

# Adaptive Routing and Recharging Policies for Electric Vehicles

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## A Appendix

### A.1 Code for procedures used in Algorithm 1

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1: procedure FindOptDirectPath( $\mathcal{G}, x, y, k$ )
2: {
3:  $G = \{(i, j) \in \mathcal{N} : 0 \leq i \leq x, 0 \leq j \leq y\}$ 
4:  $check(0, 0) = 1$ 
5: for  $diag = 0, \dots, x + y - 1$  do
6:   for  $(i, j) \in G : i + j = diag$  do
7:     if  $V(i, j) + LB(|x - i| + |y - j|) < V(x, y)$  then
8:       for all  $(\Delta i, \Delta j) : \Delta i \geq 0, \Delta j \geq 0, 0 < \Delta i + \Delta j < k, (i + \Delta i, j + \Delta j) \in G$  do
9:          $v = V(i, j) + (1 - E[A_i])\mathcal{W}_i + s + c((\Delta i + \Delta j) \cdot h, 0) + (\Delta i + \Delta j) \cdot t$ 
10:        if  $v < V(i + \Delta i, j + \Delta j)$  then
11:           $V(i + \Delta i, j + \Delta j) = v$ 
12:           $pred(i + \Delta i, j + \Delta j) = (i, j)$ 
13:           $check(i + \Delta i, j + \Delta j) = 2$ 
14:        end if
15:      end for
16:    end if
17:  end for
18: end for
19: }
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20: procedure ScanReverse( $\mathcal{G}, x, y, k, V, check, pred$ )
21: {
22:    $ScanType = null$ 
23:    $diag = MaxDiag$ 
24:   while  $diag \geq MinDiag$  do
25:      $lo = \min\{i : check(i, diag - i) > 0\}$ 
26:      $hi = \max\{i : check(i, diag - i) > 0\}$ 
27:      $i = lo$ 
28:      $j = diag - i$ 
29:     while ( $i \leq hi$  or  $check(i, j) > 0$ ) do
30:       if ( $check(i, j) > 0$  and  $V(i, j) + LB(|x - i| + |y - j|) < V(x, y)$ ) then
31:         for  $\Delta i = 1, \dots, \lfloor k/2 \rfloor$  do
32:            $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$ 
33:           if  $v < V(i + \Delta i, j - \Delta i)$  then
34:              $V(i + \Delta i, j - \Delta i) = v$ 
35:              $pred(i + \Delta i, j - \Delta i) = (i, j)$ 
36:              $check(i + \Delta i, j - \Delta i) = 2$ 
37:              $ScanType = forward$ 
38:           end if
39:         end for
40:       end if
41:        $i = i + 1$ 
42:        $j = j - 1$ 
43:     end while
44:      $i = hi$ 
45:      $j = diag - i$ 
46:     while ( $i \geq lo$  or  $check(i, j) > 0$ ) do
47:       if ( $check(i, j) > 0$  and  $V(i, j) + LB(|x - i| + |y - j|) < V(x, y)$ ) then
48:          $check(i, j) = check(i, j) - 1$ 
49:         for  $\Delta i = 1, \dots, \lfloor k/2 \rfloor$  do
50:            $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$ 
51:           if  $v < V(i - \Delta i, j + \Delta i)$  then
52:              $V(i - \Delta i, j + \Delta i) = v$ 
53:              $pred(i - \Delta i, j + \Delta i) = (i, j)$ 
54:              $check(i - \Delta i, j + \Delta i) = 2$ 
55:              $ScanType = forward$ 
56:           end if
57:         end for
58:       for all  $(\Delta i, \Delta j) : \Delta i + \Delta j < 0, |\Delta i| + |\Delta j| \leq k$  do
59:          $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c((|\Delta i| + |\Delta j|) \cdot h, 0) + (|\Delta i| + |\Delta j|) \cdot t$ 
60:         if  $v < V(i + \Delta i, j + \Delta j)$  then
61:            $V(i + \Delta i, j + \Delta j) = v$ 
62:            $pred(i + \Delta i, j + \Delta j) = (i, j)$ 
63:            $check(i + \Delta i, j + \Delta j) = 2$ 
64:            $ScanType = forward$ 
65:            $MinDiag = \min\{MinDiag, diag + \Delta i + \Delta j\}$ 
66:         end if
67:       end for
68:     end if
69:   end while
70:    $diag = diag - 1$ 
71: end while
72: }

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73: procedure ScanForward( $\mathcal{G}, x, y, k, V, check, pred$ )
74: {
75:    $ScanType = null$ 
76:    $diag = MinDiag$ 
77:   while  $diag \leq MaxDiag$  do
78:      $lo = \min\{i : check(i, diag - i) > 0\}$ 
79:      $hi = \max\{i : check(i, diag - i) > 0\}$ 
80:      $i = lo$ 
81:      $j = diag - i$ 
82:     while ( $i \leq hi$  or  $check(i, j) > 0$ ) do
83:       if ( $check(i, j) > 0$  and  $V(i, j) + LB(|x - i| + |y - j|) < V(x, y)$ ) then
84:         for  $\Delta i = 1, \dots, \lfloor k/2 \rfloor$  do
85:            $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$ 
86:           if  $v < V(i + \Delta i, j - \Delta i)$  then
87:              $V(i + \Delta i, j - \Delta i) = v$ 
88:              $pred(i + \Delta i, j - \Delta i) = (i, j)$ 
89:              $check(i + \Delta i, j - \Delta i) = 2$ 
90:              $ScanType = reverse$ 
91:           end if
92:         end for
93:       end if
94:        $i = i + 1$ 
95:        $j = j - 1$ 
96:     end while
97:      $i = hi$ 
98:      $j = diag - i$ 
99:     while ( $i \geq lo$  or  $check(i, j) > 0$ ) do
100:      if ( $check(i, j) > 0$  and  $V(i, j) + LB(|x - i| + |y - j|) < V(x, y)$ ) then
101:         $check(i, j) = check(i, j) - 1$ 
102:        for  $\Delta i = 1, \dots, \lfloor k/2 \rfloor$  do
103:           $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$ 
104:          if  $v < V(i - \Delta i, j + \Delta i)$  then
105:             $V(i - \Delta i, j + \Delta i) = v$ 
106:             $pred(i - \Delta i, j + \Delta i) = (i, j)$ 
107:             $check(i - \Delta i, j + \Delta i) = 2$ 
108:             $ScanType = reverse$ 
109:          end if
110:        end for
111:        for all ( $\Delta i, \Delta j$ ) :  $\Delta i + \Delta j > 0, |\Delta i| + |\Delta j| \leq k$  do
112:           $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij} + \mu_{ij}} + s + c((|\Delta i| + |\Delta j|) \cdot h, 0) + (|\Delta i| + |\Delta j|) \cdot t$ 
113:          if  $v < V(i + \Delta i, j + \Delta j)$  then
114:             $V(i + \Delta i, j + \Delta j) = v$ 
115:             $pred(i + \Delta i, j + \Delta j) = (i, j)$ 
116:             $check(i + \Delta i, j + \Delta j) = 2$ 
117:             $ScanType = reverse$ 
118:             $MaxDiag = \max\{MaxDiag, diag + \Delta i + \Delta j\}$ 
119:          end if
120:        end for
121:      end if
122:    end while
123:     $diag = diag + 1$ 
124:  end while
125: }

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## A.2 Omitted Proofs

### A.2.1 Proof of Lemma 1

**Proof.** Suppose  $\pi_{\mathcal{N}}^R : \mathcal{S} \rightarrow \mathcal{A}^R$  is a feasible recharging policy, and there exists a state  $(i, q_i, a_i) \in \mathcal{S}$  such that  $\pi_{\mathcal{N}}^R(i, q_i, a_i) = \sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta > 0$  for some  $k \in (i+1, \dots, n-1)$  and  $0 \leq \Delta < h_k$ . Then consider an alternate policy  $\pi_{\mathcal{N}}^{R'} : \mathcal{S} \rightarrow \mathcal{A}^R$  where

$$\pi_{\mathcal{N}}^{R'}(j, q_j, a_j) = \begin{cases} \pi_{\mathcal{N}}^R(j, q_j, a_j) - \Delta, & (j, q_j, a_j) = (i, q_i, a_i) \\ \pi_{\mathcal{N}}^R(j, q_j + \Delta, a_j) + \Delta I_{\pi_{\mathcal{N}}^R(j, q_j + \Delta, a_j) > 0}, & j \in (i+1, \dots, k) \text{ and } q_j = \sum_{\ell=j}^{k-1} h_\ell \\ \pi_{\mathcal{N}}^R(j, q_j, a_j), & \text{otherwise} \end{cases}$$

for any  $(j, q_j, a_j) \in \mathcal{S}$ , where  $I_{\pi_{\mathcal{N}}^R(j, q_j + \Delta, a_j) > 0}$  equals 1 if  $\pi_{\mathcal{N}}^R(j, q_j + \Delta, a_j) > 0$  and 0 otherwise. In this alternate policy, the vehicle's charge level when departing from node  $i$  is  $\sum_{\ell=i}^{k-1} h_\ell$  instead of  $\sum_{\ell=i}^{k-1} h_\ell + \Delta$ . The next node where the vehicle stops to recharge is the same as in  $\pi_{\mathcal{N}}^R$ , although the vehicle recharges an extra amount  $\Delta$  so that the resulting charge level is the same as in the original policy. Beyond that node, both policies are identical.

Suppose that  $j$  is the next node after  $i$  at which the vehicle stops to recharge for a given sequence  $(a_{i+1}, \dots, a_j)$  of observed station availabilities (i.e.,  $A_\ell = a_\ell$  for all  $\ell \in (i+1, \dots, j)$ ). Then the expected cost of the policy  $\pi_{\mathcal{N}}^R$  for the state  $(i, q_i, a_i)$  is

$$C_{\pi_{\mathcal{N}}^R}(i, q_i, a_i) \mid (A_\ell = a_\ell \forall \ell \in (i+1, \dots, j)) = \sum_{\ell=i}^{j-1} t_\ell + s + (1 - a_i)W_i + c \left( \sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta, q_i \right) + C_{\pi_{\mathcal{N}}^R} \left( j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j \right). \quad (1)$$

Now suppose that the same sequence of station availabilities is observed under policy  $\pi_{\mathcal{N}}^{R'}$ . Then the expected cost of the policy under such a scenario is

$$C_{\pi_{\mathcal{N}}^{R'}}(i, q_i, a_i) \mid (A_\ell = a_\ell \forall \ell \in (i+1, \dots, j)) = \sum_{\ell=i}^{j-1} t_\ell + s + (1 - a_i)W_i + c \left( \sum_{\ell=i}^{k-1} h_\ell - q_i, q_i \right) + C_{\pi_{\mathcal{N}}^{R'}} \left( j, \sum_{\ell=j}^{k-1} h_\ell, a_j \right). \quad (2)$$

When we compare the costs of the two policies for the given realization of observed station availabilities, we

find that

$$\begin{aligned}
(1) - (2) &= c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta, q_i\right) + C_{\pi_{\mathcal{N}}^R}\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) - c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i, q_i\right) - C_{\pi_{\mathcal{N}}^{R'}}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + C_{\pi_{\mathcal{N}}^R}\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) - C_{\pi_{\mathcal{N}}^{R'}}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right), \sum_{\ell=j}^{k-1} h_\ell + \Delta\right) - c\left(\pi_{\mathcal{N}}^{R'}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right), \sum_{\ell=j}^{k-1} h_\ell\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right), \sum_{\ell=j}^{k-1} h_\ell + \Delta\right) - c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) + \Delta, \sum_{\ell=j}^{k-1} h_\ell\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) - c\left(\Delta, \sum_{\ell=j}^{k-1} h_\ell\right) \\
&\geq 0 \quad (\text{by convexity and monotone increasing property of } c(\cdot)) \tag{3}
\end{aligned}$$

since  $\sum_{\ell=i}^{k-1} h_\ell > \sum_{\ell=j}^{k-1} h_\ell + \Delta$ . The expression in (3) holds for any given sequence  $(a_{i+1}, \dots, a_j)$  of observed station availabilities such that  $j$  is the next node after  $i$  where the vehicle recharges, and because the probability of the sequence occurring is the same under both policies, the (unconditional) expected cost of the policy  $\pi_{\mathcal{N}}^{R'}$  is no greater than that of  $\pi_{\mathcal{N}}^R$  for the state  $(i, q_i, a_i)$ . It follows that there exists an optimal recharging policy  $\pi_{\mathcal{N}}^{R*} : \mathcal{S} \rightarrow \mathcal{A}^R$  such that

$$\pi_{\mathcal{N}}^{R*}(i, q_i, a_i) \in \left\{ \left( \sum_{\ell=i}^{k-1} h_\ell - q_i \right)^+ : k \in (i+1, \dots, n-1), \sum_{\ell=i}^{k-1} h_\ell \leq q_{max} \right\}$$

for all  $(i, q_i, a_i) \in \mathcal{S}$ . ■

## A.2.2 Proof of Lemma 2

**Proof.** We prove the claim by induction. At node 0, we have  $q_0 = 0 \in \mathcal{H}(1)$ , and by Lemma 1, there exists an optimal recharging policy  $\pi_{\mathcal{N}}^{R*}$  such that  $\pi_{\mathcal{N}}^{R*}(0, 0, \cdot) \in \mathcal{H}(1)$ .

Then assuming the claim holds for a node  $i \in (0, \dots, n-2)$ , suppose  $q_i = \sum_{\ell=i}^{j-1} h_\ell \in \mathcal{H}(i)$  for some  $j \in (i, \dots, n-1)$  and  $\pi_{\mathcal{N}}^{R*}(i, q_i, a_i) = \sum_{\ell=j}^{j'-1} h_\ell \in \mathcal{H}(j)$  for some  $j' \in (j, \dots, n-1)$ . At node  $i+1$ , the vehicle's charge level is

$$q_{i+1} = \sum_{\ell=i}^{j-1} h_\ell + \sum_{\ell=j}^{j'-1} h_\ell - h_i = \sum_{\ell=i+1}^{j'-1} h_\ell \in \mathcal{H}(i+1),$$

and by Lemma 1, we have

$$\begin{aligned}
\pi_{\mathcal{N}}^{R^*}(i+1, q_{i+1}, \cdot) &\in \left\{ \left( \sum_{\ell=i+1}^{k-1} h_{\ell} - q_{i+1} \right)^+ : k \in (i+2, \dots, n-1), \sum_{\ell=i}^{k-1} h_{\ell} \leq q_{max} \right\} \\
&\subseteq \left\{ \sum_{\ell=j'}^{k-1} h_{\ell} : k \in (j', \dots, n-1), \sum_{\ell=i}^{k-1} h_{\ell} \leq q_{max} \right\} \\
&\subseteq H(j').
\end{aligned}$$

Thus, the claim holds for node  $i+1$  as well, and it follows that  $q_i \in \mathcal{H}(i)$  and  $\pi_{\mathcal{N}}^{R^*}(i, q_i, a_i) \in \mathcal{H}(k)$  for any realized state  $(i, q_i, a_i) \in \mathcal{S}$  given that the initial state is  $(0, 0, \cdot)$ . ■