Appendix A. Proof of Lemma 2

Suppose that user A schedules a mass departure at time t. Clearly, user B will not depart either at t or immediately after t since it would be better to depart either just before the mass or some time after t when the queue caused by the mass departure has diminished or disappeared. To prove that a mass cannot be part of a PSNE we will show that user A will never schedule a mass departure at t unless user B is scheduling vehicles either at t or immediately after t, which, as just argued, user B is not willing to do.

Thus, suppose that user A is considering a mass departure of M vehicles at time t, and assume that user B does not depart during the period $\left[t, \breve{t}\right]$, where $\breve{t} > t$. Variables M, t, and $\breve{t}$ are held fixed throughout the proof. The logic of the proof depends on whether or not a queue exists at t.

Appendix A.1. A queue exists at t

Assume $Q\left(t\right) > 0$. Pick any time $t' \in \left(t, \breve{t}\right]$ such that $Q\left(t'\right) > 0$. We will show that user A is better off postponing the mass departure from t to t'. Let $m \in [0, M]$ index vehicles in the order they are positioned in the mass, and let $c(m,t)$ denote the cost incurred by vehicle m when the mass departs at t:

$$c(m,t) = \alpha \cdot \left(T(t) + \frac{m}{s}\right) + \begin{cases} \beta \cdot \left(t^* - t - T\left(t\right) - \frac{m}{s}\right) & \text{if } \ t + T\left(t\right) + \frac{m}{s} \le t^* \\ \gamma \cdot \left(t + T\left(t\right) + \frac{m}{s} - t^*\right) & \text{if } \ t + T\left(t\right) + \frac{m}{s} \ge t^* \end{cases}.$$

If the mass is rescheduled to depart at $t^{'}$ instead, vehicle m incurs a cost:

$$c(m,t') = \alpha \cdot \left(T(t') + \frac{m}{s}\right) + \begin{cases} \beta \cdot \left(t^* - t' - T\left(t'\right) - \frac{m}{s}\right) & \text{if } \ t' + T\left(t'\right) + \frac{m}{s} \le t^* \\ \gamma \cdot \left(t' + T\left(t'\right) + \frac{m}{s} - t^*\right) & \text{if } \ t' + T\left(t'\right) + \frac{m}{s} \ge t^* \end{cases}.$$

Now, vehicle m arrives at the same time whether the mass is scheduled at t or $t^{'}$ since a queue remains at t' and user B does not depart between t and t'. Hence $t' + T(t') = t + T(t)$, and $c(m,t')$ can be written:

$$c(m,t') = \alpha \cdot \left(T(t) + \frac{m}{s}\right) - \alpha \cdot (t^{'} - t) + \begin{cases} \beta \cdot \left(t^{*} - t - T\left(t\right) - \frac{m}{s}\right) & \text{if } \ t + T\left(t\right) + \frac{m}{s} \leq t^{*} \\ \gamma \cdot \left(t + T\left(t\right) + \frac{m}{s} - t^{*}\right) & \text{if } \ t + T\left(t\right) + \frac{m}{s} \geq t^{*} \end{cases}$$

$$= c(m,t) - \alpha \cdot (t^{'} - t) < c(m,t).$$

Postponing the mass departure therefore reduces costs for all M vehicles in the mass.

Appendix A.2. No queue exists at t

Assume $Q\left(t\right) = 0$ and set $t' = \breve{t}$. If $M \leq s \cdot (t' - t)$, vehicles provisionally scheduled in the mass can instead be rescheduled to depart at rate s for a period of duration M/s. The M vehicles arrive at the same time as with the mass departure, but without incurring a queuing delay. If $M > s \cdot (t' - t)$, then $s \cdot (t' - t)$ of the vehicles can be rescheduled to depart at rate s for a period $t' - t$, and the remaining $M - s \cdot (t' - t)$ vehicles can be rescheduled to depart in a mass at $t^{'}$. All M vehicles will arrive at the same time as they do in the mass departure at t. The first $s \cdot (t' - t)$ vehicles do not queue at all, and the last $M - s \cdot (t' - t)$ vehicles do not queue during $[t, t']$. Total queuing costs are therefore reduced, and total schedule delay costs are unchanged. Postponing the mass departure again reduces fleet costs, so that the mass departure is not optimal. *QED*

Appendix B. Proposition 2: Proof of Result 4 for mass departures

Suppose that user A deviates from the candidate PSNE by scheduling multiple mass departures. We prove that such a deviation cannot reduce user A's costs. The proof is done in two steps. We first show that any deviation with multiple mass departures does not achieve lower costs than a deviation with a single mass departure launched before t^{*} (Appendix B.1). We then show that this deviation is not gainful if the bounds on departure rates stated in Proposition 2 are satisfied (Appendix B.2).

Appendix B.1. Optimality of the single mass departure deviation

We show that any deviation from the PSNE involving multiple mass departures is dominated by a single mass departure. The proof involves establishing the following three results for either user: (i) Fleet costs can be (weakly) reduced by rescheduling any vehicles that suffer queuing delay, but are not part of a mass, to a period without queuing. (ii) Fleet costs can be (weakly) reduced by rescheduling any vehicles in a mass departure after t^* to a period without queuing. (iii) Any deviation with multiple mass departures launched before t^* entails strictly higher fleet costs than a deviation with a single mass departure. These three results establish that the candidate PSNE need only be tested against a single mass departure launched before t^*.

Result i: When a queue exists, user A is willing to depart at a positive and finite rate only if condition (17) is satisfied. For early arrivals this requires $r_B(t) = \alpha \cdot s / (\alpha - \beta) > s$. Since $r_B(t) < s$ in the candidate PSNE, condition (17) is violated, and user A will not deviate by departing early when there is a queue. For late arrivals, user A is willing to depart at a positive and finite rate when there is a queue only if $r_B(t) = \alpha \cdot s / (\alpha + \gamma)$. Proposition 2 stipulates that $r_B(t) \leq \alpha \cdot s / (\alpha + \gamma)$. If $r_B(t) < \alpha \cdot s / (\alpha + \gamma)$, user A is better off scheduling its vehicles later. If $r_B(t) = \alpha \cdot s / (\alpha + \gamma)$, user A is indifferent between departing or not, and would not gain by rescheduling vehicles late when there is a queue.

Result ii: Assume that the last mass departure is launched at a time of late arrivals. We show that rescheduling vehicles in the mass to a later period in which they do not incur queuing delay is beneficial. By induction, it then follows that all mass departures launched at times of late arrivals can be gainfully rescheduled.

Suppose the last mass departure is launched at time t_L and comprises M vehicles. Assume first that there is a queue at t_L. We show that postponing the mass departure to the moment when the queue disappears (weakly) reduces fleet costs. Let $m \in [0, M]$ index vehicles in the order they are positioned in the mass, and let $c(m, t_L)$ denote the cost incurred by vehicle m:

$$c(m, t_L) = \alpha \cdot \left(T(t_L) + \frac{m}{s}\right) + \gamma \cdot \left(t_L + T(t_L) + \frac{m}{s} - t^*\right) .$$

Now suppose the mass is postponed to $t_L^{'} > t_L$ when a queue still exists. Vehicle m now incurs a cost of:

$$c(m, t_L^{'}) = \alpha \cdot \left(T(t_L^{'}) + \frac{m}{s}\right) + \gamma \cdot \left(t_L^{'} + T(t_L^{'}) + \frac{m}{s} - t^*\right) . \tag{B.1}$$

Since only user B departs during $(t_L, t_L^{'})$,

$$T(t_L^{'}) = T(t_L) + \int_{t_L}^{t_L^{'}} \frac{r_B(u) - s}{s} du = T(t_L) - (t_L^{'} - t_L) \cdot \left(1 - \lambda^B_{t_L, t_L^{'}}\right) . \tag{B.2}$$

Substituting (B.2) in (B.1), we get:

$$c(m, t_L^{'}) = c(m, t_L) - \alpha \cdot (t_L^{'} - t_L) \cdot \left(1 - \lambda^B_{t_L, t_L^{'}}\right) + \gamma \cdot (t_L^{'} - t_L) \cdot \lambda^B_{t_L, t_L^{'}} .$$

Postponing the mass departure changes costs by:

$$c(m, t_L^{'}) - c(m, t_L) = (t_L^{'} - t_L) \cdot \left[\lambda^B_{t_L, t_L^{'}} \cdot (\alpha + \gamma) - \alpha\right] \leq 0 ,$$

where the inequality follows from the condition $r_B\,(t) \leq \alpha \cdot s/(\alpha + \gamma)$ in Proposition 2.

We conclude that if there is a queue when the last mass departs, the mass departure can be postponed to the time where the queue just disappears without increasing costs (later vehicles are not affected by postponing the mass).

Now assume there is no queue at t_L when the last mass is launched. We show that vehicles in the mass can be rescheduled later to a time when they do not face a queue, and that doing so does not increase user A's fleet costs. Consider the cost of the last (i.e., M-th) vehicle in the mass:

$$c(M, t_L) = \gamma \cdot (t_L - t^*) + (\alpha + \gamma) \cdot \frac{M}{s} \ .$$

Because user A does not depart until the queue has disappeared (result i), the queue produced by the mass departure disappears at time $t_L^{'}$ where:

$$\frac{M}{s} + \int_{t_L}^{t_L^{'}} \frac{r_B(u) - s}{s} du = \frac{M}{s} - (t_L^{'} - t_L) \cdot \left(1 - \lambda^B_{t_L, t_L^{'}}\right) = 0 \ . \qquad \text{(B.3)}$$

A vehicle that departs at $t_L^{'}$ incurs a cost:

$$c(t_L^{'}) = \gamma \cdot (t_L^{'} - t^*) = \gamma \cdot (t_L^{'} - t_L) + \gamma \cdot (t_L - t^*) = \gamma \cdot (t_L - t^*) + \gamma \cdot \frac{M}{s \cdot \left(1 - \lambda^B_{t_L, t_L^{'}}\right)} \ ,$$

where the last equality uses Eq. (B.3). As removing a vehicle from the mass opens up a (queue-free) slot at $t_L^{'}$, rescheduling the last vehicle to $t_L^{'}$ yields a change in cost of:

$$c(t_L^{'}) - c(M, t_L) = \frac{M}{s} \cdot \left[\frac{\gamma}{1 - \lambda^B_{t_L, t_L^{'}}} - (\alpha + \gamma) \right] \leq 0 \ ,$$

where the inequality follows from condition $r_B\left(t\right) \leq \alpha \cdot s/(\alpha + \gamma)$ in Proposition 2.

This shows that, in any deviation from the candidate PSNE, the last mass departure launched at a time of late arrivals can be eliminated without increasing fleet costs. By induction, any mass departure launched at a time of late arrivals can be rescheduled without increasing fleet costs.

Next, we show that any deviation entailing multiple mass departures at times of early arrivals is dominated by scheduling a single mass departure before t^*.

Result iii: Suppose that more than one mass departure is scheduled before t^*. Assume the first mass is launched at time t_E with M vehicles, and the second mass is launched at time $t_E^{'} > t_E$. If the queue from the first mass disappears before $t_E^{'}$, fleet costs can be reduced by rescheduling vehicles in the first mass to depart at a rate equal to residual capacity $(s - r_B(t))$. Since user A does not depart in the original deviation until the queue has dissipated, the rescheduled vehicles in the alternative deviation escape queuing and arrive less early – thereby reducing both their queuing and schedule delay costs. If the queue from the first mass does not disappear before $t_E^{'}$, user A can still reduce its fleet costs by rescheduling $s \cdot (t_E^{'} - t_E) \cdot (1 - \lambda^B_{t_E, t_E^{'}})$ vehicles at a rate $s - r_B(t)$ during $(t_E, t_E^{'})$, and letting the remaining $M - s \cdot (t_E^{'} - t_E) \cdot (1 - \lambda^B_{t_E, t_E^{'}})$ vehicles join the head of the second mass at $t_E^{'}$. The first set of vehicles rescheduled at residual capacity avoid queuing and incur lower early arrival costs because they arrive closer to t^*. The second set of vehicles also incur lower schedule delay costs since they arrive later. They also incur lower queuing costs as well since they no longer queue between t_E and $t_E^{'}$. Vehicles in the original mass that departs at $t_E^{'}$ still depart and arrive at the same time because the same number of vehicles depart before them and the bottleneck operates at capacity throughout.

By induction, all but one of the mass departures launched before t^* can be eliminated in a way that decreases fleet costs. Thus, results (i)–(iii) show that a deviation with a single mass departure launched before t^* is the most viable deviation, of deviations entailing mass departures, from the candidate PSNE. In the following section we show that the cost-minimizing deviation does not reduce fleet costs with respect to the candidate PSNE.

Appendix B.2. The optimal mass departure

Following the logic of Appendix C.1, suppose user A launches a single mass departure of M vehicles at $t_m < t^*$. User A will not include in the mass vehicles that were scheduled to depart before t_m because this would create a queue that lasts until after t_e, and the cost of the delay imposed on A's later vehicles would outweigh any benefit. User A will include in the mass all vehicles scheduled to depart after t_m that would be delayed by the mass. However, as discussed above, this would be counterproductive if all these vehicles were scheduled to depart before t^* because they would suffer not only queuing delay but also greater early-arrival costs (the queue would disappear before t^*).

The mass departure is potentially beneficial only if it includes vehicles scheduled to depart both early and late. Therefore, the mass must be launched at a time $t_m < t^*$, and the queue must persist until a time $t_M > t^*$, where t_M is defined by the condition:

$$\int_{t_m}^{t_M} r_A(u)\, du = M, \tag{B.4}$$

where $r_A(\cdot)$ is user A's departure schedule in the candidate PSNE. Since the queuing costs incurred by the mass do not depend on when it is launched, t_m should be chosen to minimize total schedule delay costs for vehicles in the mass. The first and last vehicles should therefore incur the same schedule delay cost:

$$\beta \cdot (t^* - t_m) = \gamma \cdot \left(t_m + \frac{M}{s} - t^* \right).$$

This implies

$$t_m = t^* - \frac{\gamma}{\beta + \gamma} \cdot \frac{M}{s}.$$

Vehicles in the mass incur total queuing time costs of $\alpha \cdot M^2 / (2 \cdot s)$, and total schedule delay costs of $\delta \cdot M^2 / (2 \cdot s)$. Total costs for the mass are therefore:

$$TC^m = \frac{\alpha + \delta}{2} \cdot \frac{M^2}{s}. \tag{B.5}$$

In the candidate PSNE, where $r_A(t) + r_B(t) = s$ for $t \in (t_m, t_M)$, the M vehicles incur total costs of

$$TC^e = \beta \cdot \int_{t_m}^{t^*} r_A(u) \cdot (t^* - u)\, du + \gamma \cdot \int_{t^*}^{t_M} r_A(u) \cdot (u - t^*)\, du.$$

To complete the proof we must show that $TC^e \leq TC^m$. User A's departure rate over the period (t_m, t_M) must be consistent with condition (B.4). As shown in proving Result 3 of

Proposition 2, during late arrivals $r_A(u)$ is bounded below by $\gamma \cdot s/(\alpha+\gamma)$. Let r_E denote the minimum departure rate of user A prior to t^*. It follows by straightforward algebra that

$$TC^e \leq$$
$$M^2 \cdot \frac{\left(\beta \cdot \gamma^2 \cdot r_E \cdot s + (\alpha+\gamma) \cdot \left((\beta+\gamma)^2 \cdot s^2 + r_E^2 \cdot \gamma^2 - 2 \cdot (\beta+\gamma) \cdot \gamma \cdot s \cdot r_E\right)\right)}{2 \cdot (\beta+\gamma)^2 \cdot s^3}. \tag{B.6}$$

Setting the right-hand side of inequality (B.6) equal to Eq. (B.5) one obtains

$$r_E = \frac{s}{2} \cdot \frac{2 \cdot \alpha \cdot (\beta+\gamma) + \gamma \cdot (\beta + 2 \cdot \gamma) - \sqrt{\beta^2 \cdot \gamma^2 + 4 \cdot \alpha \cdot (\beta+\gamma)^2 \cdot (\alpha+\gamma)}}{\gamma \cdot (\alpha+\gamma)}.$$

If user A's departure rate during early arrivals is at least r_E, departing in a mass cannot reduce its fleet costs. The candidate PSNE is therefore robust to mass departure deviations.

Straightforward algebra leads to:

$$0 < r_E < \frac{\gamma}{\alpha+\gamma} \cdot s,$$

which shows that in the limit $\gamma \to 0$, $r_E \to 0$. *QED*

Appendix C. Asymmetric equilibrium costs with $\gamma \leq \alpha$

User A's fleet costs in the PSNE depicted in Figure 3 are

$$\begin{aligned} TC_A &= r_E \cdot (t_{BA} - t_s^o) \cdot \beta \cdot \left(t^* - \frac{t_s^o + t_{BA}}{2}\right) + (s - r_E) \cdot (t^* - t_{BA}) \cdot \beta \cdot \frac{t^* - t_{BA}}{2} + \\ & \quad \frac{\alpha}{\alpha+\gamma} \cdot s \cdot (t_{AB} - t^*) \cdot \gamma \cdot \frac{t_{AB} - t^*}{2} + \frac{\gamma}{\alpha+\gamma} \cdot s \cdot (t_e^o - t_{AB}) \cdot \gamma \cdot \left(\frac{t_{AB} + t_e^o}{2} - t^*\right) \\ &= \frac{\beta \cdot r_E}{2} \cdot \left((t^* - t_s^o)^2 - (t^* - t_{BA})^2\right) + \frac{\beta}{2} \cdot (s - r_E) \cdot (t^* - t_{BA})^2 \\ & \quad + \frac{\alpha \cdot \gamma \cdot s}{2 \cdot (\alpha+\gamma)} \cdot (t_{AB} - t^*)^2 + \frac{\gamma^2 \cdot s}{2 \cdot (\alpha+\gamma)} \cdot \left((t_e^o - t^*)^2 - (t_{AB} - t^*)^2\right). \end{aligned} \tag{C.1}$$

Differentiating TC_A with respect to t_{BA} and t_{AB}, and using Eqs. (7b), (7c) and condition (25) in the text, one obtains for the cost-minimizing transition times:

$$t_{BA} = t^* - \frac{\gamma}{2\cdot(\beta+\gamma)}\cdot\frac{N}{s},\ t_{AB} = t^* + \frac{\beta}{2\cdot(\beta+\gamma)}\cdot\frac{N}{s}. \tag{C.2}$$

Substituting Eqs. (7b), (7c), and (C.2) into Eq. (C.1) gives

$$TC_A = \left(\frac{3+z}{4} - \frac{1}{2}\cdot\sqrt{\frac{\alpha}{\alpha+\gamma}+z^2}\right)\cdot TC^o,$$

where $z \equiv \beta\cdot\gamma/\left((\alpha+\gamma)\cdot(\beta+\gamma)\right)$.

Appendix D. Heterogeneous desired arrival times

Appendix D.1. Case 1

User A's fleet costs in Case 1 are:

$$TC_A^1 = \frac{\beta\cdot s}{2}\cdot\left(t_A^* - t_{As}^1\right)^2 + \frac{\gamma\cdot s}{2}\cdot\left(t_{As}^1 + \frac{N_A}{s} - t_A^*\right)^2 = \frac{\beta\cdot\gamma}{2\,(\beta+\gamma)}\cdot\frac{N_A^2}{s}.$$

User B's fleet costs are:

$$\begin{aligned}
TC_B^1 &= \frac{\beta\cdot s}{2}\cdot\left(t_B^* - t_{Ae}^1\right)^2 + \frac{\gamma\cdot s}{2}\cdot\left(t_{Be}^1 - t_B^*\right)^2 \\
&= \frac{\beta\cdot s}{2}\cdot\left(t_B^* - t_A^* - \frac{\beta}{\beta+\gamma}\cdot\frac{N_A}{s}\right)^2 + \frac{\gamma\cdot s}{2}\cdot\left(t_A^* + \frac{\beta}{\beta+\gamma}\cdot\frac{N_A}{s} + \frac{N_B}{s} - t_B^*\right)^2 \\
&= \frac{\beta\cdot s}{2}\cdot\left(\frac{\gamma}{\beta+\gamma}\cdot\frac{N_B}{s} - x\right)^2 + \frac{\gamma\cdot s}{2}\left(x + \frac{\beta}{\beta+\gamma}\cdot\frac{N_B}{s}\right)^2 \\
&= \frac{\beta\cdot\gamma}{2\cdot(\beta+\gamma)}\frac{N_B^2}{s} + \frac{\beta+\gamma}{2}\cdot s\cdot x^2,
\end{aligned}$$

where $x \equiv t_A^* - t_B^* + (\beta/(\beta+\gamma))\cdot(N_A/s) + (\gamma/(\beta+\gamma))\cdot(N_B/s)$. Total system costs are:

$$TC^1 = TC_A^1 + TC_B^1 = \frac{\beta\cdot\gamma}{2\cdot(\beta+\gamma)\cdot s}\cdot\left(N_A^2 + N_B^2\right) + \frac{\beta+\gamma}{2}\cdot s\cdot x^2.$$

The price of anarchy is

$$PA = \frac{TC^1 - TC^o}{TC^o} = \frac{\frac{\beta+\gamma}{4} \cdot s \cdot x^2}{\frac{\beta \cdot \gamma}{2 \cdot (\beta+\gamma) \cdot s} \cdot \left(N_A^2 + N_B^2\right) + \frac{\beta+\gamma}{4} \cdot s \cdot x^2}. \tag{D.1}$$

PA is an increasing function of x. Condition (40) in the text, $t_B^* - t_A^* \geq (\gamma/(\beta+\gamma)) \cdot (N_B/s)$, implies that $x \leq (\beta/(\beta+\gamma)) \cdot (N_A/s)$. Substituting this value for x into Eq. (D.1), and simplifying, yields

$$PA \leq \frac{\beta \cdot N_A^2}{2 \cdot \gamma \cdot \left(N_A^2 + N_B^2\right) + \beta \cdot N_A^2}. \tag{D.2}$$

Condition (39) in the text, $t_B^* - t_A^* > (\beta/(\beta+\gamma)) \cdot (N_A/s)$, implies that $x \leq (\gamma/(\beta+\gamma)) \cdot (N_B/s)$. This in turn implies $(\beta/(\beta+\gamma)) \cdot (N_A/s) \leq (\gamma/(\beta+\gamma)) \cdot (N_B/s)$, or $N_A \leq (\gamma/\beta) \cdot N_B$. Substituting this inequality into Eq. (D.2) yields

$$PA \leq \frac{\frac{\beta}{\gamma}}{2 + \frac{\beta}{\gamma} + 2 \cdot \frac{\beta^2}{\gamma^2}}. \tag{D.3}$$

In the model variant in Section 3.1 the relative magnitudes of parameters β and γ are unrestricted so there is no upper or lower bound on β/γ. The formula in Eq. (D.3) reaches a maximum at $\beta/\gamma = 1$, for which $PA = 1/5$.

The lower bound on the efficiency index w is derived in the same way as the upper bound on PA. Imposing conditions (39) and (40) as equalities yields $x = (\beta/(\beta+\gamma)) \cdot (N_A/s) = (\gamma/(\beta+\gamma)) \cdot (N_B/s)$. Substituting these equalities into Eq. (43) gives

$$w \geq \frac{\beta \cdot \gamma + \beta^2 + \gamma^2}{\frac{3}{2}\beta \cdot \gamma + \beta^2 + \gamma^2}. \tag{D.4}$$

Eq. (D.4) achieves a minimum value of 6/7 with $\beta = \gamma$.

Appendix D.2. Case 2

The three transition times in Case 2, t_{As}^2, t_{Ae}^2, and t_{Be}^2, are solved using three conditions. First, Eq. (36) for Case 1 continues to apply, as otherwise user A could decrease its fleet costs by rescheduling a vehicle from t_{As}^2 to t_{Ae}^2, or vice versa (cf Eq. (D.5a) below). Second, the bottleneck is fully utilized from t^2_{As} to t^2_{Be} (cf Eq. (D.5b)). Finally, user B's entire fleet must depart during the period $[t^*_A, t^2_{Be}]$ (cf Eq. (D.5c)).

$$\beta \cdot \left(t^*_A - t^2_{As}\right) = \gamma \cdot \left(t^2_{Ae} - t^*_A\right) \ , \tag{D.5a}$$

$$t^2_{Be} - t^2_{As} = \frac{N_A + N_B}{s} \ , \tag{D.5b}$$

$$\frac{\alpha}{\alpha+\gamma} \cdot s\left(t^2_{Ae} - t^*_A\right) + s \cdot \left(t^2_{Be} - t^2_{Ae}\right) = N_B. \tag{D.5c}$$

Equations (D.5) resolve to Eqs. (44a)-(44c) in the text. Three consistency conditions must be satisfied. First, as in Case 1, user B cannot gain by rescheduling its last vehicle to t^2_{As}. The requisite condition is:

$$t^*_B - t^*_A \geq \frac{\gamma}{\beta+\gamma} \cdot \frac{N_B}{s} - \frac{\alpha \cdot \beta}{(\beta+\gamma)\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s}. \tag{D.6}$$

Second, user B cannot gain by rescheduling a vehicle at t^*_A to t^2_{Be}. Thus, $\beta \cdot (t^*_B - t^*_A) \leq \gamma \cdot \left(t^2_{Be} - t^*_B\right)$ which reduces to:

$$t^*_B - t^*_A \leq \frac{\beta \cdot \gamma}{(\beta+\gamma)\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s} + \frac{\gamma}{\beta+\gamma} \cdot \frac{N_B}{s}. \tag{D.7}$$

Condition (D.7) is more stringent than condition (27). Third, user A must stop departing before user B starts to arrive late (i.e., $t^2_{Ae} < t^*_B$) since otherwise the PSNE does not exist unless $\gamma \leq \alpha$.This follows from the proof of Proposition 1. In particular, Lemma 1 holds and the only possible PSNE entails queuing. Using a similar reasoning as in Lemmas 3 and 4 it is possible to show that a PSNE will not exist.Given Eq. (44b) this implies

$$t^*_B - t^*_A \geq \frac{\beta \cdot (\alpha+\gamma)}{\gamma \cdot (\alpha+\beta+\gamma)} \cdot \frac{N_A}{s}. \tag{D.8}$$

Using Eqs. (31) and (44a), the difference in timing of the system optimum and the PSNE in Case 2 can be written as

$$t^2_{As} - t^o_{As} = \frac{-\beta \cdot \alpha}{(\beta+\gamma)\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s} + \frac{x}{2}. \tag{D.9}$$

Condition (D.7) implies a minimum value of $x = (\beta \cdot (\alpha + \beta) / ((\beta + \gamma) \cdot (\alpha + \beta + \gamma))) \cdot (N_A/s)$. Condition (D.6) implies a maximum value of $x = (\beta \cdot (2\alpha + \beta + \gamma) / ((\beta + \gamma) \cdot (\alpha + \beta + \gamma))) \cdot (N_A/s)$. Finally, Condition (D.8) implies a maximum value of $x = (\gamma/(\beta + \gamma)) \cdot (N_B/s) - (\alpha \cdot \beta^2/(\gamma \cdot (\beta + \gamma) \cdot (\alpha + \beta + \gamma))) \cdot (N_A/s)$. Applying these values to Eq. (D.9) yields the feasible range for $t^2_{As} - t^o_{As}$:

$$t^2_{As} - t^o_{As} \in \left[\begin{array}{c} \frac{-\beta\cdot(\alpha-\beta)}{2\cdot(\beta+\gamma)\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s}, \\ Min\left(\frac{\beta}{2\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s}, \frac{\gamma}{2\cdot(\beta+\gamma)} \cdot \frac{N_B}{s} - \frac{\alpha\cdot\beta\cdot(\beta+2\gamma)}{2\cdot\gamma\cdot(\beta+\gamma)\cdot(\alpha+\beta+\gamma)} \cdot \frac{N_A}{s} \right) \end{array} \right].$$

The lower bound applies with $t^*_B - t^*_A$ at its maximum value consistent with condition (D.7). The upper bound applies with $t^*_B - t^*_A$ at its minimum value consistent with conditions (D.6) and (D.8).