Appendix A

Proposition 1: *In the single runway case the expression of expected delay shown in equations (13) and (14) serves as a lower bound of the true delay at optimality.*

Proof: Assuming we are dealing with a single period and runway, we shall define our parameters as follows:

$Q \equiv$ The set of all possible outcomes

$S \equiv$ The set of all slots

The exact expression for the expected number of arrivals in slot *s* can be written as

$$\sum_{q \in Q} p_q W_q^s = \overline{W}^s \quad (A1)$$

For a given outcome we can express the delay at slot *s* as

$$W_q^s = \max\left(\left(n_q^s + W_q^{s-1} + y_s - 1\right), 0\right) \quad (A2)$$

At optimality we have

$$W_q^{s*} = \max\left(\left(n_q^s + W_q^{s-1*} + y_s^* - 1\right), 0\right) \quad (A3)$$

Partition the set *Q* into two disjoint subsets as follows:

$V \equiv$ The set of all outcomes where $n_q^s + W_q^{s-1*} + y_s^* - 1$ is positive

$U \equiv$ The set of all outcomes where W_q^{s*} is zero

Substituting (A3) into (A1) we have

$$\begin{aligned}\overline{W}^{s*} &= \sum_{q \in Q} p_q W_q^{s*} = \sum_{q \in Q} p_q \left(\max\left(\left(n_q^s + W_q^{s-1*} + y_s^* - 1\right), 0\right)\right) \\ &= \sum_{q \in V} p_q \left(n_q^s + W_q^{s-1*} + y_s^* - 1\right) + \sum_{q \in U} p_q (0) \\ &= \sum_{q \in V} p_q \left(n_q^s + W_q^{s-1*} + y_s^* - 1\right)\end{aligned}$$

Since $n_q^s + W_q^{s-1*} + y_s^* - 1$ is non-positive for all $q \in U$,

$$\bar{W}^{s*} \geq \sum_{q \in Q} p_q \left(n_q^s + W_q^{s-1*} + y_s^* - 1 \right)$$
$$= \sum_{q \in Q} p_q n_q^s + \sum_{q \in Q} p_q W_q^{s-1*} + y_s^* \sum_{q \in Q} p_q - \sum_{q \in Q} p_q$$
$$= \bar{n}^s + \bar{W}^{s-1*} + y_s^* - 1$$

where $\bar{n}^s$ is the expected number of short haul arrivals during slot *s*. We also require that the expected number of arrivals be non-negative and

$$\bar{W}^{s*} \geq 0$$

So we have

$$\bar{W}^{s*} \geq \max\left(\left(\bar{n}^s + \bar{W}^{s-1*} + y_s^* - 1 \right), 0 \right)$$

Thus, the expressions in equations (13) and (14) serve as a lower bound on the true expected delay. □

Corollary 1: *In the single runway case the expression of expected delay shown in equations (13) and (14) serves as an exact representation of the true delay when all at least 1 flight arrives in a time interval (slot).*

Proof: When at least 1 flight arrives in every time interval (slot), $n_q^s + W_q^{s-1} + y_s - 1 \geq 0$, so (A2) can be rewritten as:

$$W_q^s = n_q^s + W_q^{s-1} + y_s - 1 \quad (A4)$$

Linearity of (A4) allows the following expectations to be taken:

$$\bar{W}^s = \bar{n}^s + \bar{W}^{s-1} + y_s - 1$$

Thus $\bar{W}^s$ is an exact transformation of the true expected delay. □