Appendix A: Proof of Theorem 1

Herein, we provide the full details of the proof for Theorem 1. First, we outline three lemmas that are utilized in the proof of Theorem 1.

LEMMA 1. *The function* $g_t(k|s_t, a_t) =$

$$\sum_{j=\max\{a_t^+ +1,k\}}^{s_t+a_t} p_{s_t+a_t-j} + \left[\sum_{i=s_t+a_t-j}^{\infty} p_i\right]_{\substack{j\geq k\\ j=a_t^+}}. \tag{1}$$

Proof.

$$g_t(k|s_t, a_t) = \sum_{j\in\{S|j\geq k\}} p_t(j|s_t, a_t) \tag{2}$$

$$= \sum_{\substack{j\geq k\\ a_t^+<j\leq s_t+a_t}} p_{s_t+a_t-j} + \left[q_{s_t+a_t-j}\right]_{\substack{j\geq k\\ j=a_t^+}} \tag{3}$$

$$= \sum_{j=\max\{a_t^+ +1,k\}}^{s_t+a_t} p_{s_t+a_t-j} + \left[\sum_{i=s_t+a_t-j}^{\infty} p_i\right]_{\substack{j\geq k\\ j=a_t^+}} \tag{4}$$

□

LEMMA 2. *The following two summations are equivalent.*

$$\sum_{j=k}^{s_t+a_t} p_{s_t+a_t-j} = \sum_{i=0}^{s_t+a_t-k} p_i \tag{5}$$

Proof.

$$\sum_{j=k}^{s_t+a_t} p_{s_t+a_t-j} = p_{s_t+a_t-k} + p_{s_t+a_t-(k+1)} + \ldots + p_{s_t+a_t-(s_t+a_t)} \tag{6}$$

$$= p_{s_t+a_t-k} + p_{s_t+a_t-(k+1)} + \ldots + p_0 = \sum_{i=0}^{s_t+a_t-k} p_i \tag{7}$$

□

LEMMA 3. *The following two summations are equivalent.*

$$\sum_{j=a_t^+ +1}^{s_t+a_t} p_{s_t+a_t-j} + \sum_{i=s_t+a_t-a_t^+}^{\infty} p_i = \sum_{i=0}^{\infty} p_i \tag{8}$$

Proof.

$$\sum_{j=a_t^+ +1}^{s_t+a_t} p_{s_t+a_t-j} + \sum_{i=s_t+a_t-a_t^+}^{\infty} p_i \tag{9}$$

$$= p_{s_t+a_t-(a_t^+ +1)} + p_{s_t+a_t-(a_t^+ +2)} + \ldots + p_{s_t+a_t-(s_t+a_t)} + \sum_{i=s_t-a_t^-}^{\infty} p_i \tag{10}$$

$$= p_{s_t-a_t^- -1} + p_{s_t-a_t^- -2} + \ldots + p_0 + \sum_{i=s_t-a_t^-}^{\infty} p_i \tag{11}$$

$$= \sum_{i=0}^{s_t-a_t^- -1} p_i + \sum_{i=s_t-a_t^-}^{\infty} p_i = \sum_{i=0}^{\infty} p_i \tag{12}$$

□

Utilizing these, we prove Theorem 1: *There exists optimal decision rules* $d_t^*: s_t \to A_{s_t}$ *for the EVB-SSMP which are nonincreasing in* s_t *for* $t = 1, \ldots, N-1$ *when demand* D_t *is governed by a nonincreasing discrete distribution.*

Proof. The claim is shown by demonstrating that the EVB-SSMP exhibits the following 5 conditions (Puterman 2005).

1. $r_t(s_t, a_t)$ is nondecreasing in s_t for all $a_t \in A'$.

   That $r_t(s_t, a_t)$ is nondecreasing in s_t for a fixed a_t means that for a fixed action (i.e., number of batteries charged or discharged), the expected immediate reward will be greater when the number of full batteries is greater. This coincides with intuition as more batteries can be swapped or discharged when there are more full batteries available thereby leading to more reward. Consider $s_t \geq \tilde{s}_t$, using $s_t + a_t - s_{t+1} = \min\{D_t, s_t - a_t^-\}$ for any value which D_t can assume, it suffices to show that

$$r_t(s_t, a_t) \geq r_t(\tilde{s}_t, a_t), \tag{13}$$

   using the expected immediate reward function

$$r_t(s_t, a_t) = \sum_{j=0}^{\infty} \left[P(D_t = j)\big(\rho \min\{j, s_t - a_t^-\}\big) \right] - K_t a_t^+ + J_t a_t^-. \tag{14}$$

   It suffices to show that

$$r_t(s_t, a_t) \geq r_t(\tilde{s}_t, a_t) \Leftrightarrow \tag{15}$$

$$\sum_{j=0}^{\infty} \left[P(D_t = j)\big(\rho \min\{j, s_t - a_t^-\}\big) \right] - K_t a_t^+ + J_t a_t^-$$

$$\geq \sum_{j=0}^{\infty} \left[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - a_t^-\}\big) \right] - K_t a_t^+ + J_t a_t^- \Leftrightarrow \tag{16}$$

$$\sum_{j=0}^{\infty} \left[P(D_t = j)\big(\rho \min\{j, s_t - a_t^-\}\big) \right] \geq \sum_{j=0}^{\infty} \left[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - a_t^-\}\big) \right]. \tag{17}$$

   Therefore, because $P(D_t = j)\rho$ is multiplied by both sides of the inequality in Equation (17) for all values of j, Equation (13) can be demonstrated by proving

$$\min\{j, s_t - a_t^-\} \geq \min\{j, \tilde{s}_t - a_t^-\}, \tag{18}$$

   for all possible values of j. Using a proof by cases, the three possible cases of demand $D_t = j$ with respect to $s_t - a_t^-$ and $\tilde{s}_t - a_t^-$ are considered: (a) $j \leq \tilde{s}_t - a_t^-$, $j \leq s_t - a_t^-$, (b) $j \geq \tilde{s}_t - a_t^-$, $j \leq s_t - a_t^-$, and (c) $j \geq \tilde{s}_t - a_t^-$, $j \geq s_t - a_t^-$. The case where j is greater than $s_t - a_t^-$ and less than $\tilde{s}_t - a_t^-$ does not need to be considered as it is not possible because $s_t \geq \tilde{s}_t$. In each case, Equation (18) is reduced to a valid statement.

   (a) $j \leq \tilde{s}_t - a_t^-$, $j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} \geq \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow j = j \tag{19}$$

(b) $j \geq \tilde{s}_t - a_t^-$, $j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} \geq \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow j \geq \tilde{s}_t - a_t^- \tag{20}$$

(c) $j \geq \tilde{s}_t - a_t^-$, $j \geq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} \geq \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow s_t - a_t^- \geq \tilde{s}_t - a_t^- \Leftrightarrow s_t \geq \tilde{s}_t \tag{21}$$

2. $g_t(k|s_t, a_t)$ is nondecreasing in s_t for all $k \in S$ and $a_t \in A'$.

That $g_t(k|s_t, a_t)$ is nondecreasing in s_t for a fixed a_t and k means that the probability that the number of full batteries in the next state is greater than some threshold k is higher when the number of full batteries in the current state is greater. Consider $s_t \geq \tilde{s}_t$, it suffices to show that

$$g_t(k|s_t, a_t) \geq g_t(k|\tilde{s}_t, a_t) \Leftrightarrow \tag{22}$$

$$\sum_{j \in \{S | j \geq k\}} p_t(j|s_t, a_t) \geq \sum_{j \in \{S | j \geq k\}} p_t(j|\tilde{s}_t, a_t) \Leftrightarrow \tag{23}$$

$$\sum_{\substack{j \geq k \\ a_t^+ < j \leq s_t + a_t}} p_{s_t + a_t - j} + \Big[q_{s_t + a_t - j} \Big]_{\substack{j \geq k \\ j = a_t^+}} \geq$$

$$\sum_{\substack{j \geq k \\ a_t^+ < j \leq \tilde{s}_t + a_t}} p_{\tilde{s}_t + a_t - j} + \Big[q_{\tilde{s}_t + a_t - j} \Big]_{\substack{j \geq k \\ j = a_t^+}} \Leftrightarrow \tag{24}$$

$$\sum_{j = \max\{a_t^+ + 1, k\}}^{s_t + a_t} p_{s_t + a_t - j} + \left[\sum_{i = s_t + a_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = a_t^+}} \geq$$

$$\sum_{j = \max\{a_t^+ + 1, k\}}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} + \left[\sum_{i = \tilde{s}_t + a_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = a_t^+}}. \tag{25}$$

Using a proof by cases, all cases of k with respect to a_t are considered. For each case, Equation (25) is reduced to a valid statement. Note that the second term of both the left hand side and right hand side of Equation (25) is only included when both $j \geq k$ and $j = a_t^+$, which represents when demand meets or exceeds supply. It is indicated in each case of the proof which of the terms are included in the summation based on the relationship between k and a_t.

(a) $a_t^+ \geq k \Rightarrow a_t^+ + 1 > k$

The second term of each summation appears as both $j \geq k$ and $j = a_t^+$ are satisfied. Using Lemma 3, Equation (26) is reduced to Equation (27).

$$\sum_{j = a_t^+ + 1}^{s_t + a_t} p_{s_t + a_t - j} + \sum_{i = s_t + a_t - a_t^+}^{\infty} p_i \geq \sum_{j = a_t^+ + 1}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} + \sum_{i = \tilde{s}_t + a_t - a_t^+}^{\infty} p_i \Leftrightarrow \tag{26}$$

$$\sum_{i=0}^{\infty} p_i = \sum_{i=0}^{\infty} p_i \tag{27}$$

(b) $a_t^+ < k \Rightarrow a_t^+ + 1 \geq k$

The second term of each summation does not appear as $j = a^+$ will never be satisfied. Starting from Equation (25), Lemma 2 is utilized to arrive at a known valid statement.

$$\sum_{j=k}^{s_t + a_t} p_{s_t + a_t - j} \geq \sum_{j=k}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} \Leftrightarrow \tag{28}$$

$$\sum_{i=0}^{s_t+a_t-k} p_i \geq \sum_{i=0}^{\tilde{s}_t+a_t-k} p_i \Leftrightarrow \tag{29}$$

$$\sum_{i=0}^{\tilde{s}_t+a_t-k} p_i + \sum_{i=\tilde{s}_t+a_t-k+1}^{s_t+a_t-k} p_i \geq \sum_{i=0}^{\tilde{s}_t+a_t-k} p_i \Leftrightarrow \tag{30}$$

$$\sum_{i=\tilde{s}_t+a_t-k+1}^{s_t+a_t-k} p_i \geq 0 \tag{31}$$

3. $r_t(s_t, a_t)$ is a subadditive function on $S \times A'$.

The subadditivity of $r_t(s_t, a_t)$ implies that the incremental effect on the expected total reward of charging less batteries (or discharging more batteries) is less when the number of full batteries is greater. Consider $a_t \geq \tilde{a}_t$ and $s_t \geq \tilde{s}_t$, using $s_t + a_t - s_{t+1} = \min\{D_t, s_t - a_t^-\}$ for any value which D_t can assume, it suffices to show that

$$r_t(s_t, a_t) + r_t(\tilde{s}_t, \tilde{a}_t) \leq r_t(s_t, \tilde{a}_t) + r_t(\tilde{s}_t, a_t) \Leftrightarrow \tag{32}$$

$$\begin{aligned} &\sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, s_t - a_t^-\}\big)\Big] - K_t a_t^+ + J_t a_t^- \\ &+ \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - \tilde{a}_t^-\}\big)\Big] - K_t \tilde{a}_t^+ + J_t \tilde{a}_t^- \\ &\leq \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, s_t - \tilde{a}_t^-\}\big)\Big] - K_t \tilde{a}_t^+ + J_t \tilde{a}_t^- \\ &+ \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - a_t^-\}\big)\Big] - K_t a_t^+ + J_t a_t^- \Leftrightarrow \end{aligned} \tag{33}$$

$$\begin{aligned} &\sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, s_t - a_t^-\}\big)\Big] + \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - \tilde{a}_t^-\}\big)\Big] \\ &\leq \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, s_t - \tilde{a}_t^-\}\big)\Big] + \sum_{j=0}^{\infty}\Big[P(D_t = j)\big(\rho \min\{j, \tilde{s}_t - a_t^-\}\big)\Big]. \end{aligned} \tag{34}$$

Therefore, because $P(D_t = j)\rho$ is multiplied by all terms in Equation (34), it suffices to show that

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\}, \tag{35}$$

for all values of j. Using a proof by cases, every relevant case of a_t and $\tilde{a}_t$, and each scenario for demand $D_t = j$ with respect to $s_t - a_t^-$, $\tilde{s}_t - \tilde{a}_t^-$, $s_t - \tilde{a}_t^-$, $\tilde{s}_t - a_t^-$ are considered. The case where $\tilde{a}_t \leq 0$ and $a_t \geq 0$ is excluded as this is not possible from the definition of subadditivity that $a_t \geq \tilde{a}_t$. For each case, Equation (35) is reduced to a valid statement.

(a) $\tilde{a}_t \geq 0$, $a_t \geq 0 \Rightarrow \tilde{a}_t^- = a_t^- = 0$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{36}$$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t\} = \min\{j, s_t\} + \min\{j, \tilde{s}_t\} \tag{37}$$

(b) $\tilde{a}_t \leq 0$, $a_t \geq 0 \Rightarrow \tilde{a}_t^- \geq 0$, $a_t^- = 0$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \tag{38}$$

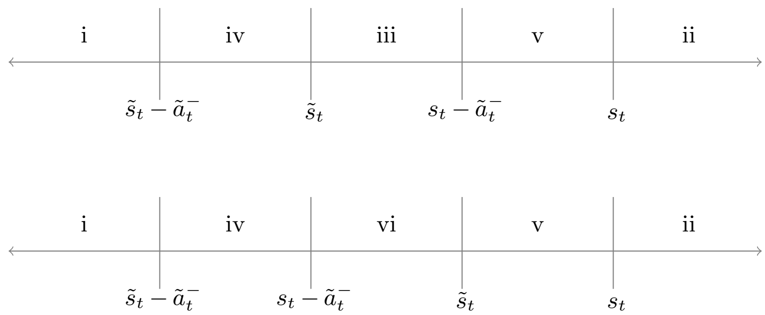

Figure 1 **Scenarios of demand with respect to inventory for case (b).**

Every possibility for demand j with respect to s_t, $\tilde{s}_t - \tilde{a}_t^-$, $s_t - \tilde{a}_t^-$, and $\tilde{s}_t$ is considered. Figure 1 is provided to aid the reader in visualizing the six possible scenarios. The ranges i-vi in the diagram correspond to the following scenarios i-vi.

i. $j \leq \tilde{s}_t - \tilde{a}_t^- \Rightarrow j \leq \tilde{s}_t,\ j \leq s_t - \tilde{a}_t^-,\ j \leq s_t$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{39}$$

$$j + j \leq j + j \Leftrightarrow 2j = 2j \tag{40}$$

ii. $j \geq s_t \Rightarrow j \geq s_t - \tilde{a}_t^-,\ j \geq \tilde{s}_t,\ j \geq \tilde{s}_t - \tilde{a}_t^-$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{41}$$

$$s_t + \tilde{s}_t - \tilde{a}_t^- = s_t - \tilde{a}_t^- + \tilde{s}_t \tag{42}$$

iii. $j \geq \tilde{s}_t,\ j \leq s_t - \tilde{a}_t^- \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-,\ j \leq s_t$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{43}$$

$$j + \tilde{s}_t - \tilde{a}_t^- \leq j + \tilde{s}_t \Leftrightarrow \tilde{a}_t \geq 0 \tag{44}$$

iv. $j \leq \tilde{s}_t,\ j \leq s_t - \tilde{a}_t^-,\ j \geq \tilde{s}_t - \tilde{a}_t^- \Rightarrow j \leq s_t$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{45}$$

$$j + \tilde{s}_t - \tilde{a}_t^- \leq j + j \Leftrightarrow \tilde{s}_t - \tilde{a}_t^- \leq j \tag{46}$$

v. $j \geq \tilde{s}_t,\ j \geq s_t - \tilde{a}_t^-,\ j \leq s_t \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{47}$$

$$j + \tilde{s}_t - \tilde{a}_t^- \leq s_t - \tilde{a}_t^- + \tilde{s}_t \Leftrightarrow j \leq s_t \tag{48}$$

vi. $j \leq \tilde{s}_t,\ j \geq s_t - \tilde{a}_t^- \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-,\ j \leq s_t$

$$\min\{j, s_t\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t\} \Leftrightarrow \tag{49}$$

$$j + \tilde{s}_t - \tilde{a}_t^- \leq s_t - \tilde{a}_t^- + j \Leftrightarrow \tilde{s}_t \leq s_t \tag{50}$$

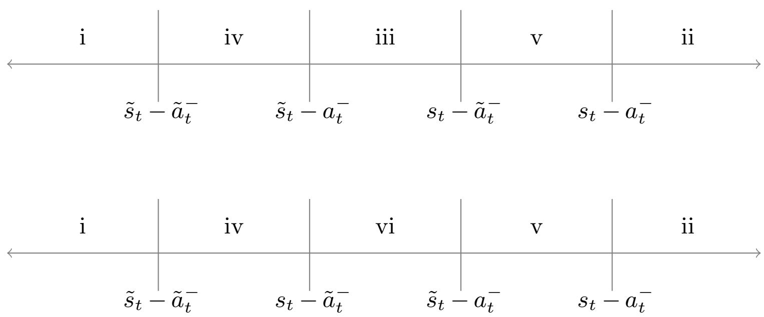

Figure 2 Scenarios of demand with respect to inventory for case (c).

(c) $\tilde{a}_t \leq 0,\ a_t \leq 0 \Rightarrow \tilde{a}_t^- \geq 0,\ a_t^- \geq 0,\ \tilde{a}_t^- \geq a_t^-$

Every possibility for demand j with respect to $s_t - a_t^-$, $\tilde{s}_t - \tilde{a}_t^-$, $s_t - \tilde{a}_t^-$, $\tilde{s}_t - a_t^-$ is considered. Figure 2 is provided to aid the reader in visualizing the six possible scenarios. The ranges i-vi in the diagram correspond to the following scenarios i-vi.

i. $j \leq \tilde{s}_t - \tilde{a}_t^- \Rightarrow j \leq \tilde{s}_t - a_t^-,\ j \leq s_t - \tilde{a}_t^-,\ j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\}$$
$$\leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{51}$$
$$j + j \leq j + j \Leftrightarrow 2j = 2j \tag{52}$$

ii. $j \geq s_t - a_t^- \Rightarrow j \geq \tilde{s}_t - a_t^-,\ j \geq s_t - \tilde{a}_t^-,\ j \geq \tilde{s}_t - \tilde{a}_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\}$$
$$\leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{53}$$
$$s_t - a_t^- + \tilde{s}_t - \tilde{a}_t^- = s_t - \tilde{a}_t^- + \tilde{s}_t - a_t^- \tag{54}$$

iii. $j \geq \tilde{s}_t - a_t^-,\ j \leq s_t - \tilde{a}_t^- \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-,\ j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\}$$
$$\leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{55}$$
$$j + \tilde{s}_t - \tilde{a}_t^- \leq j + \tilde{s}_t - a_t^- \Leftrightarrow \tilde{a}_t^- \geq a_t^- \tag{56}$$

iv. $j \leq \tilde{s}_t - a_t^-,\ j \leq s_t - \tilde{a}_t^-,\ j \geq \tilde{s}_t - \tilde{a}_t^- \Rightarrow j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\}$$
$$\leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{57}$$
$$j + \tilde{s}_t - \tilde{a}_t^- \leq j + j \Leftrightarrow \tilde{s}_t - \tilde{a}_t^- \leq j \tag{58}$$

v. $j \geq \tilde{s}_t - a_t^-,\ j \geq s_t - \tilde{a}_t^-,\ j \leq s_t - a_t^- \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\}$$
$$\leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{59}$$
$$j + \tilde{s}_t - \tilde{a}_t^- \leq s_t - \tilde{a}_t^- + \tilde{s}_t - a_t^- \Leftrightarrow j \leq s_t - a_t^- \tag{60}$$

vi. $j \leq \tilde{s}_t - a_t^-$, $j \geq s_t - \tilde{a}_t^- \Rightarrow j \geq \tilde{s}_t - \tilde{a}_t^-$, $j \leq s_t - a_t^-$

$$\min\{j, s_t - a_t^-\} + \min\{j, \tilde{s}_t - \tilde{a}_t^-\} \leq \min\{j, s_t - \tilde{a}_t^-\} + \min\{j, \tilde{s}_t - a_t^-\} \Leftrightarrow \tag{61}$$

$$j + \tilde{s}_t - \tilde{a}_t^- \leq s_t - \tilde{a}_t^- + j \Leftrightarrow \tilde{s}_t \leq s_t \tag{62}$$

4. $g_t(k|s_t, a_t)$ is a subadditive function on $S \times A'$ for all $k \in S$.

The subadditivity of $g_t(k|s_t, a_t)$ implies that the incremental effect of charging less batteries (or discharging more batteries) on the probability that the system moves to a state of full batteries above some threshold k is less when the number of full batteries is greater. Consider $a_t \geq \tilde{a}_t$ and $s_t \geq \tilde{s}_t$, it suffices to show that

$$g_t(k|s_t, a_t) + g_t(k|\tilde{s}_t, \tilde{a}_t) \leq g_t(k|s_t, \tilde{a}_t) + g_t(k|\tilde{s}_t, a_t) \Leftrightarrow \tag{63}$$

$$\begin{aligned} &\sum_{j=\max\{a_t^+ + 1, k\}}^{s_t + a_t} p_{s_t + a_t - j} + \left[\sum_{i=s_t + a_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = a_t^+}} \\ &+ \sum_{j=\max\{\tilde{a}_t^+ + 1, k\}}^{\tilde{s}_t + \tilde{a}_t} p_{\tilde{s}_t + \tilde{a}_t - j} + \left[\sum_{i=\tilde{s}_t + \tilde{a}_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = \tilde{a}_t^+}} \\ &\leq \sum_{j=\max\{\tilde{a}_t^+ + 1, k\}}^{s_t + \tilde{a}_t} p_{s_t + \tilde{a}_t - j} + \left[\sum_{i=s_t + \tilde{a}_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = \tilde{a}_t^+}} \\ &+ \sum_{j=\max\{a_t^+ + 1, k\}}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} + \left[\sum_{i=\tilde{s}_t + a_t - j}^{\infty} p_i \right]_{\substack{j \geq k \\ j = a_t^+}}. \end{aligned} \tag{64}$$

Using a proof by cases, every relevant case of k with respect to a_t and $\tilde{a}_t$ is considered. For each case, Equation (64) is reduced to a valid statement. The function $g_t(k|s_t, a_t)$ is comprised of two terms. The first term calculates the probability when demand never exceeds supply of batteries and the second calculates the probability that demand equals or exceeds supply. It is indicated in each case of the proof which of the terms are included in the summation based on the relationship between k, a_t, and $\tilde{a}_t$.

(a) $\tilde{a}_t^+ \geq k \Rightarrow a_t^+ \geq k$, $\tilde{a}_t^+ + 1 > k$, $a_t^+ + 1 > k$

For this case demand for battery swaps may exceed supply, therefore both terms of $g_t(k|s_t, a_t)$ appear.

$$\begin{aligned} &\sum_{j=a_t^+ + 1}^{s_t + a_t} p_{s_t + a_t - j} + \sum_{i=s_t + a_t - a_t^+}^{\infty} p_i + \sum_{j=\tilde{a}_t^+ + 1}^{\tilde{s}_t + \tilde{a}_t} p_{\tilde{s}_t + \tilde{a}_t - j} + \sum_{i=\tilde{s}_t + \tilde{a}_t - \tilde{a}_t^+}^{\infty} p_i \\ \leq &\sum_{j=\tilde{a}_t^+ + 1}^{s_t + \tilde{a}_t} p_{s_t + \tilde{a}_t - j} + \sum_{i=s_t + \tilde{a}_t - \tilde{a}_t^+}^{\infty} p_i + \sum_{j=a_t^+ + 1}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} + \sum_{i=\tilde{s}_t + a_t - a_t^+}^{\infty} p_i \Leftrightarrow \end{aligned} \tag{65}$$

$$\sum_{i=0}^{\infty} p_i + \sum_{i=0}^{\infty} p_i \leq \sum_{i=0}^{\infty} p_i + \sum_{i=0}^{\infty} p_i \Leftrightarrow \tag{66}$$

$$2 \sum_{i=0}^{\infty} p_i = 2 \sum_{i=0}^{\infty} p_i \tag{67}$$

(b) $\tilde{a}_t^+ < k,\, a_t^+ \geq k \Rightarrow \tilde{a}_t^+ + 1 \leq k,\, a_t^+ + 1 > k$

For this case, because $a_t^+ \geq k$, the second term of $g(k|s_t, a_t)$ does appear when action a_t is taken as demand can exceed supply. However, because $\tilde{a}_t^+ < k$, demand can never exceed supply when action $\tilde{a}_t$ is taken.

$$\sum_{j=a_t^+ + 1}^{s_t + a_t} p_{s_t + a_t - j} + \sum_{i = s_t + a_t - a_t^+}^{\infty} p_i + \sum_{j=k}^{\tilde{s}_t + \tilde{a}_t} p_{\tilde{s}_t + \tilde{a}_t - j}$$

$$\leq \sum_{j=k}^{s_t + \tilde{a}_t} p_{s_t + \tilde{a}_t - j} + \sum_{j=a_t^+ + 1}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} + \sum_{i=\tilde{s}_t + a_t - a_t^+}^{\infty} p_i \Leftrightarrow \tag{68}$$

$$\sum_{i=0}^{\infty} p_i + \sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i \leq \sum_{i=0}^{s_t + \tilde{a}_t - k} p_i + \sum_{i=0}^{\infty} p_i \Leftrightarrow \tag{69}$$

$$\sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i \leq \sum_{i=0}^{s_t + \tilde{a}_t - k} p_i \Leftrightarrow \tag{70}$$

$$\sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i \leq \sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i + \sum_{i=\tilde{s}_t + \tilde{a}_t - k + 1}^{s_t + \tilde{a}_t - k} p_i \Leftrightarrow \tag{71}$$

$$0 \leq \sum_{i=\tilde{s}_t + \tilde{a}_t - k + 1}^{s_t + \tilde{a}_t - k} p_i \tag{72}$$

(c) $a_t^+ < k \Rightarrow \tilde{a}_t^+ < k,\, \tilde{a}_t^+ + 1 \leq k,\, a_t^+ + 1 \leq k$

For this case, demand for battery swaps never exceeds supply therefore, the second term of $g_t(k|s_t, a_t)$ does not appear when either action a_t or action $\tilde{a}_t$ are taken.

$$\sum_{j=k}^{s_t + a_t} p_{s_t + a_t - j} + \sum_{j=k}^{\tilde{s}_t + \tilde{a}_t} p_{\tilde{s}_t + \tilde{a}_t - j} \leq \sum_{j=k}^{s_t + \tilde{a}_t} p_{s_t + \tilde{a}_t - j} + \sum_{j=k}^{\tilde{s}_t + a_t} p_{\tilde{s}_t + a_t - j} \Leftrightarrow \tag{73}$$

$$\sum_{i=0}^{s_t + a_t - k} p_i + \sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i \leq \sum_{i=0}^{s_t + \tilde{a}_t - k} p_i + \sum_{i=0}^{\tilde{s}_t + a_t - k} p_i \Leftrightarrow \tag{74}$$

$$\sum_{i=0}^{\tilde{s}_t + a_t - k} p_i + \sum_{i=\tilde{s}_t + a_t - k + 1}^{s_t + a_t - k} p_i + \sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i$$

$$\leq \sum_{i=0}^{\tilde{s}_t + \tilde{a}_t - k} p_i + \sum_{i=\tilde{s}_t + \tilde{a}_t - k + 1}^{s_t + \tilde{a}_t - k} p_i + \sum_{i=0}^{\tilde{s}_t + a_t - k} p_i \Leftrightarrow \tag{75}$$

$$\sum_{i=\tilde{s}_t + a_t - k + 1}^{s_t + a_t - k} p_i \leq \sum_{i=\tilde{s}_t + \tilde{a}_t - k + 1}^{s_t + \tilde{a}_t - k} p_i. \tag{76}$$

In Equation (76) the number of terms on each side are exactly the same, however because $a_t \geq \tilde{a}_t$ the start of the summation is greater on the left hand side. Therefore, Equation (76) holds when $p_j = P(D_t = j)$ is governed by a nonincreasing discrete distribution.

5. $r_N(s_N)$ is nondecreasing in s_N.

Consider $s_N \geq \tilde{s}_N$, it suffices to show that $r_N(s_N) \geq r_N(\tilde{s}_N)$. This expression is reduced to a known valid statement.

$$r_N(s_N) \geq r_N(\tilde{s}_N) \Leftrightarrow \rho s_N \geq \rho \tilde{s}_N \Leftrightarrow s_N \geq \tilde{s}_N \tag{77}$$

□

Appendix B: Full Results from the Benchmark Policies

Table 1 Benchmark policy results for Winter.

					Backward Induction	Stationary Benchmark Policy			Dynamic Benchmark Policy		
Scenario	M	$\Phi(\%M)$	γ	ρ	Time (s)	Time (s)	Opt Gap	Demand Gap	Time (s)	Opt Gap	Demand Gap
1	104	0.42	5872	15.38	62.57	0.34	0.13	-0.01	0.34	0.13	-0.02
2	185	0.83	5359	21.54	77.02	14.11	0.11	0.00	13.95	0.14	0.00
3	88	0.48	1513	17.31	7.88	0.68	0.26	0.00	0.57	0.27	0.00
4	200	0.44	3821	12.69	51.27	11.75	0.48	-0.23	10.75	0.48	-0.23
5	158	0.58	1256	16.54	30.98	5.22	0.47	-0.01	5.67	0.50	-0.01
6	173	0.56	4077	24.62	46.9	7.95	0.11	0.00	7.55	0.13	0.00
7	123	0.62	1897	10.38	13.84	2.25	0.87	-0.42	2.67	0.88	-0.42
8	169	0.54	5615	15.00	44.41	6.97	0.19	-0.08	6.23	0.20	-0.08
9	73	0.79	1000	21.92	6.62	0.51	0.24	0.00	0.39	0.28	0.00
10	192	0.85	2667	23.85	84.32	16.31	0.22	0.00	14.79	0.26	0.00
11	162	0.27	2154	13.08	17.78	3.11	0.46	-0.13	2.32	0.46	-0.13
12	65	0.40	2795	22.69	5.17	0.21	0.07	0.03	0.2	0.06	0.01
13	119	0.50	4333	20.00	18.25	1.3	0.09	0.00	1.04	0.09	0.00
14	115	0.37	4590	10.00	9.46	0.97	1.02	-0.47	0.71	1.04	-0.49
15	96	0.87	1128	14.62	10.82	0.82	0.49	-0.14	0.73	0.53	-0.14
16	196	0.46	2410	20.77	52.97	12.07	0.26	0.00	10.35	0.28	0.00
17	146	0.38	6000	23.08	26.46	2.89	0.07	0.02	2.51	0.06	0.00
18	131	0.96	3051	13.46	26.49	3.67	0.41	-0.27	3.88	0.45	-0.27
19	177	0.94	4846	14.23	65.08	12.56	0.31	-0.18	12.56	0.34	-0.18
20	135	0.29	2538	22.31	14.73	1.64	0.13	0.00	1.36	0.14	0.00
21	54	0.73	4718	15.77	5.36	0.19	0.20	0.08	0.17	0.18	0.06
22	188	0.81	2026	12.31	61.4	14.79	0.67	-0.29	13.72	0.70	-0.29
23	154	0.90	1641	18.85	38.77	6.86	0.36	0.00	6.91	0.42	0.00
24	81	0.33	1769	10.77	4.36	0.32	0.74	-0.40	0.28	0.73	-0.40
25	138	0.98	3949	20.38	35.43	4.55	0.13	0.00	4.65	0.16	0.00
26	150	0.63	1385	23.46	29.37	4.78	0.29	0.00	4.31	0.33	0.00
27	181	0.35	4462	19.23	36.77	6.01	0.14	0.00	5.1	0.14	0.00
28	58	1.00	3308	16.92	6.1	0.26	0.17	0.09	0.24	0.14	0.02
29	142	0.67	4205	11.15	25.4	4.42	0.68	-0.39	3.86	0.69	-0.39
30	127	0.77	5487	17.69	29.55	3.11	0.12	0.03	2.85	0.11	0.00
31	100	0.75	2923	18.46	14.61	1.14	0.14	0.00	0.92	0.16	0.00
32	165	0.69	3436	18.08	62.34	7.68	0.22	0.00	7.47	0.24	0.00
33	92	0.88	5744	11.54	12.98	1.1	0.54	-0.24	1.06	0.55	-0.29
34	50	0.71	2282	11.92	3.85	0.16	0.46	-0.33	0.15	0.47	-0.37
35	108	0.65	3179	25.00	17.98	1.04	0.08	0.00	0.83	0.10	0.00
36	62	0.31	4974	19.62	5.03	0.2	0.05	0.02	0.18	0.16	0.07
37	77	0.60	5231	24.23	10.98	0.26	0.16	0.12	0.25	0.11	0.07
38	69	0.92	5103	21.15	10.52	0.23	0.17	0.12	0.22	0.14	0.09
39	85	0.52	3564	13.85	10.28	0.4	0.22	-0.15	0.24	0.21	-0.17
40	112	0.25	3692	16.15	11.95	0.38	0.11	0.00	0.35	0.13	0.02

Table 2 Benchmark policy results for Spring.

					Backward Induction	Stationary Benchmark Policy			Dynamic Benchmark Policy		
Scenario	M	$\Phi(\%M)$	γ	ρ	Time (s)	Time (s)	Opt Gap	Demand Gap	Time (s)	Opt Gap	Demand Gap
1	104	0.42	5872	5.51	58.32	0.49	0.11	0.05	0.83	0.07	0.04
2	185	0.83	5359	8.38	87.46	15.07	0.07	0.00	12.83	0.05	0.00
3	88	0.48	1513	6.41	9.76	0.61	0.16	0.00	0.61	0.12	0.00
4	200	0.44	3821	4.26	64.8	11.81	0.31	0.00	9.37	0.24	0.00
5	158	0.58	1256	6.05	39.51	5.49	0.32	0.00	4.49	0.27	0.00
6	173	0.56	4077	9.82	55.9	7.88	0.07	0.00	6.7	0.05	0.00
7	123	0.62	1897	3.18	18.69	2.21	0.76	-0.37	1.98	0.67	-0.37
8	169	0.54	5615	5.33	54.77	6.96	0.12	0.00	10.85	0.08	0.00
9	73	0.79	1000	8.56	8.46	0.55	0.14	0.00	0.83	0.11	0.00
10	192	0.85	2667	9.46	87.18	16.79	0.12	0.00	17.5	0.10	0.00
11	162	0.27	2154	4.44	23.66	3.29	0.33	0.00	3.71	0.28	0.00
12	65	0.40	2795	8.92	5.96	0.22	0.06	0.03	0.22	0.03	0.01
13	119	0.50	4333	7.67	21.06	0.93	0.06	0.00	1.29	0.04	0.00
14	115	0.37	4590	3.00	8.73	0.85	1.06	-0.60	0.68	0.92	-0.62
15	96	0.87	1128	5.15	14.11	0.84	0.31	0.00	0.83	0.26	0.00
16	196	0.46	2410	8.03	61.96	11.69	0.16	0.00	11.97	0.13	0.00
17	146	0.38	6000	9.10	31.09	2.96	0.06	0.02	3.05	0.03	0.01
18	131	0.96	3051	4.62	35.82	3.69	0.23	0.00	3.77	0.19	0.00
19	177	0.94	4846	4.97	81.21	12.25	0.17	0.00	14.82	0.14	0.00
20	135	0.29	2538	8.74	16.44	1.75	0.09	0.00	2.35	0.07	0.00
21	54	0.73	4718	5.69	5.99	0.18	0.18	0.11	0.26	0.12	0.08
22	188	0.81	2026	4.08	77.64	15.06	0.48	-0.01	16.03	0.41	-0.01
23	154	0.90	1641	7.13	47.9	6.78	0.22	0.00	7.08	0.18	0.00
24	81	0.33	1769	3.36	5.71	0.35	0.56	-0.21	0.34	0.45	-0.21
25	138	0.98	3949	7.85	42.05	4.57	0.07	0.00	4.34	0.06	0.00
26	150	0.63	1385	9.28	37.25	4.77	0.18	0.00	6.64	0.14	0.00
27	181	0.35	4462	7.31	43.57	6.02	0.09	0.00	6.27	0.06	0.00
28	58	1.00	3308	6.23	7.13	0.29	0.16	0.12	0.37	0.07	0.05
29	142	0.67	4205	3.54	34.85	4.27	0.41	-0.16	3.97	0.33	-0.16
30	127	0.77	5487	6.59	35.73	3.15	0.10	0.04	3.37	0.05	0.01
31	100	0.75	2923	6.95	17.49	1.03	0.08	0.00	1.14	0.06	0.00
32	165	0.69	3436	6.77	54.1	8.06	0.13	0.00	7.99	0.10	0.00
33	92	0.88	5744	3.72	16.95	1.07	0.26	0.09	1.19	0.18	0.02
34	50	0.71	2282	3.90	4.82	0.16	0.23	0.05	0.15	0.15	0.00
35	108	0.65	3179	10.00	18.95	0.96	0.05	0.00	1.15	0.03	0.00
36	62	0.31	4974	7.49	4.79	0.21	0.05	0.02	0.18	0.12	0.07
37	77	0.60	5231	9.64	10.85	0.25	0.15	0.12	0.44	0.08	0.06
38	69	0.92	5103	8.21	10.57	0.23	0.16	0.12	0.24	0.10	0.08
39	85	0.52	3564	4.79	11.48	0.28	0.14	0.03	0.27	0.08	0.00
40	112	0.25	3692	5.87	11.97	0.44	0.10	0.04	0.34	0.08	0.04

Table 3 Benchmark policy results for Summer.

					Backward Induction	Stationary Benchmark Policy			Dynamic Benchmark Policy		
Scenario	M	$\Phi(\%M)$	γ	ρ	Time (s)	Time (s)	Opt Gap	Demand Gap	Time (s)	Opt Gap	Demand Gap
1	104	0.42	5872	10.38	60.57	0.37	0.14	-0.03	0.35	0.11	-0.04
2	185	0.83	5359	16.54	79.45	15.1	0.08	0.00	13.54	0.07	0.00
3	88	0.48	1513	12.31	8.18	0.63	0.22	-0.03	0.52	0.18	-0.03
4	200	0.44	3821	7.69	54.13	12.15	0.49	-0.26	10.22	0.43	-0.26
5	158	0.58	1256	11.54	35.62	5.57	0.42	-0.04	5.07	0.36	-0.04
6	173	0.56	4077	19.62	50.69	7.98	0.09	0.00	7	0.07	0.00
7	123	0.62	1897	5.38	17.08	2.24	1.04	-0.33	2.17	0.95	-0.33
8	169	0.54	5615	10.00	45.74	7.48	0.19	-0.10	6.74	0.16	-0.10
9	73	0.79	1000	16.92	7.36	0.52	0.18	0.00	0.45	0.15	0.00
10	192	0.85	2667	18.85	82.94	17.26	0.16	0.00	15.01	0.13	0.00
11	162	0.27	2154	8.08	18.35	3.29	0.50	-0.15	2.34	0.44	-0.15
12	65	0.40	2795	17.69	5.02	0.21	0.07	0.02	0.2	0.03	0.01
13	119	0.50	4333	15.00	17.41	1.55	0.08	-0.02	0.93	0.05	-0.02
14	115	0.37	4590	5.00	9.31	0.77	1.44	-0.41	0.81	1.37	-0.43
15	96	0.87	1128	9.62	12.08	0.87	0.43	-0.10	0.71	0.38	-0.10
16	196	0.46	2410	15.77	57.96	11.79	0.21	0.00	9.72	0.18	0.00
17	146	0.38	6000	18.08	26.27	3.01	0.06	0.02	2.38	0.03	0.00
18	131	0.96	3051	8.46	28.91	3.73	0.39	-0.22	3.86	0.35	-0.22
19	177	0.94	4846	9.23	69.32	13.01	0.29	-0.16	12.43	0.25	-0.16
20	135	0.29	2538	17.31	13.73	1.81	0.12	0.00	1.27	0.09	0.00
21	54	0.73	4718	10.77	5.06	0.18	0.19	0.06	0.16	0.14	0.01
22	188	0.81	2026	7.31	68.47	15.07	0.66	-0.28	13.85	0.60	-0.28
23	154	0.90	1641	13.85	44.08	6.84	0.29	-0.02	7.29	0.25	-0.02
24	81	0.33	1769	5.77	4.25	0.33	0.93	-0.35	0.32	0.83	-0.35
25	138	0.98	3949	15.38	36.78	4.67	0.09	0.00	4.7	0.08	0.00
26	150	0.63	1385	18.46	32.77	5.01	0.23	0.00	4.58	0.19	0.00
27	181	0.35	4462	14.23	36.48	6.26	0.12	-0.02	5.06	0.09	-0.02
28	58	1.00	3308	11.92	5.96	0.24	0.16	0.06	0.2	0.09	-0.02
29	142	0.67	4205	6.15	27.89	4.21	0.80	-0.33	3.84	0.74	-0.33
30	127	0.77	5487	12.69	30.68	3.17	0.11	-0.02	2.85	0.07	-0.05
31	100	0.75	2923	13.46	14.82	1.02	0.11	-0.03	0.92	0.09	-0.03
32	165	0.69	3436	13.08	51.22	8.7	0.17	-0.03	7.62	0.14	-0.03
33	92	0.88	5744	6.54	11.96	1.07	0.57	-0.17	1.05	0.58	-0.24
34	50	0.71	2282	6.92	3.11	0.15	0.52	-0.25	0.14	0.49	-0.31
35	108	0.65	3179	20.00	16.1	0.91	0.06	0.00	0.74	0.05	0.00
36	62	0.31	4974	14.62	4.17	0.48	0.05	0.01	0.2	0.09	0.03
37	77	0.60	5231	19.23	9.28	0.25	0.15	0.12	0.36	0.08	0.06
38	69	0.92	5103	16.15	8.95	0.23	0.16	0.12	0.22	0.09	0.07
39	85	0.52	3564	8.85	8.52	0.28	0.24	-0.14	0.28	0.20	-0.17
40	112	0.25	3692	11.15	9.4	0.37	0.13	-0.03	0.34	0.10	-0.03

Table 4 Benchmark policy results for Fall.

Scenario	M	$\Phi(\%M)$	γ	ρ	Backward Induction Time (s)	Stationary Benchmark Policy Time (s)	Opt Gap	Demand Gap	Dynamic Benchmark Policy Time (s)	Opt Gap	Demand Gap
1	104	0.42	5872	4.15	59.8	0.31	0.12	0.05	0.72	0.08	0.04
2	185	0.83	5359	6.62	78.79	13.8	0.08	0.00	13.81	0.06	0.00
3	88	0.48	1513	4.92	8.62	0.62	0.18	0.00	0.6	0.14	0.00
4	200	0.44	3821	3.08	60.82	11.03	0.38	0.00	10.65	0.30	0.00
5	158	0.58	1256	4.62	35.87	5.15	0.34	0.00	5.25	0.29	0.00
6	173	0.56	4077	7.85	50.78	8.5	0.08	0.00	6.94	0.05	0.00
7	123	0.62	1897	2.15	13.36	3.19	1.11	-0.71	2.02	0.98	-0.71
8	169	0.54	5615	4.00	48.23	6.64	0.14	0.00	6.28	0.11	0.00
9	73	0.79	1000	6.77	7.23	0.48	0.15	0.00	0.45	0.12	0.00
10	192	0.85	2667	7.54	80.56	16.02	0.14	0.00	15.71	0.11	0.00
11	162	0.27	2154	3.23	21.65	2.96	0.38	0.00	2.46	0.31	0.00
12	65	0.40	2795	7.08	5.15	0.21	0.07	0.03	0.2	0.04	0.01
13	119	0.50	4333	6.00	18.26	0.81	0.07	0.00	1.05	0.05	0.00
14	115	0.37	4590	2.00	5.12	0.87	2.23	-0.80	0.72	2.07	-0.81
15	96	0.87	1128	3.85	12.68	0.85	0.37	0.00	0.77	0.30	0.00
16	196	0.46	2410	6.31	58.1	12.52	0.17	0.00	10.29	0.13	0.00
17	146	0.38	6000	7.23	26.69	2.89	0.06	0.02	2.37	0.04	0.00
18	131	0.96	3051	3.38	31.55	3.66	0.29	0.00	3.71	0.24	0.00
19	177	0.94	4846	3.69	74.59	12.54	0.21	0.00	12.46	0.17	0.00
20	135	0.29	2538	6.92	14.81	1.73	0.09	0.00	1.23	0.07	0.00
21	54	0.73	4718	4.31	5.28	0.17	0.18	0.11	0.16	0.12	0.08
22	188	0.81	2026	2.92	71.83	15.52	0.59	-0.01	13.69	0.50	-0.01
23	154	0.90	1641	5.54	43.97	6.87	0.25	0.00	6.84	0.20	0.00
24	81	0.33	1769	2.31	3.6	0.32	0.94	-0.58	0.32	0.78	-0.58
25	138	0.98	3949	6.15	38.18	4.98	0.09	0.00	4.6	0.06	0.00
26	150	0.63	1385	7.38	33.18	4.91	0.19	0.00	4.24	0.15	0.00
27	181	0.35	4462	5.69	40.36	6.06	0.10	0.00	4.87	0.08	0.00
28	58	1.00	3308	4.77	6.25	0.28	0.16	0.12	0.28	0.08	0.05
29	142	0.67	4205	2.46	27.47	4.38	0.72	-0.45	3.91	0.60	-0.45
30	127	0.77	5487	5.08	32.26	3	0.10	0.04	2.86	0.05	0.01
31	100	0.75	2923	5.38	15.35	1.06	0.10	0.00	0.95	0.07	0.00
32	165	0.69	3436	5.23	48.88	7.97	0.15	0.00	7.31	0.12	0.00
33	92	0.88	5744	2.62	13.49	1.07	0.42	-0.08	1.02	0.34	-0.15
34	50	0.71	2282	2.77	3.9	0.15	0.34	-0.05	0.15	0.25	-0.10
35	108	0.65	3179	8.00	16.61	0.78	0.06	0.00	0.83	0.04	0.00
36	62	0.31	4974	5.85	4.29	4.89	0.05	0.02	0.32	0.12	0.06
37	77	0.60	5231	7.69	9.52	0.23	0.15	0.12	0.24	0.08	0.06
38	69	0.92	5103	6.46	9.37	0.21	0.17	0.12	0.24	0.09	0.07
39	85	0.52	3564	3.54	10.08	0.27	0.17	0.03	0.27	0.11	0.00
40	112	0.25	3692	4.46	10.7	0.34	0.11	0.04	0.35	0.09	0.04