Appendices

A Derivation of the four regions in Figure 2(a)

When $(\delta_1(t), \delta_2(t)) = (1,0)$, $g_1 = \min\{D_1, \frac{S_1}{\xi_1}, \frac{S_2}{1-\xi_1}\}$, and $g_2 = 0$. Then Equation (5) can be written as

$$\frac{dk_1}{dt} = -\frac{1-\xi_1}{L}\min\{D_1, \frac{S_1}{\xi_1}, \frac{S_2}{1-\xi_1}\} \tag{22}$$

Depending on the selection of (k_1, k), traffic dynamics vary a lot since the minimum function in Equation (22) will take one of the following three values: D_1, $\frac{S_1}{\xi_1}$, and $\frac{S_2}{1-\xi_1}$.

(i) When $k_1 < k_c$, we have $D_1 < C < \frac{S_1}{\xi_1}$. Then the bold dashed line to separate Region 1 and Region 4 occurs when $D_1 = \frac{S_2}{1-\xi_1} < C$. In this case, $D_1 = v_f k_1$ and $S_2 = v_f k_c \frac{k_j - 2k + k_1}{k_j - k_c}$, which leads to

$$k = \frac{k_j}{2} - \frac{[(1-\xi_1)k_j - (2-\xi_1)k_c]k_1}{2k_c}. \tag{23}$$

(ii) When $k_1 > k_c$ and $D_1 = C < \frac{S_1}{\xi_1}$, the bold dashed line to separate Region 1 and Region 2 is

$$k_1 = k_c. \tag{24}$$

The bold dashed line to separate Region 2 and Region 4 occurs when $D_1 = \frac{S_2}{1-\xi_1} = C$. In this case, $S_2 = v_f k_c \frac{k_j - 2k + k_1}{k_j - k_c}$, which leads to

$$k = \frac{\xi_1 k_j + (1-\xi_1)k_c + k_1}{2}. \tag{25}$$

(iii) When $k_1 > k_c$ and $D_1 = C > \frac{S_1}{\xi_1}$, the bold dashed line to separate Region 2 and Region 3 is when $D_1 = C = \frac{S_1}{\xi_1}$, which leads to

$$k_1 = k_j - \xi_1(k_j - k_c). \tag{26}$$

The bold dashed line to separate Region 3 and Region 4 occurs when $\frac{S_1}{\xi_1} = \frac{S_2}{1-\xi_1} < C$. In this case, $S_1 = C\frac{k_j - k_1}{k_j - k_c}$ and $S_2 = C\frac{k_j - 2k + k_1}{k_j - k_c}$, which leads to

$$k = \frac{(2\xi_1 - 1)k_j + k_1}{2\xi_1}. \tag{27}$$

Therefore, based on the above analysis, the bold dashed lines in Figure 2(a) divide the $k_1 - k$ space into four different regions, and the minimum function in Equation (22) will choose: (i) $D_1 < C$ in Region 1; (ii) $D_1 = C$ in Region 2; (iii) $\frac{S_1}{\xi_1}$ in Region 3; and (iv) $\frac{S_2}{1-\xi_1}$ in Region 4.

B Derivation of an approximate closed-form MFD

Here we derive the approximate closed-form MFD in (16) when $0.5 < \xi < 1$ from the stationary states provided in Table 3 under different retaining ratios. For $\xi > 0.5$, the possible combinations of regions having stationary states are: $(1,5)$, $(1,7)$, $(2,6)$, $(4,7)$, $(3,5)$, $(3,7)$, and $(3,8)$. For regions $(4,7)$ and $(3,8)$, the stationary states are gridlock states, and therefore, the average network average flow-rates are zero. For the rest of the regions, we can approximate the average network flow-rate using Equation (15).

(1) For regions $(1,5)$, the fixed point is $k_1^* = \frac{2k}{1+e^{-\gamma_1 \pi T}}$. Starting with $k_1(nT) = k_1^*$, we can get

$k_1(nT+\pi T) = k_1(nT)e^{-\gamma_1 \pi T}$. Since ring 1 is uncongested, $g_1(k_1) = v_f k_1$. Therefore, the average network flow-rate is

$$\begin{aligned} q(k) \approx \pi \frac{g_1(k_1^*) + g_1(k_1(nT+\pi T))}{2} &= \tfrac{1}{2}\pi v_f (k_1^* + k_1(nT+\pi T)) \\ &= \tfrac{1}{2}\pi v_f 2k \tfrac{1+e^{-\gamma_1 \pi T}}{1+e^{-\gamma_1 \pi T}} = \pi v_f k. \end{aligned} \tag{28}$$

(2) For regions $(1,7)$, the fixed point is $k_1^* = \frac{(k_j-2k)(e^{\gamma_5 \pi T}-1)}{1-e^{(\gamma_5-\gamma_1)\pi T}}$. Starting with $k_1(nT) = k_1^*$, we can get $k_1(nT+\pi T) = k_1(nT)e^{-\gamma_1 \pi T}$. Since ring 1 is uncongested, $g_1(k_1) = v_f k_1$. In addition, since T is small, $-\gamma_1 \pi T$, $\gamma_5 \pi T$, and $(\gamma_5 - \gamma_1)\pi T$ are also small. Therefore, the average network flow-rate is

$$\begin{aligned} q(k) &\approx \tfrac{1}{2}\pi v_f (k_1^* + k_1(nT+\pi T)) = \tfrac{1}{2}\pi v_f (k_j - 2k) \tfrac{(e^{\gamma_5 \pi T}-1)(1+e^{-\gamma_1 \pi T})}{1-e^{(\gamma_5-\gamma_1)\pi T}} \\ &\approx \tfrac{1}{2}\pi v_f (k_j - 2k) \tfrac{\gamma_5 \pi T(2-\gamma_1 \pi T)}{(\gamma_1-\gamma_5)\pi T} \approx \pi C \tfrac{(k_j-2k)}{\xi(k_j-k_c)-k_c}. \end{aligned} \tag{29}$$

(3) For regions $(2,6)$, the fixed point is $k_1^* = k_1(t)$. In this combination of regions, the out-fluxes are restricted by the capacity. Therefore, the average network flow-rate is

$$q(k) \approx \pi \frac{g_1(k_1^*) + g_1(k_1(nT+\pi T))}{2} = \pi C. \tag{30}$$

(4) For regions $(3,5)$, the fixed point is $k_1^* = \frac{2k(1-e^{-\gamma_4 \pi T})-k_j(e^{\gamma_2 \pi T}-1)e^{-\gamma_4 \pi T}}{1-e^{(\gamma_2-\gamma_4)\pi T}}$. Starting with $k_1(nT) = k_1^*$, we can get $k_1(nT+\pi T) = k_j(1-e^{\gamma_2 \pi T}) + k_1(nT)e^{\gamma_2 \pi T}$. In this combination of regions, the out-flux is governed by the supply in ring 1, i.e., $g_1(k_1) = \frac{S_1(k_1)}{\xi} = \frac{C(k_j-k_1)}{\xi(k_j-k_c)}$. Therefore, the average network flow-rate is

$$\begin{aligned} q(k) &\approx \pi C \tfrac{2k_j-(k_j(1-e^{\gamma_2 \pi T})+k_1^*(1+e^{\gamma_2 \pi T}))}{2\xi(k_j-k_c)} \approx \pi C \tfrac{2k_j - 2\frac{2k*\gamma_4 - k_j*\gamma_2}{\gamma_4-\gamma_2}}{2\xi(k_j-k_c)} \\ &= \pi C \tfrac{(k_j-2k)\frac{\gamma_4}{\gamma_4-\gamma_2}}{\xi(k_j-k_c)} = \pi C \tfrac{(k_j-2k)}{\xi(k_j-k_c)-k_c}. \end{aligned} \tag{31}$$

(5) For regions $(3,7)$, the fixed point is $k_1^* = \frac{2k+k_j(e^{\gamma_2 \pi T}-1)}{e^{\gamma_2 \pi T}+1}$. Starting with $k_1(nT) = k_1^*$, we can get $k_1(nT+\pi T) = k_j(1-e^{\gamma_2 \pi T}) + k_1(nT)e^{\gamma_2 \pi T}$. In this combination of regions, the out-flux is governed by the supply in ring 1, i.e., $g_1(k_1) = \frac{S_1(k_1)}{\xi} = \frac{C(k_j-k_1)}{\xi(k_j-k_c)}$. Therefore, the average

network flow-rate is

$$\begin{aligned} q(k) &\approx \pi C \tfrac{2k_j-(k_j(1-e^{\gamma_2 \pi T})+k_1^*(1+e^{\gamma_2 \pi T}))}{2\xi(k_j-k_c)} = \pi C \tfrac{2k_j-(k_j(1-e^{\gamma_2 \pi T})+2k+k_j(e^{\gamma_2 \pi T}-1))}{2\xi(k_j-k_c)} \\ &= \pi C \tfrac{k_j-k}{\xi(k_j-k_c)}. \end{aligned} \tag{32}$$

From the average network flow-rates calculated above, we obtain (16) for the macroscopic fundamental diagram. ■

C A secant algorithm of calculating stationary states with general signal settings and turning ratios

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Inputs: $k$, $T$, $\Delta$, $\xi$, and $\Phi(k_1)$

Initialization: vector of stationary states, i.e., SS=[]; minimum value of $k_1$, i.e., $k_{1,min} = \max\{2k-k_j, 0\}$; maximum value of $k_1$, i.e., $k_{1,max} = \min\{2k, k_j\}$; the threshold of $k_1$, i.e., $e_k$; searching step, i,e., $\Delta k$; maximum number of iterations, i.e., $n_{max}$

For $k_1 = k_{1,min} : \Delta k : k_{1,max}$
    Set $k_1^0 = k_1$ and calculate $\Phi(k_1^0)$
    If $\Phi(k_1^0) == 0$
        $k_1^0$ is a root, and add it to SS
    Else
        Set $k_1^1 = P k_1^0$ and calculate $\Phi(k_1^1)$
        For n=1 : $n_{max}$
            If $|k_1^1 - k_1^0| < e_k$ or $\Phi(k_1^1) == 0$, add $k_1^1$ to SS and break
            If $\Phi(k_1^1) = \Phi(k_1^0) \neq 0$, continue
            $k_1^{tmp} = k_1^1 - \Phi(k_1^1)[\frac{k_1^1 - k_1^0}{\Phi(k_1^1) - \Phi(k_1^0)}]$, and calculate $\Phi(k_1^{tmp})$
            $k_1^0 = k_1^1$ and $\Phi(k_1^0) = \Phi(k_1^1)$
            $k_1^1 = k_1^{tmp}$ and $\Phi(k_1^1) = \Phi(k_1^{tmp})$
        End for
    End if
End for
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