

The first inequality holds true since $G_{\mathrm{barge}}(T-t_0) \leq G_{\mathrm{barge}}(T-t)$ for $0 \leq t \leq t_0$ and the second inequality follows from $G_{\mathrm{barge}} \leq G_{\mathrm{rctruck}} \leq G_{\mathrm{truck}}$. $\square$

A.3 Proof of Lemma 3

When the shipper has no information, then the costs per container are equal to c_{rctruck}, so $\sigma_1 = 0$. The expected fraction of containers that reach their destination in time is equal to $\rho_1 = \mathrm{Prob}(t + \tau_{\mathrm{rctruck}} \leq T) = \int_0^T G_{\mathrm{rctruck}}(T-t) f(t)\, dt$. $\square$

A.4 Proof of Lemma 4

First observe that if we substitute (8) in (4), we get

$$\rho_2(\alpha, \gamma) = \gamma \int_{t_0}^T G_{\mathrm{rctruck}}(T-t) f(t)\, dt + \gamma G_{\mathrm{barge}}(T-t_0) F(t_0) +$$

$$(1-\gamma)\int_0^T G_{\text{truck}}(T-t)f(t)\,dt =$$

$$\sum_{k=1}^n w_k\gamma\int_{t_0}^T G_{\text{rctruck}}(T-t)f_k(t)\,dt + \sum_{k=1}^n w_k\gamma G_{\text{barge}}(T-t_0)F_k(t_0) +$$

$$\sum_{k=1}^n w_k(1-\gamma)\int_0^T G_{\text{truck}}(T-t)f_k(t)\,dt.$$

The decision maker is able to plan the amount of containers to be shipped by barge per category of containers. In other words, we may introduce decision parameters $\vec{\gamma} = (\gamma_1, \ldots, \gamma_n)$ and define

$$\begin{aligned}\rho_3(\alpha,\vec{\gamma}) = \sum_{k=1}^n w_k\gamma_k\int_{t_0}^T G_{\text{rctruck}}(T-t)f_k(t)\,dt &+ \\ \sum_{k=1}^n w_k\gamma_k G_{\text{barge}}(T-t_0)F_k(t_0) &+ \\ \sum_{k=1}^n w_k(1-\gamma_k)\int_0^T G_{\text{truck}}(T-t)f_k(t)\,dt&.\end{aligned}$$

By substituting (8) in (3), we may now write

$$\sigma_2(\alpha,\gamma) = 1-\theta+\sum_{k=1}^n w_k\gamma(\theta-\alpha_k),$$

and we define

$$\sigma_3(\alpha,\vec{\gamma}) = 1-\theta+\sum_{k=1}^n w_k\gamma_k(\theta-\alpha_k). \quad \square$$

A.5 Proof of Lemma 5

The expected costs read $\gamma(1-\alpha)c_{\text{barge}} + (1-\gamma(1-\alpha))c_{\text{truck}}$, so expected efficiency equals $\sigma_4(\alpha,\gamma) = 1-\theta+\theta\gamma(1-\alpha)$. The expected fraction of containers that arrive in time at the customer reads

$$\rho_4(\alpha,\gamma) = \gamma(1-\alpha)G_{\text{barge}}(T-t_0)+$$

$$\int_0^{t_\gamma} G_{\text{truck}}(T-t)f(t)\,dt + \int_{t_0}^{T} G_{\text{truck}}(T-t)f(t)\,dt \quad \square$$

A.6 Proof of Lemma 6

Observe that

$$\frac{\partial\rho_4(\alpha,\gamma)}{\partial\gamma} = (1-\alpha)G_{\text{barge}}(T-t_0) + G_{\text{truck}}(T-t_\gamma)f(t_\gamma)\frac{dt_\gamma}{d\gamma} =$$

$$(1-\alpha)\{G_{\text{barge}}(T-t_0) - G_{\text{truck}}(T-t_\gamma)\} \leq 0$$

because $\frac{dt_\gamma}{d\gamma} = -\frac{F(t_0)}{f(t_\gamma)}$ and $G_{\text{truck}}(T-t_\gamma) \geq G_{\text{truck}}(T-t_0) \geq G_{\text{barge}}(T-t_0)$ while $0 \leq t_\gamma \leq t_0$. Moreover, $\rho_4(\alpha,0) = \rho_0$. $\square$

A.7 Proof of Theorem 1

We first prove two lemmas.

Lemma 8 *Let $g(t) \geq 0$ be decreasing and assume that $M(t) = \int_0^t \mu(x)\,dx \geq 0$ for* *all* $0 \leq t \leq t_0$ *and that* $M(t_0) = 0$. *Then*

$$\int_0^{t_0} g(x)\mu(x)\,dx \geq 0.$$

Proof. We may approximate $g(t)$ by nonnegative decreasing step functions, so it suffices to show the lemma for $g(t) = \sum_{k=1}^m g_k E_k(t)$, where $0 = a_0 < a_1 < \cdots < a_m = t_0$ is a partition and $E_k(t)$ equals 1 for $a_{k-1} \leq t < a_k$ and equals 0 elsewhere. As a result,

$$\int_0^{t_0} g(x)\mu(x)\,dx =$$

$$\sum_{k=1}^m g_k\{M(a_k) - M(a_{k-1})\} = g_m M(a_m) + \sum_{k=1}^{m-1}(g_k - g_{k+1})M(a_k) - g_1 M(0) \geq 0,$$

since $g_k \geq g_{k+1} \geq 0$, $M(a_k) \geq 0$, and $M(0) = 0$. □

Lemma 9 *If we put*

$$\gamma = \frac{1}{1-\alpha}\sum_{k=1}^n w_k\gamma_k(1-\alpha_k), \tag{19}$$

then $\rho_3(\alpha, \vec{\gamma}) \leq \rho_4(\alpha, \gamma)$ *and* $\sigma_3(\alpha, \vec{\gamma}) \leq \sigma_4(\alpha, \gamma)$.

Proof. If we assume that γ is defined as in (19), then

$$\sigma_4(\alpha,\gamma) - \sigma_3(\alpha,\vec{\gamma}) = \gamma\theta(1-\alpha) - \sum_{k=1}^n w_k\gamma_k(\theta-\alpha_k) = \theta\sum_{k=1}^n w_k\gamma_k(1-\alpha_k) - \sum_{k=1}^n w_k\gamma_k(\theta-\alpha_k) \geq 0.$$

We now consider, using the expression for γ as in (19),

$$\rho_4(\alpha,\gamma)-\rho_3(\alpha,\vec{\gamma}) = \int_0^{t_\gamma} G_{\mathrm{truck}}(T-t)f(t)\,dt+\gamma(1-\alpha)G_{\mathrm{barge}}(T-t_0)+\int_{t_0}^{T} G_{\mathrm{truck}}(T-t)f(t)\,dt$$

$$-\sum_{k=1}^{n} w_k\gamma_k \int_{t_0}^{T} G_{\mathrm{rctruck}}(T-t)f_k(t)\,dt - \sum_{k=1}^{n} w_k\gamma_k(1-\alpha_k)G_{\mathrm{barge}}(T-t_0)$$

$$-\sum_{k=1}^{n} w_k(1-\gamma_k)\int_0^{T} G_{\mathrm{truck}}(T-t)f_k(t)\,dt =$$

$$\sum_{k=1}^{n} w_k \left\{\int_0^{t_\gamma} G_{\mathrm{truck}}(T-t)f_k(t)\,dt - (1-\gamma_k)\int_0^{t_0} G_{\mathrm{truck}}(T-t)f_k(t)\,dt\right\}$$

$$+\sum_{k=1}^{n} w_k\gamma_k \int_{t_0}^{T} \left\{G_{\mathrm{truck}}(T-t)-G_{\mathrm{rctruck}}(T-t)\right\} f_k(t)\,dt \geq$$

$$\sum_{k=1}^{n} w_k \left\{\int_0^{t_\gamma} G_{\mathrm{truck}}(T-t)f_k(t)\,dt - (1-\gamma_k)\int_0^{t_0} G_{\mathrm{truck}}(T-t)f_k(t)\,dt\right\}.$$

If we put

$$h(t) = \sum_{k=1}^{n} w_k(1-\gamma_k)f_k(t), \tag{20}$$

then $0 \leq h(t) \leq f(t)$ and we get

$$\rho_4(\alpha,\gamma)-\rho_3(\alpha,\vec{\gamma}) \geq \int_0^{t_\gamma} G_{\mathrm{truck}}(T-t)f(t)\,dt - \int_0^{t_0} G_{\mathrm{truck}}(T-t)h(t)\,dt.$$

Let $E_\gamma(t)$ be the function which equals 1 for $0 \leq t < t_\gamma$ and equals 0 elsewhere, and define $\mu(t) = f(t)E_\gamma(t) - h(t)$. Then we obtain

$$M(t) = \int_0^{t} \mu(x)\,dx \geq 0, \quad 0 \leq t \leq t_0, \tag{21}$$

and that $M(t_0) = 0$. Indeed,

$$M(t_0) = \int_0^{t_\gamma} f(t)\,dt - \int_0^{t_0} h(t)\,dt = F(t_\gamma) - \sum_{k=1}^{n} w_k(1-\gamma_k)F_k(t_0) =$$

$$(1-\gamma)(1-\alpha) - \sum_{k=1}^{n} w_k(1-\gamma_k)(1-\alpha_k) = 0.$$

Consequently we get for $t_\gamma \le t \le t_0$

$$M(t) = \int_0^{t_\gamma} \mu(x)\,dx - \int_{t_\gamma}^{t} h(x)\,dx \ge \int_0^{t_\gamma} \mu(x)\,dx - \int_{t_\gamma}^{t_0} h(x)\,dx = M(t_0) = 0.$$

If $0 \le t \le t_\gamma$, we arrive at

$$M(t) = \int_0^{t} \mu(x)\,dx = \int_0^{t} \{f(x) - h(x)\}\,dx \ge 0.$$

The lemma is proved applying Lemma 8 to

$$\rho_4(\alpha, \vec{\gamma}) - \rho_3(\alpha, \gamma) \ge \int_0^{t_0} G_{\text{truck}}(T-t)\mu(t)\,dt. \ \square$$

Proof of Theorem 1. We may verify directly that $\mathcal{E}_1 \preceq \mathcal{E}_2$ by showing that $\rho_1 \le \rho_2(\alpha, 0)$ and $\sigma_1 \le \sigma_2(\alpha, 0)$. Further, $\mathcal{E}_2 \preceq \mathcal{E}_3$ follows trivially from $\mathcal{A}_2 \subseteq \mathcal{A}_3$. The set $\mathcal{A}_2$ emerges as a subset of $\mathcal{A}_3$ by considering the special case $\vec{\gamma} = (\gamma, \gamma, \ldots, \gamma)$. The assertion $\mathcal{E}_3 \preceq \mathcal{E}_4$ follows from Lemma 9. $\square$

A.8 Proof of Lemma 7

We establish the efficiency frontiers for the specific case of $\theta = 0$, i.e., the case when $c_{\mathrm{truck}} = c_{\mathrm{barge}}$. Information Scenario 1 is not affected by the value of θ, so we again find that $\sigma_1 = 0$ and $\rho_1 = \int_0^T G_{\mathrm{rctruck}}(T-t)f(t)\,dt$, and that $\mathcal{E}_1 = \{(\rho_1, 0)\}$.

Lemma 1 for $\theta = 0$ results in $\sigma_2(\alpha,\gamma) = 1 - \gamma\alpha$ and $\rho_2(\alpha,\gamma) = \rho_0 - \gamma\rho(\alpha)$, where $\rho(\alpha) \geq 0$ by Lemma 2. As a result, the efficiency frontier under Information Scenario 2 is a single point corresponding with $\gamma = 0$, i.e., $\mathcal{E}_2 = \{(\rho_0, 1)\}$.

In a similar fashion, we may infer from Lemma 4 that $\sigma_3(\alpha,\vec{\gamma}) = 1 - \sum_{k=1}^n w_k\gamma_k\alpha_k$ and $\rho_3(\alpha,\vec{\gamma}) = \rho_0 - \sum_{k=1}^n \gamma_k\rho_k(\alpha)$ are optimized for $\vec{\gamma} = \vec{0}$, so $\mathcal{E}_3 = \{(\rho_0, 1)\}$.

Under Information Scenario 4, Lemma 5 provides $\sigma_4 = 1$, and $\rho_4(\alpha,\gamma)$ is decreasing in γ by Lemma 6. As a result, $\mathcal{E}_4 = \{(\rho_0, 1)\}$. $\square$

A.9 Proof of Theorem 2

Recall that (3) provides $\sigma_2(\alpha,\gamma) = 1 - \theta + \gamma(\theta - \alpha)$ and that (4) can be rewritten as $\rho_2(\alpha,\gamma) = \rho_0 - \gamma\rho(\alpha)$ with $0 \leq \rho(\alpha) \leq \rho_0$ and $\alpha \mapsto \rho(\alpha)$ decreasing; see Lemma 2.

In other words, for fixed α, the functions $\gamma \mapsto \rho_2(\alpha,\gamma)$ and $\gamma \mapsto \sigma_2(\alpha,\gamma)$ are linear. As a consequence, the function (ρ_2, σ_2) maps the line segment $L_\alpha = \{(\alpha,\gamma) : 0 \leq \gamma \leq 1\}$ onto another line segment M_α given by (15). Each line segment M_α emanates from the same point (ρ_0, σ_0) at $\gamma = 0$ and extends to the point $(\rho_0 - \rho(\alpha), 1 - \alpha)$ at $\gamma = 1$. See also Figure 1.

Since $[0,1]^2 = \bigcup_\alpha L_\alpha$, we obtain $\mathcal{A}_2 = \bigcup_\alpha M_\alpha$. We proceed to construct $\mathcal{E}_2$ based on the line segments M_α. First, we observe that M_α for $\alpha > \theta$ do not contribute to $\mathcal{E}_2$, so we may restrict our analysis of M_α to $0 \leq \alpha \leq \theta$. Next, denote the magnitude of the negative slope of the line M_α by $\Phi(\alpha)$ as defined in (16). Note that $\Phi(0) = \frac{\theta}{\rho_0} > 0$ with $\rho(0) = \rho_0 > 0$. On the other hand, $\rho(\theta) \geq \rho(1) = \rho_0 - \rho_1 > 0$, which implies $\Phi(\theta) = 0$.

Fix $0 \leq \lambda \leq \theta$ and write $z(\lambda) = (\rho_0 - \frac{\theta-\lambda}{\Psi(\lambda)}, 1-\lambda)$. To prove the theorem, we need to show that $z(\lambda)$ for $0 \leq \lambda \leq \theta$ constitute all maximal elements in $\mathcal{A}_2$. There are two cases to be discerned.

Case 1: $\Psi(\lambda) = \Phi(\lambda)$. In this case, $z(\lambda) = (\rho_2(\lambda,1), \sigma_2(\lambda,1)) \in \mathcal{A}_2$. Next, write $x = (\rho_0 - \gamma\rho(\alpha), 1-\theta+\gamma(\theta-\alpha))$ and assume that $x \geq z(\lambda)$. This implies $1-\theta+\gamma(\theta-\alpha) \geq 1-\lambda$, so $\lambda \geq \gamma\alpha + (1-\gamma)\theta$. Further, $\rho_0 - \gamma\rho(\alpha) \geq \rho_0 - \frac{\theta-\lambda}{\Psi(\lambda)}$ or $\gamma\rho(\alpha) \leq \frac{\theta-\lambda}{\Psi(\lambda)}$ or $\gamma\Psi(\lambda) \leq \frac{\theta-\lambda}{\rho(\alpha)} \leq \frac{\gamma(\theta-\alpha)}{\rho(\alpha)}$. In other words, $\Psi(\lambda) \leq \Phi(\alpha)$ which can only be true when $\lambda = \alpha$ and $\gamma = 1$, so $x = z(\lambda)$. This implies that $z(\lambda)$ is a maximal element in $\mathcal{A}_2$.

Case 2: $\Psi(\lambda) > \Phi(\lambda)$. We may write $\Psi(\lambda) = \Phi(\alpha)$ for some $0 \leq \alpha < \lambda$. Write $\lambda = \gamma\alpha + (1-\gamma)\theta$ for some $0 < \gamma < 1$. Then, with $1-\lambda = 1-\theta+\gamma(\theta-\alpha)$, we get

$$z(\lambda) = (\rho_0 - \frac{\theta-\lambda}{\Psi(\lambda)}, 1-\lambda) = (\rho_0 - \frac{\theta-\lambda}{\Phi(\alpha)}, 1-\theta+\gamma(\theta-\alpha)) \in M_\alpha \subseteq \mathcal{A}_2.$$

Write $x = (\rho_0 - \gamma\rho(\beta), 1-\theta+\gamma(\theta-\beta))$ and $x \geq z(\lambda)$. This implies $\rho(\alpha) \geq \rho(\beta)$ and hence $\alpha \leq \beta$ since ρ is decreasing. On the other hand, $1-\theta+\gamma(\theta-\beta) \geq 1-\theta+\gamma(\theta-\alpha)$ so $\alpha \geq \beta$. This implies $\alpha = \beta$ and that $z(\lambda)$ is a maximal element in $\mathcal{A}_2$.

Finally, we need to show that all maximal elements have been described. This follows from the fact that for $(x_1, x_2) \in \mathcal{A}_2$, we have the following assertions. First of all, if $x_2 \leq 1-\theta$, then $x_1 \leq \rho_0$, so $(x_1, x_2) \leq (\rho_0, 1-\theta)$. Secondly, if $1-\theta \leq x_2 \leq 1$, then according to the analysis given above, $(x_1, x_2) \leq z(\lambda)$ with $\lambda = 1 - x_2$. $\Box$

A.10 Construction of $\mathcal{E}_3$

We construct the Pareto frontier $\mathcal{E}_3$ in Theorem 3, while Proposition 2 describes the Pareto optimal decisions on $\mathcal{E}_3$.

Theorem 3 *A function* $(\alpha, \gamma) \mapsto \tilde{\rho}_3(\alpha, \gamma)$*, with* $\gamma \mapsto \tilde{\rho}_3(\alpha, \gamma)$ *decreasing for each* $\alpha \in [0, \theta]$*, can be constructed in such a way that*

$$\rho_3^*(\lambda) = \max\left\{\tilde{\rho}_3\left(\alpha, \frac{\theta - \lambda}{\theta - \alpha}\right) : 0 \leq \alpha \leq \lambda\right\} \tag{22}$$

provides $\mathcal{E}_3 = \{(\rho_3^*(\lambda), 1 - \lambda) : 0 \leq \lambda \leq \theta\}$.

We construct the Pareto frontier $\mathcal{E}_3$ by first establishing the following proposition.

Proposition 1 *The mapping* $\vec{\gamma} \mapsto (\rho_3, \sigma_3)$ *as defined by (10) and (9) is an affine mapping. It maps the* n*-dimensional cube* $L_\alpha = \{(\alpha, \vec{\gamma}) : \vec{\gamma} \in [0, 1]^n\}$ *onto the polygon*

$$\mathcal{M}_\alpha = \{(\rho_3(\alpha, \vec{\gamma}), \sigma_3(\alpha, \vec{\gamma})) : \vec{\gamma} \in [0, 1]^n\}. \tag{23}$$

A mapping $(\tilde{\rho}_3, \tilde{\sigma}_3)$ *can be constructed which maps* $\{(\alpha, \gamma) : 0 \leq \gamma \leq 1\}$ *onto the Pareto frontier of* $\mathcal{M}_\alpha$ *for each* $0 \leq \alpha \leq \theta$.

This result is useful for the construction of the Pareto frontier $\mathcal{E}_3$ as only Pareto optimal points of $\mathcal{M}_\alpha$ for $0 \leq \alpha \leq \theta$ will constitute $\mathcal{E}_3$.

Proof of Proposition 1. It can readily be verified that the mapping (ρ_3, σ_3) is affine with ρ_3 as given by (10) and σ_3 as given by (9). Write $\vec{e} = (1, 1, \cdots, 1)$ and consider the unit n-cube $\{\vec{x} \in \mathbb{R}^n : \vec{0} \leq \vec{x} \leq \vec{e}\}$. As the polygon $\mathcal{M}_\alpha = \{(\rho_3(\alpha, \vec{\gamma}), \sigma_3(\alpha, \vec{\gamma})) : \vec{0} \leq \vec{\gamma} \leq \vec{e}\}$ is the image of an affine transformation of the n-cube, the extreme points of the polygon are the images under the mapping of the extreme points of the n-cube. Moreover, the edges of the polygon are the images of edges of the n-cube. Consequently, we may construct the Pareto frontier of $\mathcal{M}_\alpha$ by considering the slopes of the polygon edges $\Phi_k(\alpha) = \frac{\theta - \alpha_k}{\rho_k(\alpha)}$, and establish distinct $k_1, k_2, \ldots, k_n$ such that $\Phi_{k_1}(\alpha) \geq \Phi_{k_2}(\alpha) \geq \cdots \geq \Phi_{k_n}(\alpha)$. The extreme points that define the piecewise linear Pareto frontier of $\mathcal{M}_\alpha$ are given by

$$\vec{a}_p = (\rho_0 - \sum_{i=1}^{p} w_{k_i} \rho_{k_i}(\alpha), 1 - \theta + \sum_{i=1}^{p} w_{k_i}(\theta - \alpha_{k_i})), \quad p = 0, \ldots, n, \tag{24}$$

where in particular $\vec{a}_0 = (\rho_0, 1 - \theta)$ and $\vec{a}_n = (\rho_0 - \rho(\alpha), 1 - \alpha)$. We therefore define the function

$$\tilde{\sigma}_3(\alpha, \gamma) = 1 - \theta + \gamma(\theta - \alpha), \tag{25}$$

and with $r_p = \rho_0 - \sum_{i=1}^{p} w_{k_i} \rho_{k_i}(\alpha)$ for $p = 1, \ldots, n$, we define

$$\tilde{\rho}_3(\alpha, \gamma) = r_p + \frac{\gamma - g_p}{g_{p+1} - g_p}(r_{p+1} - r_p), \quad g_p \leq \gamma \leq g_{p+1}, \tag{26}$$

where $0 = g_0 < g_1 < \cdots < g_n = 1$ are given by $(p = 0, .., n)$

$$g_p = \frac{\sum_{i=1}^{p} w_{k_i}(\theta - \alpha_{k_i})}{\sum_{i=1}^{n} w_{k_i}(\theta - \alpha_{k_i})} = \frac{\sum_{i=1}^{p} w_{k_i}(\theta - \alpha_{k_i})}{\theta - \alpha}. \tag{27}$$

In this manner, the Pareto frontier of $\mathcal{M}_\alpha$ is given by $\{(\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma)) : 0 \leq \gamma \leq 1\}$ for each $0 \leq \alpha \leq \theta$. $\square$

We now use Proposition 1 to identify the Pareto optimal decisions in the following proposition.

Proposition 2 *The element* $(\rho_3^*(\lambda), 1-\lambda)$ *of the Pareto optimal frontier* $\mathcal{E}_3$ *gives rise to a Pareto optimal decision* $(\alpha, \vec{\gamma})$*, where the value of* $\alpha \in [0, \lambda]$ *satisfies* $\rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \frac{\theta-\lambda}{\theta-\alpha})$*, and* $\vec{\gamma}$ *is given by*

$$\begin{cases} \gamma_{k_i} = 1, & 1 \leq i \leq p \\ \gamma_{k_{p+1}} = \frac{\lambda-\lambda_p}{\lambda_{p+1}-\lambda_p}, & \\ \gamma_{k_i} = 0, & p+1 < i \leq n \end{cases}, \tag{28}$$

where $\lambda_p = 1 - \theta + \sum_{i=1}^{p} w_{k_i}(\theta - \alpha_{k_i})$*,* $\lambda_p \leq \lambda < \lambda_{p+1}$*, and* $k_1, \ldots, k_n$ *distinct such that* $\Phi_{k_1}(\alpha) \geq \Phi_{k_2}(\alpha) \geq \cdots \geq \Phi_{k_n}(\alpha)$*, with* $\Phi_k(\alpha) = \frac{\theta-\alpha_k}{\rho_k(\alpha)}$ *and* $\rho_k(\alpha)$ *as in (11).*

Proposition 2 states that Pareto optimal decisions assign container categories k to barge for which the trade-off between efficiency gain and reliability loss are favorable i.e for which $\Phi_k(\alpha)$ are larger. Note that the function $\Phi_k(\alpha)$ is similar to the function $\Phi(\alpha)$ as defined in (16). We find in particular that the departure time of the barge is given by $t_0 = F^{-1}(1-\alpha)$ and that the fraction of containers of category k planned for barge is equal to γ_k as given in Proposition 2.

Proof of Proposition 2 In case $(\rho_3^*(\lambda), 1-\lambda) \in \mathcal{E}_3$, is given, we construct the corresponding Pareto optimal solution $(\alpha, \vec{\gamma})$ as follows. First of all, we determine α by $(\rho_3^*(\lambda), 1-\lambda) = \tilde{\rho}_3(\alpha, \gamma)$ with $\gamma = \frac{\theta-\lambda}{\theta-\alpha}$. We then construct $\vec{\gamma} = (\gamma_1, \ldots, \gamma_n)$ by either identifying $0 \leq p < n$ such that $g_p \leq \gamma < g_{p+1}$ or by identifying that $\lambda = \alpha$, i.e., $\gamma = 1$. Based on the proof of Proposition 1, we may conclude that $\vec{\gamma}$ can be established as identified in the statement of the proposition, with the special case of $\gamma = 1$ giving rise to $\vec{\gamma} = (1, \ldots, 1)$. $\square$

We are now ready to prove the theorem.

Proof of Theorem 3. Fix $0 \leq \lambda \leq \theta$ and put $z(\lambda) = (\rho_3^*(\lambda), 1 - \lambda)$.

Case 1: $\rho_3^*(\lambda) = \tilde{\rho}_3(\lambda, 1)$

Then $z(\lambda) = (\tilde{\rho}_3(\lambda, 1), \tilde{\sigma}_3(\lambda, 1)) \in M_\lambda \subseteq \mathcal{A}_3$. If $x = (\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma))$ satisfies $x \geq z(\lambda)$, then $1 - \theta + \gamma(\theta - \alpha) \geq 1 - \lambda$, so $\lambda \geq \gamma\alpha + (1 - \gamma)\theta$. Further, $\tilde{\rho}_3(\alpha, \gamma) \geq \tilde{\rho}_3(\lambda, 1) = \rho_3^*(\lambda)$. On the other hand, since $\gamma \geq \frac{\theta - \lambda}{\theta - \alpha}$, we get $\tilde{\rho}_3(\alpha, \gamma) \leq \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha}) \leq \rho_3^*(\lambda)$ since $\alpha \leq \lambda$. This implies $\tilde{\rho}_3(\alpha, \gamma) = \tilde{\rho}_3(\lambda, 1) = \rho_3^*(\lambda)$ and $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$, so $\tilde{\sigma}_3(\alpha, \gamma) = 1 - \lambda$ and hence $x = z(\lambda)$. We have obtained that $z(\lambda) \in \mathcal{A}_3$ is a maximal element.

Case 2: $\rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \frac{\theta - \lambda}{\theta - \alpha})$ for some $0 \leq \alpha \leq \lambda$

Then $z(\lambda) = (\tilde{\rho}_3(\alpha, \gamma), \tilde{\sigma}_3(\alpha, \gamma)) \in M_\lambda \subseteq \mathcal{A}_3$ with $\gamma = \frac{\theta - \lambda}{\theta - \alpha}$. If $x = (\tilde{\rho}_3(\beta, \delta), \tilde{\sigma}_3(\beta, \delta))$ satisfies $x \geq z(\lambda)$, then $1 - \theta + \delta(\theta - \beta) \geq 1 - \lambda$, so $\lambda \geq \delta\beta + (1 - \delta)\theta$, and $\tilde{\rho}_3(\beta, \delta) \geq \tilde{\rho}_3(\alpha, \gamma) = \rho_3^*(\lambda)$. On the other hand, since $\delta \geq \frac{\theta - \lambda}{\theta - \beta}$, we get $\tilde{\rho}_3(\beta, \delta) \leq \tilde{\rho}_3(\beta, \frac{\theta - \lambda}{\theta - \beta}) \leq \rho_3^*(\lambda) = \tilde{\rho}_3^*(\alpha, \frac{\theta - \lambda}{\theta - \alpha})$ since $\beta \leq \lambda$. This implies $\tilde{\rho}_3(\beta, \delta) = \rho_3^*(\lambda) = \tilde{\rho}_3(\alpha, \gamma)$, and $\tilde{\sigma}_3(\beta, \delta) = 1 - \lambda = \tilde{\sigma}_3(\alpha, \gamma)$ and hence $x = z(\lambda)$. We have obtained that $z(\lambda) \in \mathcal{A}_3$ is a maximal element. We may argue as in Theorem 2 that we have captured all maximal elements in $\mathcal{A}_3$ in this manner. $\square$

A.11 Construction of $\mathcal{E}_4$

We construct the Pareto frontier $\mathcal{E}_4$ using the following theorem, The proof of which is similar to the proof of Theorem 3 and is not provided in the appendix.

Theorem 4 *Consider the mapping* $\sigma_4 : (\alpha, \gamma) \mapsto 1 - \theta + \gamma\theta(1-\alpha)$ *and the mapping* $\rho_4 : (\alpha, \gamma) \mapsto \rho_4(\alpha, \gamma)$ *as defined in (13). If we set*

$$\rho_4^*(\lambda) = \max\left\{\rho_4\left(\alpha, \frac{\theta - \lambda}{\theta(1-\alpha)}\right) : 0 \leq \alpha \leq \frac{\lambda}{\theta}\right\}, \tag{29}$$

then $\mathcal{E}_4 = \{(\rho_4^*(\lambda), 1 - \lambda) : 0 \leq \lambda \leq \theta\}$.

For each $0 \leq \lambda \leq \theta$ with $\rho_4^*(\lambda) = \rho_4\left(\alpha, \frac{\theta-\lambda}{\theta(1-\alpha)}\right)$, the Pareto optimal decision sets the barge departure time equal to $t_0 = F^{-1}(1-\alpha)$ and the fraction of containers planned for barge equal to $\gamma = \frac{\theta-\lambda}{\theta(1-\alpha)}$. The Pareto optimal decisions $\mathcal{E}_4$ need to be interpreted a bit different as compared to the Pareto optimal decisions $\mathcal{E}_2$: Under Information Scenario 4, the fraction γ of containers assigned to barge are the ones with the release times as late as possible before barge departure, as discussed in Section 3.4.