

## Appendix 1: Cycling under degeneracy in the origin-based TAP

A single-origin TAP is distinctive from a generic MCF problem in that: (1) it has a single origin, (2) the arc costs can be flow dependent, and (3) the arc costs are nonnegative. In a cycling example, the variables in the basis with positive values in this solution do not play any role among cycling iterations. Cunningham (1979) showed that the positive variables in the basis, and the equations used to determine their values, can be eliminated from the problem in a cycling example. The smaller problem is “totally degenerate”. For the network simplex method, in a totally degenerate problem, all demand is zero and all flows remain zero. This fact tells that: (1) the single origin property of TAP does not matter in a cycling example; and (2) the flow-dependent arc travel time does not matter either because the flow is zero in a totally degenerate problem. Hence, a cycling example in the network simplex with nonnegative costs could be transformed for the TAP in a similar context.

Below, we show a totally degenerate example, which was modified from the originally reported example in (Gassner 1964). Similar to the MCF, we assume that there is not a specific rule for determining the entering (except the negative reduced cost) and leaving arcs in the pivot steps.

Given a network as shown in Figure 11, node 1 is the origin, which sends 1 unit of flow to node 5. The arc costs are given in Table 5.

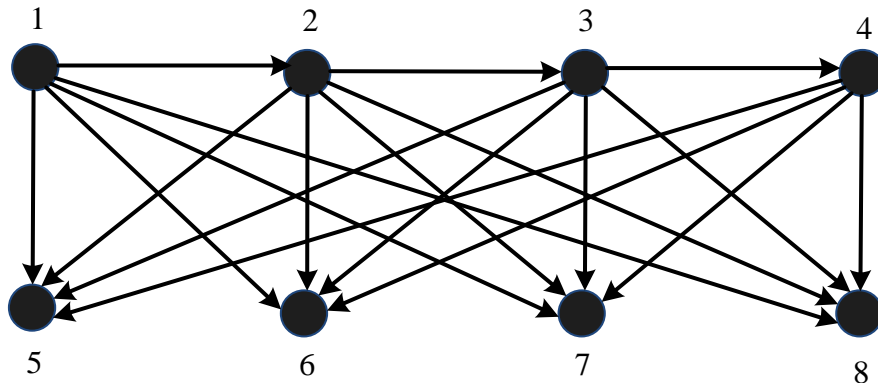


Figure 11 Example of the 8-node network

Table 5 Arc costs in the 8-node network

$c_{ij}$		$j$							
		1	2	3	4	5	6	7	8
$i$	1	100				3	5	5	11
	2	100				9	7	9	15
	3			100		7	7	11	13
	4					13	13	13	17

All arcs are uncapacitated. We construct the initial basis as shown in Figure 12(a), where the arc with the hollow arrow indicates that it carries positive flow. Arcs with solid arrows have zero flow. The subgraph without arc (1,5) is totally degenerate. A similar totally degenerate case is not unusual in a TAP, particularly where arcs with positive flows are sparse. The arc costs in the table, except arc (1,5), can be understood as the free flow travel time. The actual travel time could include an extra flow-dependent term that does not affect cycling since the flow is zero in the totally degenerate subgraph. It is easy to verify that arcs (3,8), (2,7) and (1,6) have negative reduced costs; thus they are qualified to enter the basis that leads to the degenerate basis in Figure 12(b).

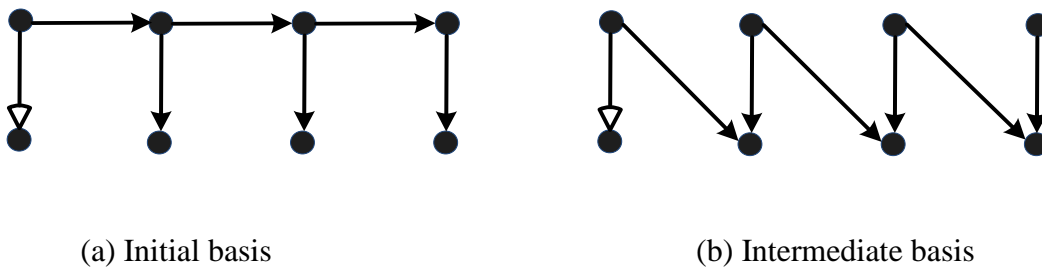


Figure 12 Illustration of pivots

The remaining pivot steps are the same as in Gassner's example, as shown in Figure 13. A dashed arc indicates an entering arc; a gray arc indicates a leaving arc. One can verify that the entering arc in each step includes the reduced cost -2, and the leaving arc in each step is an inactive backward arc in the pivot cycle (in the same direction as the entering arc). The basis returns to the initial basis after 12 pivot

steps. Notably, Nguyen’s pivot rule does not admit cycling in this example due to the directed tree structure; Danzig’s selection rule does not admit cycling either.

Given the example, it is conjectured that the algorithm for the TAP without the SFST pivot rule (e.g., NS-SPT) is possible to converge slowly, or behave somewhat pathologically when the problem is highly degenerate.

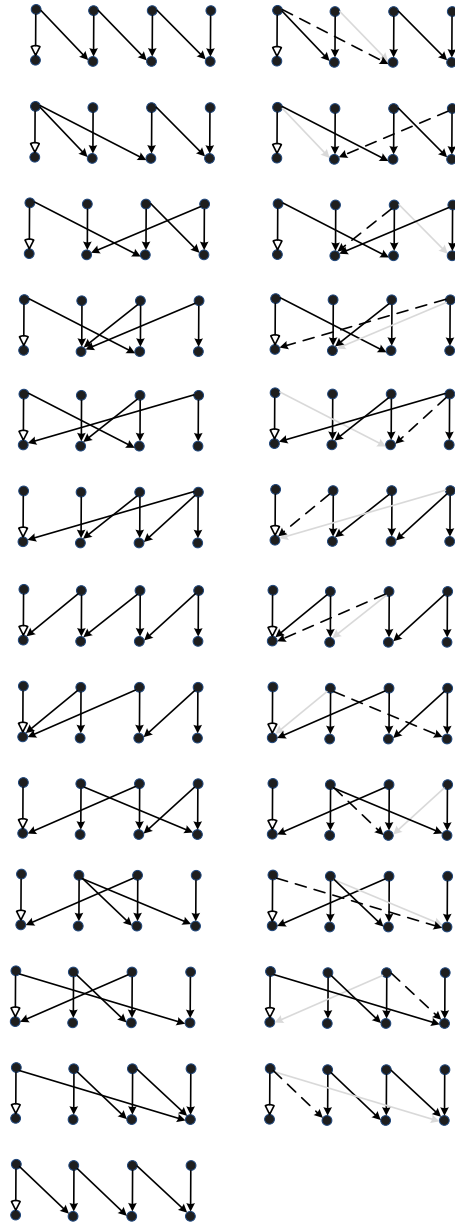


Figure 13 Pivot steps in Gassner’s cycling example