Adaptive Routing and Recharging Policies for Electric Vehicles

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A Appendix

A.1 Code for procedures used in Algorithm 1

```
procedure FindOptDirectPath($\mathcal{G}, x, y, k$)
{
$G = \{(i,j) \in \mathcal{N} : 0 \leq i \leq x,\ 0 \leq j \leq y\}$
$check(0,0) = 1$
for $diag = 0, ..., x + y - 1$ do
    for $(i,j) \in G : i + j = diag$ do
        if $V(i,j) + \text{LB}(|x - i| + |y - j|) < V(x,y)$ then
            for all $(\Delta i, \Delta j) : \Delta i \geq 0,\ \Delta j \geq 0,\ 0 < \Delta i + \Delta j < k,\ (i + \Delta i, j + \Delta j) \in G$ do
                $v = V(i,j) + (1 - E[A_i])\mathcal{W}_i + s + c((\Delta i + \Delta j) \cdot h, 0) + (\Delta i + \Delta j) \cdot t$
                if $v < V(i + \Delta i, j + \Delta j)$ then
                    $V(i + \Delta i, j + \Delta j) = v$
                    $pred(i + \Delta i, j + \Delta j) = (i,j)$
                    $check(i + \Delta i, j + \Delta j) = 2$
                end if
            end for
        end if
    end for
end for
}
```

```
**procedure** ScanReverse($\mathcal{G}, x, y, k, V, check, pred$)
{
$ScanType = null$
$diag = MaxDiag$
**while** $diag \geq MinDiag$ **do**
  $lo = \min\{i : check(i, diag - i) > 0\}$
  $hi = \max\{i : check(i, diag - i) > 0\}$
  $i = lo$
  $j = diag - i$
  **while** ($i \leq hi$ **or** $check(i,j) > 0$) **do**
    **if** ($check(i,j) > 0$ **and** $V(i,j) + \mathrm{LB}(|x-i| + |y-j|) < V(x,y)$) **then**
      **for** $\Delta i = 1, ..., \lfloor k/2 \rfloor$ **do**
        $v = V(i,j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$
        **if** $v < V(i+\Delta i, j-\Delta i)$ **then**
          $V(i+\Delta i, j-\Delta i) = v$
          $pred(i+\Delta i, j-\Delta i) = (i,j)$
          $check(i+\Delta i, j-\Delta i) = 2$
          $ScanType = forward$
        **end if**
      **end for**
    **end if**
    $i = i + 1$
    $j = j - 1$
  **end while**
  $i = hi$
  $j = diag - i$
  **while** ($i \geq lo$ **or** $check(i,j) > 0$) **do**
    **if** ($check(i,j) > 0$ **and** $V(i,j) + \mathrm{LB}(|x-i| + |y-j|) < V(x,y)$) **then**
      $check(i,j) = check(i,j) - 1$
      **for** $\Delta i = 1, ..., \lfloor k/2 \rfloor$ **do**
        $v = V(i,j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$
        **if** $v < V(i-\Delta i, j+\Delta i)$ **then**
          $V(i-\Delta i, j+\Delta i) = v$
          $pred(i-\Delta i, j+\Delta i) = (i,j)$
          $check(i-\Delta i, j+\Delta i) = 2$
          $ScanType = forward$
        **end if**
      **end for**
      **for all** $(\Delta i, \Delta j) : \Delta i + \Delta j < 0,\ |\Delta i| + |\Delta j| \leq k$ **do**
        $v = V(i,j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c((|\Delta i| + |\Delta j|) \cdot h, 0) + (|\Delta i| + |\Delta j|) \cdot t$
        **if** $v < V(i+\Delta i, j+\Delta j)$ **then**
          $V(i+\Delta i, j+\Delta j) = v$
          $pred(i+\Delta i, j+\Delta j) = (i,j)$
          $check(i+\Delta i, j+\Delta j) = 2$
          $ScanType = forward$
          $MinDiag = \min\{MinDiag, diag + \Delta i + \Delta j\}$
        **end if**
      **end for**
    **end if**
  **end while**
  $diag = diag - 1$
**end while**
}
```

```
**procedure** ScanForward($\mathcal{G}, x, y, k, V, check, pred$)
{
$ScanType = null$
$diag = MinDiag$
**while** $diag \le MaxDiag$ **do**
    $lo = \min\{i : check(i, diag - i) > 0\}$
    $hi = \max\{i : check(i, diag - i) > 0\}$
    $i = lo$
    $j = diag - i$
    **while** $(i \le hi$ **or** $check(i, j) > 0)$ **do**
        **if** $(check(i, j) > 0$ **and** $V(i, j) + \mathrm{LB}(|x - i| + |y - j|) < V(x, y))$ **then**
            **for** $\Delta i = 1, ..., \lfloor k/2 \rfloor$ **do**
                $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$
                **if** $v < V(i + \Delta i, j - \Delta i)$ **then**
                    $V(i + \Delta i, j - \Delta i) = v$
                    $pred(i + \Delta i, j - \Delta i) = (i, j)$
                    $check(i + \Delta i, j - \Delta i) = 2$
                    $ScanType = reverse$
                **end if**
            **end for**
        **end if**
        $i = i + 1$
        $j = j - 1$
    **end while**
    $i = hi$
    $j = diag - i$
    **while** $(i \ge lo$ **or** $check(i, j) > 0)$ **do**
        **if** $(check(i, j) > 0$ **and** $V(i, j) + \mathrm{LB}(|x - i| + |y - j|) < V(x, y))$ **then**
            $check(i, j) = check(i, j) - 1$
            **for** $\Delta i = 1, ..., \lfloor k/2 \rfloor$ **do**
                $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c(2\Delta i \cdot h, 0) + 2\Delta i \cdot t$
                **if** $v < V(i - \Delta i, j + \Delta i)$ **then**
                    $V(i - \Delta i, j + \Delta i) = v$
                    $pred(i - \Delta i, j + \Delta i) = (i, j)$
                    $check(i - \Delta i, j + \Delta i) = 2$
                    $ScanType = reverse$
                **end if**
            **end for**
            **for all** $(\Delta i, \Delta j) : \Delta i + \Delta j > 0,\ |\Delta i| + |\Delta j| \le k$ **do**
                $v = V(i, j) + \frac{\lambda_{ij}/\mu_{ij}}{\lambda_{ij}+\mu_{ij}} + s + c((|\Delta i| + |\Delta j|) \cdot h, 0) + (|\Delta i| + |\Delta j|) \cdot t$
                **if** $v < V(i + \Delta i, j + \Delta j)$ **then**
                    $V(i + \Delta i, j + \Delta j) = v$
                    $pred(i + \Delta i, j + \Delta j) = (i, j)$
                    $check(i + \Delta i, j + \Delta j) = 2$
                    $ScanType = reverse$
                    $MaxDiag = \max\{MaxDiag, diag + \Delta i + \Delta j\}$
                **end if**
            **end for**
        **end if**
    **end while**
    $diag = diag + 1$
**end while**
}
```

A.2 Omitted Proofs

A.2.1 Proof of Lemma 1

Proof. Suppose $\pi_{\mathcal{N}}^{R} : \mathcal{S} \to \mathcal{A}^{R}$ is a feasible recharging policy, and there exists a state $(i, q_i, a_i) \in \mathcal{S}$ such that $\pi_{\mathcal{N}}^{R}(i, q_i, a_i) = \sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta > 0$ for some $k \in (i+1, ..., n-1)$ and $0 \leq \Delta < h_k$. Then consider an alternate policy ${\pi_{\mathcal{N}}^{R}}' : \mathcal{S} \to \mathcal{A}^{R}$ where

$$
{\pi_{\mathcal{N}}^{R}}'(j, q_j, a_j) = \begin{cases} \pi_{\mathcal{N}}^{R}(j, q_j, a_j) - \Delta, & (j, q_j, a_j) = (i, q_i, a_i) \\ \pi_{\mathcal{N}}^{R}(j, q_j + \Delta, a_j) + \Delta I_{\pi_{\mathcal{N}}^{R}(j, q_j + \Delta, a_j) > 0}, & j \in (i+1, ..., k) \text{ and } q_j = \sum_{\ell=j}^{k-1} h_\ell \\ \pi_{\mathcal{N}}^{R}(j, q_j, a_j), & \text{otherwise} \end{cases}
$$

for any $(j, q_j, a_j) \in \mathcal{S}$, where $I_{\pi_{\mathcal{N}}^{R}(j, q_j + \Delta, a_j) > 0}$ equals 1 if $\pi_{\mathcal{N}}^{R}(j, q_j + \Delta, a_j) > 0$ and 0 otherwise. In this alternate policy, the vehicle's charge level when departing from node i is $\sum_{\ell=i}^{k-1} h_\ell$ instead of $\sum_{\ell=i}^{k-1} h_\ell + \Delta$. The next node where the vehicle stops to recharge is the same as in $\pi_{\mathcal{N}}^{R}$, although the vehicle recharges an extra amount Δ so that the resulting charge level is the same as in the original policy. Beyond that node, both policies are identical.

Suppose that j is the next node after i at which the vehicle stops to recharge for a given sequence $(a_{i+1}, ..., a_j)$ of observed station availabilities (i.e., $A_\ell = a_\ell$ for all $\ell \in (i+1, ..., j)$). Then the expected cost of the policy $\pi_{\mathcal{N}}^{R}$ for the state (i, q_i, a_i) is

$$
\begin{aligned} C_{\pi_{\mathcal{N}}^{R}}(i, q_i, a_i) \mid (A_\ell = a_\ell \ \forall\ \ell \in (i+1, ..., j)) = \sum_{\ell=i}^{j-1} t_\ell + s + (1 - a_i)\mathcal{W}_i + c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta, q_i\right) + \\ C_{\pi_{\mathcal{N}}^{R}}\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right). \end{aligned} \tag{1}
$$

Now suppose that the same sequence of station availabilities is observed under policy ${\pi_{\mathcal{N}}^{R}}'$. Then the expected cost of the policy under such a scenario is

$$
\begin{aligned} C_{{\pi_{\mathcal{N}}^{R}}'}(i, q_i, a_i) \mid (A_\ell = a_\ell \ \forall\ \ell \in (i+1, ..., j)) = \sum_{\ell=i}^{j-1} t_\ell + s + (1 - a_i)\mathcal{W}_i + c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i, q_i\right) + \\ C_{{\pi_{\mathcal{N}}^{R}}'}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right). \end{aligned} \tag{2}
$$

When we compare the costs of the two policies for the given realization of observed station availabilities, we find that

$$\begin{aligned}
(1)-(2) &= c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i + \Delta, q_i\right) + C_{\pi_{\mathcal{N}}^R}\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) - c\left(\sum_{\ell=i}^{k-1} h_\ell - q_i, q_i\right) - C_{\pi_{\mathcal{N}}^R{}'}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + C_{\pi_{\mathcal{N}}^R}\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) - C_{\pi_{\mathcal{N}}^R{}'}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right), \sum_{\ell=j}^{k-1} h_\ell + \Delta\right) - c\left(\pi_{\mathcal{N}}^{R\prime}\left(j, \sum_{\ell=j}^{k-1} h_\ell, a_j\right), \sum_{\ell=j}^{k-1} h_\ell\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) + c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right), \sum_{\ell=j}^{k-1} h_\ell + \Delta\right) - c\left(\pi_{\mathcal{N}}^R\left(j, \sum_{\ell=j}^{k-1} h_\ell + \Delta, a_j\right) + \Delta, \sum_{\ell=j}^{k-1} h_\ell\right) \\
&= c\left(\Delta, \sum_{\ell=i}^{k-1} h_\ell\right) - c\left(\Delta, \sum_{\ell=j}^{k-1} h_\ell\right) \\
&\geq 0 \quad \text{(by convexity and monotone increasing property of } c(\cdot)\text{)} \qquad (3)
\end{aligned}$$

since $\sum_{\ell=i}^{k-1} h_\ell > \sum_{\ell=j}^{k-1} h_\ell + \Delta$. The expression in (3) holds for any given sequence $(a_{i+1}, ..., a_j)$ of observed station availabilities such that j is the next node after i where the vehicle recharges, and because the probability of the sequence occurring is the same under both policies, the (unconditional) expected cost of the policy $\pi_{\mathcal{N}}^{R\prime}$ is no greater than that of $\pi_{\mathcal{N}}^R$ for the state (i, q_i, a_i). It follows that there exists an optimal recharging policy $\pi_{\mathcal{N}}^{R*} : \mathcal{S} \to \mathcal{A}^R$ such that

$$\pi_{\mathcal{N}}^{R*}(i, q_i, a_i) \in \left\{ \left(\sum_{\ell=i}^{k-1} h_\ell - q_i\right)^+ \;:\; k \in (i+1, ..., n-1),\; \sum_{\ell=i}^{k-1} h_\ell \leq q_{max} \right\}$$

for all $(i, q_i, a_i) \in \mathcal{S}$. ■

A.2.2 Proof of Lemma 2

Proof. We prove the claim by induction. At node 0, we have $q_0 = 0 \in \mathcal{H}(1)$, and by Lemma 1, there exists an optimal recharging policy $\pi_{\mathcal{N}}^{R*}$ such that $\pi_{\mathcal{N}}^{R*}(0, 0, \cdot) \in \mathcal{H}(1)$.

Then assuming the claim holds for a node $i \in (0, ..., n-2)$, suppose $q_i = \sum_{\ell=i}^{j-1} h_\ell \in \mathcal{H}(i)$ for some $j \in (i, ..., n-1)$ and $\pi_{\mathcal{N}}^{R*}(i, q_i, a_i) = \sum_{\ell=j}^{j'-1} h_\ell \in \mathcal{H}(j)$ for some $j' \in (j, ..., n-1)$. At node $i+1$, the vehicle's charge level is

$$q_{i+1} = \sum_{\ell=i}^{j-1} h_\ell + \sum_{\ell=j}^{j'-1} h_\ell - h_i = \sum_{\ell=i+1}^{j'-1} h_\ell \in \mathcal{H}(i+1),$$

and by Lemma 1, we have

$$\begin{aligned}\pi_{\mathcal{N}}^{R^*}(i+1, q_{i+1}, \cdot) &\in \left\{ \left(\sum_{\ell=i+1}^{k-1} h_\ell - q_{i+1} \right)^+ \;:\; k \in (i+2, ..., n-1),\; \sum_{\ell=i}^{k-1} h_\ell \le q_{max} \right\} \\ &\subseteq \left\{ \sum_{\ell=j'}^{k-1} h_\ell :\; k \in (j', ..., n-1),\; \sum_{\ell=i}^{k-1} h_\ell \le q_{max} \right\} \\ &\subseteq H(j').\end{aligned}$$

Thus, the claim holds for node $i+1$ as well, and it follows that $q_i \in \mathcal{H}(i)$ and $\pi_{\mathcal{N}}^{R^*}(i, q_i, a_i) \in \mathcal{H}(k)$ for any realized state $(i, q_i, a_i) \in \mathcal{S}$ given that the initial state is $(0, 0, \cdot)$. ■