Enabling Urban Logistics Services at *La Poste* through Multi-Echelon Location-Routing

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Online Supplement

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1. Capacitated Vehicle Routing

For the vehicle routing aspect of this work we consider a special case of the capacitated vehicle routing problem (CVRP) in which vehicle routes are capacity restricted in two ways: (i) vehicle carrying capacities may not be exceeded; and (ii) the total travel time of each vehicle may not exceed the global maximum service time (MST) constraint. The CVRP is a hard combinatorial optimization problem, and only instances of limited size can be solved exactly to optimality (Cordeau et al. 2002). Thus, since the 1960s, numerous heuristics and meta-heuristics for the CVRP have been proposed (for recent surveys, see e.g. Golden et al. (2008) and Laporte (2009)).

Early works on solving the CVRP, often referred to as *classical heuristics*, generally perform a rather limited exploration of the search space and thus yield feasible solutions within very short computation times; see, for example, Clarke and Wright (1964), Gillett and Miller (1974), Fisher and Jaikumar (1981). The high solution speed of these heuristics comes at the cost of solution values that are from 2% to 10% higher than the optimum or the best known solution (Laporte et al. 2000, Cordeau et al. 2002, Gendreau et al. 2001).

Since the 1990s, research has mainly focused on developing *meta-heuristics* for the CVRP that build on exploring promising regions of the solution space more intensively by incorporating principles such as local or neighborhood search, population search, memory structures, and solution recombination (cf. Cordeau et al. 2002, Laporte et al. 2000). Although these meta-heuristics produce solution values that are often within 0.5% of the optimum or the best known solution (Gendreau et al. 2001), they typically require substantial amounts of computation time (Cordeau et al. 2002).

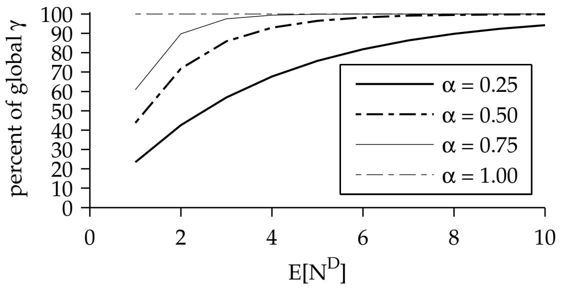
In their reviews of vehicle routing problem (VRP) meta-heuristics, Laporte et al. (2000) and Gendreau et al. (2001) conclude that tabu search (TS) meta-heuristics have proven most effective in the past and have significant potential for future improvements, e.g. by exploiting the principle of granularity to form more compact yet more promising local search neighborhoods (cf. Toth and Vigo 2003), as well as by the use parallel computing (for a survey, see e.g. Crainic 2008). More recent applications of parallel solution approaches to the CVRP include Doerner et al. (2006), Groër et al. (2011), Cordeau and Maischberger (2012), and Jin et al. (2012).

2. Stop Density Dilution and Stop Multiplication

Equations (36) and (37) in the main document reveal that the extent to which architecture-specific expected stop densities $\hat{\gamma_i}^S$ and $\hat{\gamma_i}^L$ deteriorate relative to the global stop density γ_i is closely linked to the nature of N_i^P and N_i^D, the probability distributions of the numbers of pickup and delivery items per stop in the respective city segment. If the expected values of these numbers are sufficiently large and if their coefficients of variation (c_i^P and c_i^D) are sufficiently small, then $\hat{\gamma_i}^S$ and $\hat{\gamma_i}^L$ remain relatively close to the level of γ_i as $\lim_{E[N_i^P],E[N_i^D]\to\infty} p_i^{\gamma,S}, p_i^{\gamma,L} = 1$ holds. The lower the expected values of N_i^P and N_i^D and the higher their coefficients of variation, the stronger the stop density dilution effect. We can show in particular that, as $c_i^P, c_i^D \to 0$,

$$\lim_{E[N_i^P],E[N_i^D]\to 1} p_i^{\gamma,S} = 1 - p_i^{P,L} p_i^{D,L},$$
$$\lim_{E[N_i^P],E[N_i^D]\to 1} p_i^{\gamma,L} = p_i^{P,S} + p_i^{D,S} - p_i^{P,S} p_i^{D,S},$$
$$\lim_{E[N_i^P],E[N_i^D]\to 0} p_i^{\gamma,S}, p_i^{\gamma,L} = 0.$$

In Figure 1 we plot the stop density dilution effect as a function of the expected number of items per stop for different values of α. The figure also shows that the volume-weighted average of the network-specific stop densities is always less than or equal to the global stop density γ_i. That average's convexity over α is decreasing in the expected number of items per stop, $E[N_i^P] + E[N_i^D]$. The stop density dilution effect is a major contributor to the total cost of operation's concavity



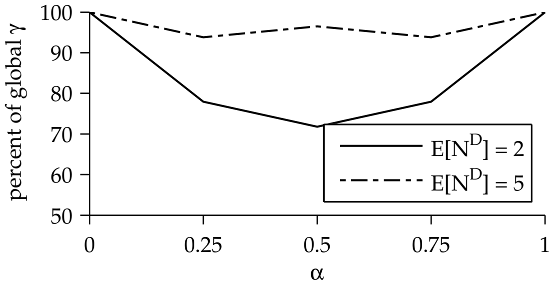


Figure 1 **Stop density dilution. Left: $3/T/T$-network–specific stop density as a percentage of global stop density. Right: Volume-weighted average of network-specific stop densities as a percentage of global stop density.**

over α because it makes cost components that are sensitive to the stop density (e.g., the cost for an inter-stop transfer) a concave function over α.

Our numerical analyses suggest also that the overall number of stops—and likewise some major contributors to the total cost of operation—are concave functions over α. Hence the stop multiplication effect, unlike stop density dilution, is increasing in the expected number of items per stop. As a result, the net effect of the expected number of items per stop on the concavity (over α) of total operational costs is ambiguous.

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