

Appendix

A. Theoretical Results

A.1. Theoretical Results for the DMCFLP_CR formulations

We now prove the dominance relationships between the three formulations presented for the DMCFLP_CR. For any integer linear programming model P , let \bar{P} denote the corresponding LP relaxation. For any model P , we denote by $v(P)$ its optimal value.

A.1.1. CR-GMC and CR-1I are equally strong

We prove that the LP relaxations of the formulations CR-GMC and CR-1I provide the same lower bound.

THEOREM 4. $v(\overline{CR-GMC}) = v(\overline{CR-1I})$.

Proof. The proof consists of two parts: First, we show how to construct a feasible solution for $\overline{CR-1I}$ from any feasible solution for $\overline{CR-GMC}$ and that both solutions have the same objective function value. Then, we show the same, constructing an equivalent and feasible $\overline{CR-GMC}$ solution from any feasible $\overline{CR-1I}$ solution.

To facilitate the proof, we first write the CR-GMC in its explicit form as it is defined in Section 3.2. As previously defined, we let $L = \{0, 1, 2, \dots, q\}$ be the set of available capacity levels to define the facility size. For each open capacity level $\ell \in L \setminus \{0\}$, we let $\bar{\ell}$ denote a closed facility of capacity level ℓ . The model is:

$$\begin{aligned}
 \text{(CR-GMC)} \quad \min & \sum_{j \in J} \sum_{\ell_2 \in L} \sum_{t \in T} (f_{j\ell_2}^o + F_{j\ell_2}^o) y_{j0\ell_2 t} + \sum_{i \in I} \sum_{j \in J} \sum_{\ell \in L} \sum_{t \in T} g_{ij\ell t} d_{it} x_{ij\ell t} \\
 & + \sum_{j \in J} \sum_{\ell_1 \in L \setminus \{0\}} \sum_{t \in T} c_{j\ell_1}^c y_{j\ell_1 \bar{\ell}_1 t} + \sum_{j \in J} \sum_{\ell_1 \in L \setminus \{0\}} \sum_{t \in T} (c_{j\ell_1}^o + F_{j\ell_1}^o) y_{j\bar{\ell}_1 \ell_1 t} \\
 \text{s.t.} \quad & \sum_{j \in J} \sum_{\ell \in L} x_{ij\ell t} = 1 \quad \forall i \in I, \quad \forall t \in T \tag{40}
 \end{aligned}$$

$$\sum_{i \in I} d_{it} x_{ij\ell t} \leq u_{j\ell} (y_{j\ell t} + y_{j\bar{\ell} t} + y_{j0\ell t}) \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T \tag{41}$$

$$y_{j0\ell(t-1)} + y_{j\ell\ell(t-1)} + y_{j\bar{\ell}\ell(t-1)} = y_{j\ell t} + y_{j\bar{\ell} t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T \setminus \{1\} \tag{42}$$

$$y_{j\bar{\ell}\ell(t-1)} + y_{j\bar{\ell}\bar{\ell}(t-1)} = y_{j\bar{\ell} t} + y_{j\bar{\ell} t} \quad \forall j \in J, \quad \forall \ell \in L \setminus \{0\}, \quad \forall t \in T \setminus \{1\} \tag{43}$$

$$\sum_{\ell_2 \in L} y_{j0\ell_2 1} = 1 \quad \forall j \in J \tag{44}$$

$$x_{ij\ell t} \geq 0 \quad \forall i \in I, \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T \tag{45}$$

$$y_{j0\ell_2 t} \in \{0, 1\} \quad \forall j \in J, \quad \forall \ell_2 \in L, \quad \forall t \in T \tag{46}$$

$$y_{j\ell_1 \ell_1 t} \in \{0, 1\} \quad \forall j \in J, \quad \forall \ell_1 \in L \setminus \{0\}, \quad \forall t \in T \tag{47}$$

$$y_{j\bar{\ell}_1\bar{\ell}_1t} \in \{0,1\} \quad \forall j \in J, \quad \forall \ell_1 \in L \setminus \{0\}, \quad \forall t \in T \quad (48)$$

$$y_{j\ell_1\bar{\ell}_1t} \in \{0,1\} \quad \forall j \in J, \quad \forall \ell_1 \in L \setminus \{0\}, \quad \forall t \in T \quad (49)$$

$$y_{j\bar{\ell}_1\ell_1t} \in \{0,1\} \quad \forall j \in J, \quad \forall \ell_1 \in L \setminus \{0\}, \quad \forall t \in T. \quad (50)$$

(A) Construction of a feasible $\overline{\text{CR-II}}$ solution from any $\overline{\text{CR-GMC}}$ solution

Consider any solution $\{x_{ij\ell t}, y_{j\ell_1\ell_2t}\}$ that is feasible in $\overline{\text{CR-GMC}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, v_{j\ell t}^o, v_{j\ell t}^c\}$ that is feasible in $\overline{\text{CR-II}}$ and has the same objective function value.

We set the values for the $x_{ij\ell t}$ variables identical to those in the CR-GMC solution. The values for the variables $y_{j\ell t}$, $s_{j\ell t}$, $v_{j\ell t}^o$ and $v_{j\ell t}^c$ are set by establishing the following relations $\forall j \in J, \forall \ell \in L, \forall t \in T$:

$$s_{j\ell t} = y_{j0\ell t} \quad (\text{CR.R1})$$

$$v_{j\ell t}^o = y_{j\bar{\ell}t} \quad (\text{CR.R2})$$

$$v_{j\ell t}^c = y_{j\bar{\ell}t} \quad (\text{CR.R3})$$

$$y_{j\ell t} = y_{j0\ell t} + y_{j\ell\ell t} + y_{j\bar{\ell}t}. \quad (\text{CR.R4})$$

According to these relations and the way the objective function coefficients are composed (see Section 3.2), it can easily be verified that both solutions have the same objective function value. Constraints (11) are satisfied, as they contain the same variables with the same values in both models. Replacing the right-hand side in constraints (12) by (CR.R4) results in constraints (41). Therefore, constraints (12) are also satisfied. We show that constraints (13) hold by using the relationships defined above:

$$y_{j\ell t} = y_{j\ell(t-1)} + s_{j\ell t} + v_{j\ell t}^o - v_{j\ell t}^c \quad (13)$$

$$\begin{aligned} & \stackrel{(\text{CR.R1}) \text{ \& } (\text{CR.R4})}{\Leftrightarrow} y_{j0\ell t} + y_{j\ell\ell t} + y_{j\bar{\ell}t} = y_{j0\ell(t-1)} + y_{j\ell\ell(t-1)} + y_{j\bar{\ell}(t-1)} + y_{j0\ell t} + y_{j\bar{\ell}t} - y_{j\bar{\ell}t} \\ & \stackrel{\text{cancel } y_{j0\ell t} \text{ \& } y_{j\bar{\ell}t}}{\Leftrightarrow} y_{j\ell\ell t} = y_{j0\ell(t-1)} + y_{j\ell\ell(t-1)} + y_{j\bar{\ell}(t-1)} - y_{j\bar{\ell}t}. \end{aligned} \quad (42)$$

As equalities (42) necessarily hold, constraints (13) are also satisfied. In a similar way, we show that constraints (14) hold:

$$\begin{aligned} & \sum_{t'=1}^t v_{j\ell t'}^o \leq \sum_{t'=1}^t v_{j\ell t'}^c \quad (14) \\ & \stackrel{(\text{CR.R2}) \text{ \& } (\text{CR.R3})}{\Leftrightarrow} \sum_{t'=1}^t y_{j\bar{\ell}t'} \leq \sum_{t'=1}^t y_{j\bar{\ell}t'} \\ & \stackrel{\text{replace LHS by (42)}}{\Leftrightarrow} \sum_{t'=1}^t y_{j\bar{\ell}(t'-1)} + \sum_{t'=1}^t y_{j\bar{\ell}(t'-1)} - \sum_{t'=1}^t y_{j\bar{\ell}t'} \leq \sum_{t'=1}^t y_{j\bar{\ell}t'} \\ & \stackrel{\text{cancel } y_{j\bar{\ell}t'} \text{ \& } y_{j\bar{\ell}t'}}{\Leftrightarrow} -y_{j\bar{\ell}t} \leq y_{j\bar{\ell}t}, \end{aligned}$$

which is true, since the y variables are non-negative. Finally, to show that constraints (15) are also satisfied, note that constraints (42) - (44) ensure that the $y_{j\ell_1\ell_2t}$ variables sum to 1 at each location j and time period t . This flow also contains the case where no facility exits, i.e., $\ell = 0$. Furthermore, once a facility is constructed, the flow in the GMC network cannot return to capacity level 0, since, according to (46) - (50), $y_{j\ell_1\ell_2t}$ is not defined for $\ell_2 = 0$. Therefore, $\sum_{t=1}^{|T|} y_{j\ell_1\ell_2t} \leq 1$ which proves that constraints (15) are satisfied.

If the SIs are used, they are also feasible in the CR-II model. They can be deduced by replacing (CR.R4) in the SIs of the CR-GMC.

(B) Construction of a feasible $\overline{\text{CR-GMC}}$ solution from any $\overline{\text{CR-II}}$ solution

Consider any solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, v_{j\ell t}^o, v_{j\ell t}^c\}$ that is feasible in $\overline{\text{CR-II}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y_{j\ell_1\ell_2t}\}$ that is feasible in $\overline{\text{CR-GMC}}$ and has the same objective function value.

The values for the $x_{ij\ell t}$ variables are set identical to those in the CR-II solution. The arcs for constructing a facility ($y_{j0\ell t}$), closing an open facility ($y_{j\ell\bar{\ell}t}$) and reopening a closed facility ($y_{j\bar{\ell}\ell t}$) are set by using the equalities (CR.R1) - (CR.R3) and therefore satisfy their domain constraints. The solution values for the arcs to keep a facility open ($y_{j\ell\ell t}$) are set by replacing (CR.R1) and (CR.R2) in equality (CR.R4):

$$\begin{aligned}
y_{j\ell t} &= y_{j0\ell t} + y_{j\ell\ell t} + y_{j\bar{\ell}\ell t} & (\text{CR.R4}) \\
\stackrel{(\text{CR.R1}), (\text{CR.R2})}{\Leftrightarrow} & y_{j\ell t} = s_{j\ell t} + y_{j\ell\ell t} + v_{j\ell t}^o \\
\Leftrightarrow & y_{j\ell\ell t} = y_{j\ell t} - s_{j\ell t} - v_{j\ell t}^o.
\end{aligned}$$

The variables are non-negative, as can be verified in equalities (13). According to inequalities (15), the total flow for capacity construction does not exceed 1 for the entire planning horizon. For the first time period, $y_{j0\ell 1}$ is set to $s_{j\ell 1}$, resulting in $\sum_{\ell_2 \in L \setminus \{0\}} y_{j0\ell_2 1} \leq 1$ for each location j . Furthermore, we set $y_{j001} = 1 - \sum_{\ell_2 \in L \setminus \{0\}} y_{j0\ell_2 1}$. Therefore, the flow initialization constraints (44) are satisfied.

Constraints (40) and (41) are satisfied, as they contain the same variables with the same values in both models. As shown above in part (A), we can transform equalities (13) into equalities (42), and vice-versa, by using (CR.R1)-(CR.R2). This proves the feasibility of constraints (42). Finally, we compute the values for $y_{j\bar{\ell}\ell t}$ by using equalities (43), sequentially from time period 1 to $|T|$:

$$\begin{aligned}
y_{j\bar{\ell}\ell t} &= y_{j\bar{\ell}\ell(t-1)} + y_{j\bar{\ell}\bar{\ell}(t-1)} - y_{j\bar{\ell}\ell t} & (43) \\
\stackrel{(\text{CR.R2}), (\text{CR.R3})}{\Leftrightarrow} & y_{j\bar{\ell}\ell t} = y_{j\bar{\ell}\ell(t-1)} + v_{j\bar{\ell}\ell(t-1)}^c - v_{j\bar{\ell}\ell t}^o.
\end{aligned}$$

Note that, due to (14), the variables have non-negative values. Furthermore, their sum never exceeds 1, because the only way how to insert flow into the v^o and v^c variables is by using the s variables, whose total sum is strictly limited to 1 by inequalities (15).

We note that the constructed solution has the same value, as can be verified by the used relationships (CR.R1)-(CR.R4) as well as the way the variables' coefficients are composed (see Section 3.2).

From the two parts (A) and (B) above, it follows that $v(\overline{CR-GMC}) = v(\overline{CR-1I})$. Q.E.D.

A.1.2. CR-GMC and CR-1I are stronger than CR-2I

We next prove that the CR-GMC and CR-1I formulations provide stronger LP bounds than the CR-2I formulation.

THEOREM 5. $v(\overline{CR-1I}) \geq v(\overline{CR-2I})$.

Proof. The proof consists of two parts: First, we show how to construct a feasible solution for $\overline{CR-2I}$ from any feasible solution of $\overline{CR-1I}$ and that both solutions have the same objective function value. Then, we describe a small problem instance where the $\overline{CR-1I}$ formulation provides a better LP relaxation bound than the $\overline{CR-2I}$ formulation.

(A) Construction of a feasible $\overline{CR-2I}$ solution from any feasible $\overline{CR-1I}$ solution

We first set the solution values for the $s_{j\ell t_1 t_2}$ and $y_{j\ell t_1 t_2}$ variables. For each j and ℓ , we consider the diagram that describes the opening schedule of a facility of size ℓ in the CR-1I solution. We separate the opening schedules for each capacity level ℓ into blocks, as described by the following algorithm:

Algorithm 1.

Input: A facility opening schedule, consisting of a value between 0 and 1, indicating the fraction at which the facility is open for each of the $|T|$ time periods (indicated by the value of $y_{j\ell t}$).

Output: The opening schedule horizontally cut into blocks. Each block is defined by a starting and ending period as well as a value between 0 and 1, indicating the fraction at which the block represents the open facility.

Description: The opening schedule, as shown in Figure 1 (a), is horizontally cut into blocks whenever the value of the opening fraction increases or decreases. Doing this, the increase and/or decrease of capacity may be split into several increases and/or decreases, respectively. This results in a representation as in Figure 1 (b). In this example, the capacity increase at the beginning of period 3 is split into two capacity increases of half size each, while the capacity decrease at the beginning of period 6 is split into two capacity decreases. To be precise, the algorithm separates the opening schedules into three capacity blocks: the first block spans periods 2 to 5, the second block spans periods 3 to 5 and the third block spans periods 3 to 4. It is easy to see that this kind of division is unambiguous, i.e., there is only one way to separate into blocks. The design of an algorithm to find this division is straightforward. We therefore do not explicitly state such an algorithm.

Note that, in the opening schedule, a capacity increase at time period t is always caused by the use of the variables $s_{j\ell t}$ or $v_{j\ell t}^o$. A capacity decrease at time period t is caused by the use of variable $v_{j\ell t}^c$.

After division, we have a number of separate blocks (each spanning one or more time periods). We divide these blocks into two groups: blocks where the capacity increase is originated from a variable $s_{j\ell t}$ and blocks where the capacity increase is originated from reopening variables $v_{j\ell t}^o$. Each block originated from a variable $s_{j\ell t}$ represents a $s_{j\ell t_1 t_2}$ variable and each block originated from a $v_{j\ell t}^o$ variable represents a variable $y_{j\ell t_1 t_2}$. The value for these variables is set equal to the fraction of value represented by the corresponding variables $s_{j\ell t}$ and $v_{j\ell t}^o$.

The following relationships then hold, since the solution value of $y_{j\ell t}$ is the sum of all capacity blocks at time period t :

$$\sum_{t_1=1}^t \sum_{t_2=t}^{|T|} (s_{j\ell t_1 t_2} + y_{j\ell t_1 t_2}) = y_{j\ell t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T. \quad (\text{CR.R5})$$

Furthermore, the following relationships hold, since the solution value of $s_{j\ell t}$ is distributed over all $s_{j\ell t_1 t_2}$ variables that originate from $t_1 = t$. The same relation is valid between the variables $v_{j\ell t}^o$ and $y_{j\ell t_1 t_2}$:

$$\sum_{t_2=t}^{|T|} s_{j\ell t t_2} = s_{j\ell t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T. \quad (\text{CR.R6})$$

All $x_{ij\ell t}$ variables are set as given in the CR-1I solution. Thus, constraints (19) are satisfied. Inequalities (14) guarantee that, at any time period t , variable $v_{j\ell t}^o$ does not hold more capacity than has been previously constructed. Thus, constraints (20) also hold. Using (CR.R6) in (15) shows that constraints (21) are satisfied. Inequalities (22) are also satisfied. To show this, first replace the terms by (CR.R5). Then, recognize that (15) limits the entire facility construction to 1. As $y_{j\ell t}$ is linked to the facility construction in equalities (13), its sum over all capacity levels can never exceed 1. Finally, the capacity constraints (23) are feasible. This is shown by replacing (CR.R5) in constraints (23), which then equal the capacity constraints of the CR-1I. If SIs are used, the feasibility of the SIs in the CR-2I formulation can be shown by replacing its RHS terms by (CR.R5).

We note that the constructed solution has the same value as the CR-1I solution. This can be seen by recognizing that the $s_{j\ell t_1 t_2}$ and $y_{j\ell t_1 t_2}$ blocks in the CR-2I solution have been constructed following the corresponding solution values of $s_{j\ell t}$ and $y_{j\ell t}$ and considering how the cost coefficients are set as described in Section 4.1.2.

(B) Problem instance where CR-1I is stronger

Consider the following example instance. We consider a planning over three time periods. A single customer exists with demands of 15, 15 and 20 units for each of the time periods, respectively. Two locations can be used to construct facilities. A single capacity level is available, providing a capacity of 10 units. The construction costs are 100\$ and the maintenance costs for an open facility are set to 500\$ for one time period. Facility closing and reopening is free. The same holds for the production and transportation of the commodity. For the given instance, the CR-1I provides a better bound than the CR-2I formulation. The solution of the CR-1I model is $y_{j_0\ell_1 t_0} = 1.0$, $y_{j_0\ell_1 t_1} = 0.5$, $y_{j_0\ell_1 t_2} = 1.0$, $y_{j_1\ell_1 t_0} = 0.5$, $y_{j_1\ell_1 t_1} = 1.0$, $y_{j_1\ell_1 t_2} = 1.0$, $s_{j_0\ell_1 t_0} = 1.0$, $s_{j_1\ell_1 t_0} = 0.5$, $s_{j_1\ell_1 t_1} = 0.5$, $v_{j_0\ell_1 t_1}^c = 0.5$, $v_{j_0\ell_1 t_2}^o = 0.5$. The demand allocation variables have the values $x_{i_0 j_0 \ell_1 t_0} = 0.66$, $x_{i_0 j_1 \ell_1 t_0} = 0.33$, $x_{i_0 j_0 \ell_1 t_1} = 0.33$, $x_{i_0 j_1 \ell_1 t_1} = 0.66$, $x_{i_0 j_0 \ell_1 t_2} = 0.5$, $x_{i_0 j_1 \ell_1 t_2} = 0.5$. The cost of this solution is 2700\$.

In the solution of the CR-2I model, the binary decision variables have the following values: $s_{j_0\ell_1t_0t_2} = 1.0$, $s_{j_1\ell_1t_0t_0} = 0.5$, $y_{j_1\ell_1t_1t_2} = 0.5$, $y_{j_1\ell_1t_2t_2} = 0.5$. The demand allocation variables are as follows: $x_{i_0j_0\ell_1t_0} = 0.66$, $x_{i_0j_1\ell_1t_0} = 0.33$, $x_{i_0j_0\ell_1t_1} = 0.66$, $x_{i_0j_1\ell_1t_1} = 0.33$, $x_{i_0j_0\ell_1t_2} = 0.5$, $x_{i_0j_1\ell_1t_2} = 0.5$. The cost of this solution is 2650\$.

From the two parts (A) and (B) above it follows that $v(\overline{\text{CR-1I}}) \geq v(\overline{\text{CR-2I}})$. Q.E.D.

THEOREM 6. $v(\overline{\text{CR-GMC}}) \geq v(\overline{\text{CR-2I}})$.

Proof. The result follows by transitivity from Theorems 4 and 5. Q.E.D.

A.1.3. CR-2I+ is equally strong as CR-GMC and CR-1I

THEOREM 7. $v(\overline{\text{CR-1I}}) = v(\overline{\text{CR-2I+}})$.

Proof. It has already been shown that we can construct an equivalent and feasible $\overline{\text{CR-2I}}$ solution from any $\overline{\text{CR-1I}}$ solution. Due to the way the described algorithm assigns the values to the $s_{j\ell t_1 t_2}$ and $y_{j\ell t_1 t_2}$ variables as well as the direct relationship between these variables and the v^c and v^o variables, it can be shown that the new constraints (26) are also satisfied.

(A) Construction of a feasible $\overline{\text{CR-1I}}$ solution from any feasible $\overline{\text{CR-2I+}}$ solution

Consider any solution $\{x_{ij\ell t}, s_{j\ell t_1 t_2}, y_{j\ell t_1 t_2}\}$ that is feasible in $\overline{\text{CR-2I+}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, v_{j\ell t}^o, v_{j\ell t}^c\}$ that is feasible in $\overline{\text{CR-1I}}$ and that has the same value.

We set the values for the $x_{ij\ell t}$ variables identical to those in the CR-2I+ solution. The values for the variables $y_{j\ell t}$, $s_{j\ell t}$, $v_{j\ell t}^o$ and $v_{j\ell t}^c$ are set by establishing the following relations $\forall j \in J, \forall \ell \in L, \forall t \in T$:

$$s_{j\ell t} = \sum_{t_2=t}^{|T|} s_{j\ell t t_2} \quad (\text{CR.R7})$$

$$y_{j\ell t} = \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} s_{j\ell t_1 t_2} + \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} y_{j\ell t_1 t_2} \quad (\text{CR.R8})$$

$$v_{j\ell t}^o = \sum_{t_2=t}^{|T|} y_{j\ell t t_2} \quad (\text{CR.R9})$$

$$v_{j\ell t}^c = \sum_{t_1=1}^{t-1} s_{j\ell t_1(t-1)} + \sum_{t_1=1}^{t-1} y_{j\ell t_1(t-1)} \quad (\text{CR.R10a})$$

$$\Leftrightarrow v_{j\ell(t+1)}^c = \sum_{t_1=1}^t s_{j\ell t_1 t} + \sum_{t_1=1}^t y_{j\ell t_1 t} \quad (\text{CR.R10})$$

Constraints (11) are equivalent to constraints (19) and are thus satisfied. By using (CR.R8), constraints (12) correspond to constraints (23). Constraints (15) correspond to constraints (21) by using (CR.R7).

Replacing (CR.R9) and (CR.R10) in constraints (14) gives, $\forall j \in J, \forall \ell \in L, \forall t \in T$:

$$\begin{aligned}
& \sum_{t'=1}^t v_{j\ell t'}^o \leq \sum_{t'=1}^t v_{j\ell t'}^c \tag{14} \\
& \Leftrightarrow \sum_{t'=1}^t v_{j\ell t'}^o \leq \sum_{t'=0}^{t-1} v_{j\ell(t'+1)}^c \\
& \stackrel{(CR.R9) \& (CR.R10)}{\Leftrightarrow} \sum_{t'=1}^t \sum_{t_2=t'}^{|T|} y_{j\ell t' t_2} \leq \sum_{t'=0}^{t-1} \sum_{t_1=1}^{t'} s_{j\ell t_1 t'} + \sum_{t'=0}^{t-1} \sum_{t_1=1}^{t'} y_{j\ell t_1 t'} \\
& \Leftrightarrow \sum_{t'=1}^t \sum_{t_2=t'}^{|T|} y_{j\ell t' t_2} \leq \sum_{t_1=1}^{t-1} \sum_{t'=t_1}^{t-1} s_{j\ell t_1 t'} + \sum_{t_1=1}^{t-1} \sum_{t'=t_1}^{t-1} y_{j\ell t_1 t'},
\end{aligned}$$

which is true due to constraints (26). Thus, constraints (14) also hold. The feasibility of the flow conservation constraints (13) can be shown by replacing the variables by the terms given in the relations (CR.R7), (CR.R9) and (CR.R10a). By doing so, all terms on the LHS and RHS will cancel each other.

Given the relations (CR.R7)-(CR.10a) and the way the variable coefficients are composed in both formulations, it can easily be verified that both solutions have the same value. Both formulations are thus equally strong. Q.E.D.

THEOREM 8. $v(\overline{CR-GMC}) = v(\overline{CR-2I+})$.

Proof. The result follows by transitivity from Theorems 4 and 7. Q.E.D.

A.2. Theoretical Results for the DMCFLP_ER formulations

We now prove the dominance relationships for the three formulations presented for the DMCFLP_ER. Let $\overline{ER-GMC}$ be the linear programming relaxation of ER-GMC. In the same way, we denote $\overline{ER-1I}$ the linear programming relaxation of ER-1I and $\overline{ER-2I}$ the linear programming relaxation of ER-2I.

A.2.1. ER-1I and ER-2I are equally strong

We first prove that the LP relaxations of the formulations ER-1I and ER-2I provide the same lower bound.

THEOREM 9. $v(\overline{ER-1I}) = v(\overline{ER-2I})$.

Proof. The proof consists of two parts: First, we show how to construct a feasible solution for $\overline{ER-1I}$ from any feasible solution of $\overline{ER-2I}$ and that both solutions have the same objective function value. Then, we show the same, constructing an equivalent $\overline{ER-2I}$ solution based on any feasible $\overline{ER-1I}$ solution.

(A) Construction of a feasible $\overline{\text{ER-1I}}$ solution from any $\overline{\text{ER-2I}}$ solution

Consider any solution $\{x_{ij\ell t}, y'_{j\ell t_1 t_2}, y_{j\ell t}, s_{j\ell t}, w_{j\ell t}\}$ that is feasible in $\overline{\text{ER-2I}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, w_{j\ell t}\}$ that is feasible in $\overline{\text{ER-1I}}$ and has the same objective function value.

We set all variables $x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}$ and $w_{j\ell t}$ in the $\overline{\text{ER-1I}}$ formulation as given in the $\overline{\text{ER-2I}}$ solution. Given that both formulations have the same objective function, the solution value is also the same. Also observe that the formulations have the same constraints except for constraints (28) in the ER-1I and constraints (35) - (37) in the ER-2I formulation. The constraints that are part of both models (including the SIs) have the same variables with the same solution values in both solutions and are thus feasible. Therefore, we only have to show that constraints (28) are also feasible. We do so by replacing (35) - (37) in (28) for $\forall j \in J, \forall t \in T$:

$$\begin{aligned}
 & \sum_{\ell \in L} \ell y_{j\ell t} = \sum_{\ell \in L} \ell y_{j\ell(t-1)} + \sum_{\ell \in L} \ell s_{j\ell t} - \sum_{\ell \in L} \ell w_{j\ell t} \quad (28) \\
 \stackrel{(35)-(37)}{\Leftrightarrow} & \sum_{\ell \in L} \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} \ell y'_{j\ell t_1 t_2} = \sum_{\ell \in L} \sum_{t_1=1}^{t-1} \sum_{t_2=t-1}^{|T|} \ell y'_{j\ell t_1 t_2} + \sum_{\ell \in L} \sum_{t_2=t}^{|T|} \ell y'_{j\ell t t_2} - \sum_{\ell \in L} \sum_{t_1=1}^{t-1} \ell y'_{j\ell t_1(t-1)} \\
 \Leftrightarrow & \sum_{\ell \in L} \sum_{t_1=1}^{t-1} \sum_{t_2=t}^{|T|} \ell y'_{j\ell t_1 t_2} + \sum_{\ell \in L} \sum_{t_2=t}^{|T|} \ell y'_{j\ell t t_2} \\
 & = \sum_{\ell \in L} \sum_{t_1=1}^{t-1} \sum_{t_2=t-1}^{|T|} \ell y'_{j\ell t_1 t_2} + \sum_{\ell \in L} \sum_{t_2=t}^{|T|} \ell y'_{j\ell t t_2} - \sum_{\ell \in L} \sum_{t_1=1}^{t-1} \ell y'_{j\ell t_1(t-1)}.
 \end{aligned}$$

The remaining terms now cancel each other and therefore constraints (28) hold.

(B) Construction of a feasible $\overline{\text{ER-2I}}$ solution from any $\overline{\text{ER-1I}}$ solution

Consider any solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, w_{j\ell t}\}$ that is feasible in $\overline{\text{ER-1I}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y'_{j\ell t_1 t_2}, y_{j\ell t}, s_{j\ell t}, w_{j\ell t}\}$ that is feasible in $\overline{\text{ER-2I}}$ and has the same objective function value.

We set the values for the $x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}$ and $w_{j\ell t}$ variables in the $\overline{\text{ER-2I}}$ formulation as given in the $\overline{\text{ER-1I}}$ solution. Given that both formulations have the same objective function, the solution value in the objective function is also the same. All constraints (including the SIs), except for constraints (35) - (37), are the same as in formulation ER-1I and are therefore feasible.

We now set the solution values for the $y'_{j\ell t_1 t_2}$ variables. For each $\ell \in L$, we consider the diagram that describes the opening schedule of a facility of size ℓ . Each opening schedule is horizontally cut into blocks as described by Algorithm 1. Note that in the optimal solution, due to (28), an increase in $y_{j\ell t}$ by an amount of α necessarily means that $s_{j\ell t} = \alpha$ and $w_{j\ell t} = 0$, whereas a decrease in $y_{j\ell t}$ by an amount of α necessarily

means that $w_{j\ell t} = \alpha$ and $s_{j\ell t} = 0$. After separation, each of the separated blocks represents a variable $y'_{j\ell t_1 t_2}$ with a solution value greater than 0. The solution value of $s_{j\ell t}$ will be distributed over all $y'_{j\ell t_1 t_2}$ variables that start at $t_1 = t$, the solution value of $w_{j\ell t}$ will be distributed over all $y'_{j\ell t_1 t_2}$ variables that terminate at the end of $t_2 = t$ and the solution value of $y_{j\ell t}$ will be distributed over all $y'_{j\ell t_1 t_2}$ variables that start at or before t and terminate at or after t . Therefore, the following relationships hold:

$$\begin{aligned} \sum_{t_2=t}^{|T|} y'_{j\ell t t_2} &= s_{j\ell t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T \\ \sum_{t_1=1}^{t-1} y'_{j\ell t_1 (t-1)} &= w_{j\ell t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T \\ \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} y'_{j\ell t_1 t_2} &= y_{j\ell t} \quad \forall j \in J, \quad \forall \ell \in L, \quad \forall t \in T. \end{aligned}$$

Replacing these relationships in the constraints (35) - (37), respectively, shows that these constraints remain feasible.

From the two parts (A) and (B) above it follows that $v(\overline{ER-1I}) = v(\overline{ER-2I})$. Q.E.D.

A.2.2. ER-GMC is stronger than ER-1I and ER-2I

We now compare the strength of the ER-GMC and ER-1I formulations. We will prove that the ER-GMC formulation is at least as strong (strictly stronger for some instances) as the ER-1I formulation in the sense that its linear programming relaxations provides a better bound. By transitivity, the same result follows for the relation between the ER-GMC and ER-2I.

The ER-GMC formulation has the following form. Its objective function contains the costs for capacity expansion, capacity reduction and remaining at the same capacity level. Its constraints are the same as defined by the GMC:

$$\begin{aligned} \text{(ER-GMC)} \quad \min \quad & \sum_{j \in J} \sum_{\substack{\ell_1 \in L \\ \ell_2 \in L, \\ \ell_1 > \ell_2}} \sum_{t \in T} (f_{j(\ell_1 - \ell_2)}^c + F_{j\ell_2}^o) y_{j\ell_1 \ell_2 t} \\ & \sum_{j \in J} \sum_{\substack{\ell_1 \in L \\ \ell_2 \in L, \\ \ell_1 < \ell_2}} \sum_{t \in T} (f_{j(\ell_2 - \ell_1)}^o + F_{j\ell_2}^o) y_{j\ell_1 \ell_2 t} \\ & \sum_{j \in J} \sum_{\ell_1 \in L} \sum_{t \in T} F_{j\ell_1}^o y_{j\ell_1 \ell_1 t} + \sum_{i \in I} \sum_{j \in J} \sum_{\ell \in L} \sum_{t \in T} g_{ij\ell t} d_{it} x_{ij\ell t} \\ \text{s.t.} \quad & (2) - (7). \end{aligned}$$

THEOREM 10. $v(\overline{ER-GMC}) \geq v(\overline{ER-1I})$.

Proof. The proof consists of two parts: First, we show how to construct a feasible solution for $\overline{\text{ER-II}}$ from any feasible solution of $\overline{\text{ER-GMC}}$ and that both solutions have the same objective function value. Second, we provide a problem instance where $\overline{\text{ER-GMC}}$ provides a better bound than $\overline{\text{ER-II}}$.

(A) Construction of a feasible $\overline{\text{ER-II}}$ solution from any $\overline{\text{ER-GMC}}$ solution

Consider any solution $\{x_{ij\ell t}, y_{j\ell_1\ell_2 t}\}$ that is feasible in $\overline{\text{ER-GMC}}$. We now construct an equivalent solution $\{x_{ij\ell t}, y_{j\ell t}, s_{j\ell t}, w_{j\ell t}\}$ that is feasible in $\overline{\text{ER-II}}$ and has the same objective function value.

We deduce the values for the new variables from those of the existing solution variables $y_{j\ell_1\ell_2 t}$. Equalities (4) in the ER-GMC formulation conserve the flow for open facilities as it is found at the end of each planning period. They can be used to deduce the values for the y variables $\forall j \in J, \forall \ell \in L, \forall t \in T$:

$$y_{j\ell t} = \sum_{\ell_1 \in L} y_{j\ell_1 \ell t}. \quad (\text{ER.R1})$$

The same equalities (4) also lead to the following result:

$$y_{j\ell(t-1)} \stackrel{(\text{ER.R1})}{=} \sum_{\ell_1 \in L} y_{j\ell_1 \ell(t-1)} \stackrel{(4)}{=} \sum_{\ell_2 \in L} y_{j\ell \ell_2 t}. \quad (\text{ER.R2})$$

Furthermore, we set $s_{j\ell t}$ and $w_{j\ell t}$ as follows:

$$s_{j\ell t} = \sum_{\ell_1 \in L} y_{j\ell_1(\ell_1+\ell)t} \quad (\text{ER.R3})$$

$$w_{j\ell t} = \sum_{\ell_1 \in L} y_{j\ell_1(\ell_1-\ell)t}. \quad (\text{ER.R4})$$

Having set the variables for the ER-1I formulation, we now show that the equalities (28) still hold. We replace the variables by the deduced values according to (ER.R1)-(ER.R4):

$$\begin{aligned} \sum_{\ell \in L} \ell y_{j\ell t} + \sum_{\ell \in L} \ell w_{j\ell t} &= \sum_{\ell \in L} \ell y_{j\ell(t-1)} + \sum_{\ell \in L} \ell s_{j\ell t} \quad \forall j \in J, \quad \forall t \in T \quad (28) \\ &\stackrel{(\text{ER.R1})-(\text{ER.R4})}{\Leftrightarrow} \sum_{\ell_1 \in L} \sum_{\ell_2 \in L} \ell_2 y_{j\ell_1 \ell_2 t} + \sum_{\ell_1 \in L} \sum_{\ell \in L} \ell y_{j\ell_1(\ell_1-\ell)t} \\ &= \sum_{\ell_1 \in L} \sum_{\ell_2 \in L} \ell_1 y_{j\ell_1 \ell_2 t} + \sum_{\ell_1 \in L} \sum_{\ell \in L} \ell y_{j\ell_1(\ell_1+\ell)t} \quad \forall j \in J, \quad \forall t \in T. \quad (51) \end{aligned}$$

In the following, we prove that (51) is true by using the principle of induction:

Proposition: Equalities (51) are true for all sizes of L .

Basic cases: We start with the trivial case of $q = 1$, i.e., $L = \{0, 1\}$. Note that, for the sake of simplicity, we suppress the variable indices j and t , but indicate only the values for the indices ℓ_1 and ℓ_2 :

$$\begin{aligned}
LHS: & \sum_{\ell_1 \in L} \sum_{\ell_2 \in L} \ell_2 y_{j\ell_1\ell_2 t} \rightarrow 0y_{00} + 1y_{01} + 0y_{10} + 1y_{11} \\
& \sum_{\ell_1 \in L} \sum_{\ell \in L} \ell y_{j\ell_1(\ell_1-\ell)t} \rightarrow 1y_{10} \\
RHS: & \sum_{\ell_1 \in L} \sum_{\ell_2 \in L} \ell_1 y_{j\ell_1\ell_2 t} \rightarrow 0y_{00} + 0y_{01} + 1y_{10} + 1y_{11} \\
& \sum_{\ell_1 \in L} \sum_{\ell \in L} \ell y_{j\ell_1(\ell_1+\ell)t} \rightarrow 1y_{01}
\end{aligned}$$

It can be easily verified that the terms on the LHS equal the terms on the RHS. The proposition is thus true for $q = 1$.

Inductive step: We now show that the proposition also holds for $q = q + 1$. For $q + 1$, the LHS and RHS include the same terms as in the previous step. In addition, the following terms are added:

$$\begin{aligned}
LHS: & \sum_{\ell_2=0}^{q+1} \ell_2 y_{j(q+1)\ell_2 t} + \sum_{\ell_1=0}^q (q+1) y_{j\ell_1(q+1)t} \\
& + \sum_{\ell_2=0}^{q+1} (q+1-\ell_2) y_{j(q+1)\ell_2 t} \\
RHS: & \sum_{\ell_2=0}^{q+1} (q+1) y_{j(q+1)\ell_2 t} + \sum_{\ell_1=0}^q \ell_1 y_{j\ell_1(q+1)t} \\
& + \sum_{\ell_1=0}^{q+1} (q+1-\ell_1) y_{j\ell_1(q+1)t}
\end{aligned}$$

Summing up all terms on the LHS and all terms on the RHS shows that both sides are equivalent.

Hence the result follows by induction.

Therefore, constraints (28) are satisfied. For the $x_{ij\ell t}$ variables, we choose the same solution values as in the ER-GMC solution. Constraints (11) are therefore necessarily satisfied. In addition, the demand allocation contributes equally to the objective function in both formulations. Constraints (12) are also satisfied, as can

be verified by replacing the $y_{j\ell_1\ell_2t}$ variables in constraints (3) by (ER.R1). The limiting constraints (see Section 4.2) are also satisfied by noting that each of the variables can be replaced by corresponding $y_{j\ell_1\ell_2t}$ variables and the sum of all $y_{j\ell_1\ell_2t}$ variables never exceeds 1. Finally, the SIs are feasible due to relationship (ER.R1).

The contribution of the variables $y_{j\ell t}$, $s_{j\ell t}$ and $w_{j\ell t}$ to the total solution costs is equivalent to that of the $y_{j\ell_1\ell_2t}$ variables. This can be easily shown by verifying the equalities (ER.R1)-(ER.R4) and the costs attributed to the $y_{j\ell_1\ell_2t}$ in Section 3.2.

(B) Problem instance where ER-GMC is stronger

We now explain, by the use of a small problem instance, under which circumstances the ER-GMC provides a better LP bound than the ER-1I and ER-2I formulations.

This instance contains one potential facility location and one client. The planning horizon contains one time period, in which the customer has a demand of 10 units. Production and transportation of the commodities is free. The maximum capacity level is 2. The capacity expansion costs 200\$ for one capacity level and 350\$ for two capacity levels. The capacity reduction costs are set to 20\$ for one capacity level and to 35\$ for two capacity levels. The maintenance costs for a facility is 300\$ at capacity level 1 and 500\$ at capacity level 2. The facility capacity is 10 at level 1 and 11 at level 2. Therefore, the costs to provide and maintain a certain amount of capacity do not follow the principle of economies of scale.

The ER-GMC formulation provides a better bound than the other formulations. The LP relaxation for the ER-GMC sets $y_{j_0\ell_0\ell_1t_0} = 1.0$ and $x_{i_0j_0\ell_1t_0} = 1.0$, resulting in a solution with a total cost of 500\$. For the ER-1I and ER-2I formulations, with and without the SIs, the optimal LP relaxation solution constructs half a level 2 facility, while allocating demand to a full level 1 facility. The decision variables linked to the objective function thus have the solution values $y_{j_0\ell_1t_0} = 1.0$ and $s_{j_0\ell_2t_0} = 0.5$. This solution has a total cost of 475\$.

From the two parts (A) and (B) above, it follows: $v(\overline{\text{ER-GMC}}) \geq v(\overline{\text{ER-1I}})$. Q.E.D.

THEOREM 11. $v(\overline{\text{ER-GMC}}) \geq v(\overline{\text{ER-2I}})$.

Proof. The result follows by transitivity from Theorems 9 and 10. Q.E.D.

B. Test Instances

Instances for multi-period facility location problems essentially contain information about the customer demand for each time period, the construction costs of the facilities and the costs to allocate demand between customers and facilities. The DFLPG and the three special cases additionally involve capacity changes. Due to the lack of openly available instance sets that include these properties, a large set of problem instances has been generated to test the proposed models. The instances can be divided into five different sets, each with a different number of time periods. Each of these sets contains a total of 288 instances, 96 for each capacity level.

In the following we present how the instance properties are generated and which parameters are used.

B.1. Number of time periods

Instances have been generated with different lengths of the planning horizon $|T|$, chosen such that $|T| \in \{6, 8, 10, 12, 14\}$. Most of the computational results in this work are based on the instances with $|T| = 12$, as this is a very common discretization in practice, for example for a monthly representation of an entire year.

B.2. Problem dimension

Instances have been generated with different numbers of customers $|I|$ and candidate facility locations $|J|$. We considered the following dimensions $(|J|/|I|)$, always assuming that $|J| \leq |I|$: $(10/20)$, $(10/50)$, $(50/50)$, $(50/100)$, $(50/250)$, $(100/250)$, $(100/500)$ and $(100/1000)$.

B.3. Number of capacity levels

The number of capacity levels q also impacts on the size of the models. Instances are generated with a maximum of 3, 5 and 10 capacity levels, which are assumed to be reasonable values for a broad variety of different application contexts.

The capacities $u_{j\ell}$ are generated based on the total number of customers and are chosen such that a considerably large number of facilities (about half of the candidate locations) is selected. The larger the set of customers, the higher is the capacity of each level. To be precise, we set $u_{j1} = 150$ if the instance covers 20 customers, $u_{j1} = 300$ if the instance covers 50 customers, $u_{j1} = 600$ if the instance covers 100 customers, $u_{j1} = 1200$ if the instance covers 250 customers, $u_{j1} = 2500$ if the instance covers 500 customers and $u_{j1} = 5000$ if the instance covers 1000 customers. The capacities of higher capacity levels $\ell \geq 2$ are set

as multiples of the first capacity level, i.e., $u_{j\ell} = \ell \cdot u_{j1}$. Note that we assume that the problem instances do not contain initially existing facilities, i.e., the initial capacity level of each facility is 0.

B.4. Customer/facility locations

For each of the different problem sizes, $|I|$ customer demand points have been randomly generated following a continuous uniform distribution, rounding the x and y coordinates to the next lowest integer value. The first $|J|$ points of $|I|$ customer locations have additionally been defined as candidate facility locations and therefore coincide with the customer demand points. The networks were generated on squares of the following three sizes: $300km$, $380km$ and $450km$.

B.5. Demand allocation costs

Costs are divided into fixed and variable costs. Fixed costs are given by the construction of facilities and the change of their capacity levels. Variable costs are composed of the costs to produce and transport the commodities.

Transportation costs have been computed based on the Euclidean distance between the points, including a small modification that results in a slight clustering effect of the customers close to a facility. The transportation costs are composed of two components:

1. A cost that depends on the total distance, referred to as the vehicle cost. The vehicle cost is linear in function of the Euclidean distance between the two points on the network (5\$/km).
2. A cost that depends on the travel time, referred to as the driver's payment. The driver's payment is 0 if the two points are within one hour of transportation distance (assuming an average vehicle speed of 62km/h) and linear in function of the Euclidean distance if the two points are at more than one hour of driving distance (50\$/h).

Let $dist_{ij}$ denote the distance between facility location j and customer i . The costs to transport one unit of demand from facility j to customer i is therefore set to:

$$g_{ij}^t = 5 \cdot dist_{ij} + 50 \cdot \max\left(0, \frac{dist_{ij}}{62} - 1\right)$$

The variable and fixed costs include economies of scale in function of the size of the facility. These costs are therefore described by concave cost functions, as explained in the following. The production costs for each unit served from a facility to a customer is defined as the cost to operate a facility and depends on

the size of the facility. The cost to produce one commodity unit at capacity level 1 is set to 20.90\$. At each higher capacity level, the production cost is 3% cheaper than at the previous level:

$$g_{j0}^p = 20.90$$

$$g_{j\ell}^p = 0.97 \cdot g_{j(\ell-1)}^p$$

Note that the production costs are added to the transportation costs to determine the total demand allocation costs $g_{ij\ell t}$ to serve the customer demands:

$$g_{ij\ell t} = g_{ij}^t + g_{j\ell}^p$$

In addition to the demand allocation costs as discussed above, a second set of instances was generated with five times higher transportation costs.

B.6. Fixed costs

The construction cost, also referred to as capacity expansion cost, is set to 100,000\$ for a facility of level 1. Each additional capacity level is 10% cheaper than the previous one. The construction costs for facilities of different capacity levels are therefore computed according to the following formula:

$$f_{j1}^o = 100,000$$

$$f_{j2}^o = 190,000$$

$$f_{j\ell}^o = f_{j(\ell-1)}^o + 0.9 \cdot (f_{j(\ell-1)}^o - f_{j(\ell-2)}^o)$$

The maintenance costs for a facility of a certain size are computed in a similar fashion. They are set relatively high to motivate capacity changes. The maintenance costs for a facility of capacity level 1 are set to 51,000\$. The maintenance costs for each additional capacity level are 15% cheaper than the previous ones:

$$F_{j1}^o = 51,000$$

$$F_{j2}^o = 94,350$$

$$F_{j\ell}^o = F_{j(\ell-1)}^o + 0.85 \cdot (F_{j(\ell-1)}^o - F_{j(\ell-2)}^o)$$

The cost to reduce the capacity of a facility by ℓ capacity levels is set to 10% of the costs to expand the capacity of a facility by ℓ capacity levels. Finally, the costs for reopening and closing existing facilities have been adopted from the input data of an industrial application (Jena et al. 2012). Although

being strictly increasing, these costs do not necessarily represent economies of scale. The costs to reopen a closed facility of capacity level $1, \dots, 10$ are 3,138.34\$, 4,084.69\$, 4,924.58\$, 5,693.26\$, 7,085.07\$, 7,727.50\$, 8,342.34\$, 8,933.68\$, 10,057.70\$ and 10,594.80\$, respectively. The costs to temporarily close an open facility of capacity level $1, \dots, 10$ are 8,624.93\$, 11,595.80\$, 14,305.60\$, 16,836.50\$, 21,524.10\$, 23,727.90\$, 25,858.30\$, 27,925.70\$, 31,901.10\$ and 33,820.70\$, respectively.

B.7. Demand distribution

We consider two different demand scenarios. In both scenarios, the demand for each of the customers is randomly generated and randomly distributed over time. The two scenarios differ in their total demand summed over all customers in each time period. In the first scenario (*regular*), the total demand is similar in each time period. We set the average demand for a customer to 12 units per time period. The total demand for all customers is therefore approximately $12 \cdot |I|$ units at each time period. The second scenario (*irregular*) assumes that the total demand follows strong variations along time and therefore varies at each time period. In this scenario, the total demand for all customers is multiplied by a random distortion factor at each time period. This random distortion factor represents the variation of the total customer demand when compared to the reference value given by $12 \cdot |I|$ units. It is set to the absolute value of a normal random variable with mean value 1.0 and standard deviation 0.6 (note that this procedure produced distortion factors from 0.14 to 2.24). Let $totDem_t$ be the total customer demand for time period t , computed as explained above for one of the two scenarios.

We now explain how the individual demands for each of the customers are generated and distributed on the different time periods such that its total sum equals approximately the value of $totDem_t$ at each of the time periods. For all customers and all time periods, the total demand covers approximately $12 \cdot |I| \cdot |T|$ units. In a first step, this total demand is randomly distributed on each of the customers. In a second step, each customer demand is distributed on different time periods:

1. Let $totRemDem$ denote the total demand for all customers and time periods that has not yet been allocated to any customer. Furthermore, let $numRemCust$ indicate the number of customers that have not yet been allocated any demand. For each customer, its total demand for all time periods, denoted to $totJDem_j$, is computed as a random normal variable with a mean $\mu = totRemDemand/numRemCust$ and standard deviation $\sigma = \mu/2$. Note that this method did not produce any negative value in the course of our instance generation.

2. The total demand for each customer, $totJDem_j$ is then divided into four equal parts. One part of the demand is allocated to a time period that is randomly selected following a uniform distribution. Each of the other three parts is allocated to the time period t that has the highest gap between the total demand yet allocated to period t and its value $totDem_t$.

Note that the choice of allocating demand to only a few of the time periods is motivated by the aforementioned industrial application in the forest industry, where each logging region is harvested, on average, about four seasons over a ten-period planning horizon. Furthermore, it results in a geographically more dispersed distribution of the demand which creates the need to adjust capacities at the facilities.

C. Model Sizes

This section summarizes the model sizes expected for each of the problem variants and formulations. Tables 10, 11 and 12 show the model sizes for problem variants DMCFLP_CR, DMCFLP_ER and DMCFLP_CR_ER, respectively. All instances possess twelve time periods.

The number of continuous x variables (*Cont. vars / SIs*) is identical for all formulations of the same problem variant. This number also represents the number of Strong Inequalities, as there is exactly one SI for each x variable. For each of the formulations, the number of additional binary variables (*Bin. var.*) and the number of constraints (*Constr.*) is given.

When the instances are generated, the total demand for each customer is randomly distributed on four time periods. For some customers, a time period may be selected more than once. Therefore, some customers may have less than four time periods with non-zero demands and the total number of non-negative demands may not necessarily equal $4 \cdot |I| \cdot |T|$. The total number of non-negative customer demands directly impacts the number of continuous x variables and the number of demand constraints. Thus, for some of the instances, the number of continuous variables and constraints may be slightly smaller than the numbers indicated in the tables. As demands are generated independently for each of the three networks (see Section B.4), instances with the same numbers of customers may have different numbers of x variables and demand constraints for each network. An analysis showed that each of the instances possesses at least 90% of the $4 \cdot |I| \cdot |T|$ non-negative customer demands.

The number of Aggregated Demand Constraints is always equal to the number of time periods (i.e., in this case 12).

Note that, even though some of the instances are very large, the small integrality gaps for large instances enable CPLEX to efficiently solve many of such instances. From the memory point of view, a large (100/1000) instance for the problem variant CRER consumed about 20 GB of memory for the CRER-GMC formulation.

q	Instance size	Cont. vars / SIs	CR-GMC		CR-1I		CR-2I+	
			Bin. var.	Constr.	Bin. var.	Constr.	Bin. var.	Constr.
3	10/20	2,400	1,920	1,280	1,440	1,160	4,680	930
	10/50	6,000	1,920	1,400	1,440	1,280	4,680	1,050
	50/50	30,000	9,600	6,200	7,200	5,600	23,400	4,450
	50/100	60,000	9,600	6,400	7,200	5,800	23,400	4,650
	50/250	150,000	9,600	7,000	7,200	6,400	23,400	5,250
	100/250	300,000	19,200	13,000	14,400	11,800	46,800	9,500
	100/500	600,000	19,200	14,000	14,400	12,800	46,800	10,500
	100/1000	1,200,000	19,200	16,000	14,400	14,800	46,800	12,500
5	10/20	4,000	3,120	2,000	2,400	1,880	7,800	1,410
	10/50	10,000	3,120	2,120	2,400	2,000	7,800	1,530
	50/50	50,000	15,600	9,800	12,000	9,200	39,000	6,850
	50/100	100,000	15,600	10,000	12,000	9,400	39,000	7,050
	50/250	250,000	15,600	10,600	12,000	10,000	39,000	7,650
	100/250	500,000	31,200	20,200	24,000	19,000	78,000	14,300
	100/500	1,000,000	31,200	21,200	24,000	20,000	78,000	15,300
	100/1000	2,000,000	31,200	23,200	24,000	22,000	78,000	17,300
10	10/20	8,000	6,120	3,800	4,800	3,680	15,600	2,610
	10/50	20,000	6,120	3,920	4,800	3,800	15,600	2,730
	50/50	100,000	30,600	18,800	24,000	18,200	78,000	12,850
	50/100	200,000	30,600	19,000	24,000	18,400	78,000	13,050
	50/250	500,000	30,600	19,600	24,000	19,000	78,000	13,650
	100/250	1,000,000	61,200	38,200	48,000	37,000	156,000	26,300
	100/500	2,000,000	61,200	39,200	48,000	38,000	156,000	27,300
	100/1000	4,000,000	61,200	41,200	48,000	40,000	156,000	29,300

Table 10 Model sizes for the formulations CR-GMC, CR-1I and CR-2I+.

q	Instance size	Cont. vars / SIs	ER-GMC		ER-1I		ER-2I	
			Bin. var.	Constr.	Bin. var.	Constr.	Bin. var.	Constr.
3	10/20	2,400	1,920	920	1,080	920	3,420	1,160
	10/50	6,000	1,920	1,400	1,080	2,000	3,420	1,280
	50/50	30,000	9,600	6,200	5,400	9,200	17,100	5,600
	50/100	60,000	9,600	6,400	5,400	9,400	17,100	5,800
	50/250	150,000	9,600	7,000	5,400	10,000	17,100	6,400
	100/250	300,000	19,200	13,000	10,800	19,000	34,200	11,800
	100/500	600,000	19,200	14,000	10,800	20,000	34,200	12,800
	100/1000	1,200,000	19,200	16,000	10,800	22,000	34,200	14,800
5	10/20	4,000	4,320	2,000	1,800	1,160	5,700	1,400
	10/50	10,000	4,320	2,120	1,800	3,200	5,700	1,520
	50/50	50,000	21,600	9,800	9,000	15,200	28,500	6,800
	50/100	100,000	21,600	10,000	9,000	15,400	28,500	7,000
	50/250	250,000	21,600	10,600	9,000	16,000	28,500	7,600
	100/250	500,000	43,200	20,200	18,000	31,000	57,000	14,200
	100/500	1,000,000	43,200	21,200	18,000	32,000	57,000	15,200
	100/1000	2,000,000	43,200	23,200	18,000	34,000	57,000	17,200
10	10/20	8,000	6,120	3,800	3,600	1,760	11,400	2,000
	10/50	20,000	6,120	3,920	3,600	6,200	11,400	2,120
	50/50	100,000	30,600	18,800	18,000	30,200	57,000	9,800
	50/100	200,000	30,600	19,000	18,000	30,400	57,000	10,000
	50/250	500,000	30,600	19,600	18,000	31,000	57,000	10,600
	100/250	1,000,000	61,200	38,200	36,000	61,000	114,000	20,200
	100/500	2,000,000	61,200	39,200	36,000	62,000	114,000	21,200
	100/1000	4,000,000	61,200	41,200	36,000	64,000	114,000	23,200

Table 11 Model sizes for the formulations ER-GMC, ER-1I and ER-2I.

q	Instance size	Cont. vars / SIs	CRER-GMC		CRER-1I	
			Bin. var.	Constr.	Bin. var.	Constr.
3	10/20	2,400	3,720	1,280	2,160	1,040
	10/50	6,000	3,720	1,400	2,160	1,160
	50/50	30,000	18,600	6,200	10,800	5,000
	50/100	60,000	18,600	6,400	10,800	5,200
	50/250	150,000	18,600	7,000	10,800	5,800
	100/250	300,000	37,200	13,000	21,600	10,600
	100/500	600,000	37,200	14,000	21,600	11,600
	100/1000	1,200,000	37,200	16,000	21,600	13,600
5	10/20	4,000	8,520	2,000	3,600	1,280
	10/50	10,000	8,520	2,120	3,600	1,400
	50/50	50,000	42,600	9,800	18,000	6,200
	50/100	100,000	42,600	10,000	18,000	6,400
	50/250	250,000	42,600	10,600	18,000	7,000
	100/250	500,000	85,200	20,200	36,000	13,000
	100/500	1,000,000	85,200	21,200	36,000	14,000
	100/1000	2,000,000	85,200	23,200	36,000	16,000
10	10/20	8,000	28,920	3,800	7,200	1,880
	10/50	20,000	28,920	3,920	7,200	2,000
	50/50	100,000	144,600	18,800	36,000	9,200
	50/100	200,000	144,600	19,000	36,000	9,400
	50/250	500,000	144,600	19,600	36,000	10,000
	100/250	1,000,000	289,200	38,200	72,000	19,000
	100/500	2,000,000	289,200	39,200	72,000	20,000
	100/1000	4,000,000	289,200	41,200	72,000	22,000

Table 12 Model sizes for the formulations CRER-GMC and CRER-1I.