

Compact Formulations from the Literature

The Compact Formulation of Avriel and Penn (1993). The first formulation for the CSPP is owed to Avriel and Penn (1993). It was proposed for both full-loading and non-full-loading transportation matrices. The version provided in the following is slightly simplified with respect to the one found in Avriel and Penn (1993) to exploit the full-loading assumption. This translates into a formulation with fewer variables and fewer constraints.

Let $\theta_{ijk}^{rc} \in \{0, 1\}$ be a binary variable equal to 1 if there is a container in tier $r \in R$ of stack $c \in C$ loaded on board in port $i \in P_1^{n-1}$ with final destination $j \in P_{i+1}^n$, and unloaded in port $k \in P_{i+1}^j$ (0 otherwise). Avriel and Penn (1993) formulated the CSPP as follows:

$$z_{AP} = \min \sum_{i=1}^{n-1} \sum_{c \in C} \sum_{k=i+1}^{j-1} \sum_{r \in R} \sum_{c \in C} \theta_{ijk}^{rc} \quad (\text{EC.1})$$

$$\text{s.t.} \sum_{r \in R} \sum_{c \in C} \left(\sum_{k=i+1}^j \theta_{ijk}^{rc} - \sum_{k=1}^{i-1} \theta_{kji}^{rc} \right) = t_{ij} \quad (i, j) \in \mathcal{W} \quad (\text{EC.2})$$

$$\sum_{k=1}^i \sum_{j=i+1}^n \sum_{v=i+1}^j \theta_{kji}^{rc} = 1 \quad i \in P_1^{n-1} \quad r \in R \quad c \in C \quad (\text{EC.3})$$

$$\sum_{i=1}^{j-1} \sum_{k=j}^n \theta_{ikj}^{rc} + \sum_{i=1}^{j-1} \sum_{k=j+1}^n \sum_{v=j+1}^k \theta_{ikv}^{r+1c} \leq 1 \quad j \in P_2^n \quad r \in R^- \quad c \in C \quad (\text{EC.4})$$

$$\theta_{ijk}^{rc} \in \{0, 1\} \quad i \in P_1^{n-1} \quad j \in P_{i+1}^n \quad k \in P_{i+1}^j \quad r \in R \quad c \in C \quad (\text{EC.5})$$

The objective function (EC.1) aims to minimize the total number of shifts. Constraints (EC.2) state that, for each pair $(i, j) \in \mathcal{W}$, the difference between the number of j -containers unloaded in any port $k = i + 1, \dots, j$ minus the number of j -containers originated from $k = 1, \dots, i - 1$, and unloaded at i must be equal to t_{ij} . Constraints (EC.3) stipulate that exactly one container has to be stowed in tier $r \in R$ of stack $c \in C$ upon leaving each port $i \in P_1^{n-1}$. Constraints (EC.4) allow to count the number of shifts: if the container stowed in tier $r \in R^-$ of stack $c \in C$ at port $j \in P_2^n$ is unloaded, then even the container stowed in tier $r + 1$ must be unloaded at port j . Integrality on the variables is imposed with constraints (EC.5).

The relationship between the optimal value of (1)-(5) and the optimal value z_{AP} of formulation (EC.1)-(EC.5) is $z_{AP} = z_{RP} - nCont$ as the number of discharges (i.e., $nCont$), which are included in the costs \tilde{c}_s of stack plans, is constant.

A weakness of formulation (EC.1)-(EC.5) is that its linear relaxation has optimal value zero (see Avriel et al. (1998)).

The Compact Formulation of Ding and Chou (2015). An alternative compact formulation has been introduced by Ding and Chou (2015), in the online supplement of their paper. As in Avriel and Penn (1993), the formulation was proposed for both full-loading and non-full-loading

transportation matrices. In the following, we report a slightly simplified version of the original formulation of Ding and Chou (2015), where the objective function has been adapted and a set of constraints has been removed thanks to the full-loading assumption.

Consider the two sets of binary variables: $\mu_{ij}^{rc} \in \{0, 1\}$ and $\phi_{ij}^{rc} \in \{0, 1\}$, where μ_{ij}^{rc} equals 1 if, upon leaving port i , a j -container (with $(i, j) \in \mathcal{W}$) is stowed in tier $r \in R$ of stack $c \in C$ (0 otherwise), and ϕ_{ij}^{rc} equals 1 if, before any loading operation at port $i \in P_2^{n-1}$ begins, a j -container ($j \in P_{i+1}^n$) is stowed in tier $r \in R$ of stack $c \in C$ (0 otherwise). The CSPP can be formulated as:

$$z_{\text{DC}} = \max \sum_{i=2}^{n-1} \sum_{j=i+1}^n \sum_{r \in R} \sum_{c \in C} \phi_{ij}^{rc} \quad (\text{EC.6})$$

$$\text{s.t.} \quad \sum_{r \in R} \sum_{c \in C} \mu_{1j}^{rc} = t_{1j} \quad j \in P_2^n \quad (\text{EC.7})$$

$$\sum_{r \in R} \sum_{c \in C} (\mu_{ij}^{rc} - \mu_{i-1,j}^{rc}) = t_{ij} \quad i \in P_2^{n-1} \quad j \in P_{i+1}^n \quad (\text{EC.8})$$

$$\sum_{j=i+1}^n \mu_{ij}^{rc} = 1 \quad i \in P_1^{n-1} \quad r \in R \quad c \in C \quad (\text{EC.9})$$

$$\sum_{j=i+1}^n \phi_{ij}^{rc} \leq 1 \quad i \in P_2^{n-1} \quad r \in R \quad c \in C \quad (\text{EC.10})$$

$$\sum_{j=i+1}^n (\phi_{ij}^{rc} - \phi_{ij}^{r+1,c}) \geq 0 \quad i \in P_2^{n-1} \quad r \in R^- \quad c \in C \quad (\text{EC.11})$$

$$\phi_{ij}^{rc} \leq \mu_{ij}^{rc} \quad i \in P_2^{n-1} \quad j \in P_{i+1}^n \quad r \in R \quad c \in C \quad (\text{EC.12})$$

$$\phi_{i+1,j}^{rc} \leq \mu_{ij}^{rc} \quad i \in P_1^{n-2} \quad j \in P_{i+2}^n \quad r \in R \quad c \in C \quad (\text{EC.13})$$

$$\mu_{ij}^{rc} \in \{0, 1\} \quad (i, j) \in \mathcal{W} \quad r \in R \quad c \in C \quad (\text{EC.14})$$

$$\phi_{ij}^{rc} \in \{0, 1\} \quad i \in P_2^{n-1} \quad j \in P_{i+1}^n \quad r \in R \quad c \in C \quad (\text{EC.15})$$

The objective function (EC.6) maximizes the number of containers still on board after all unloading operations have taken place in each port; notice that the total number of shifts (i.e., the optimal value z_{AP} of (EC.1)-(EC.5)) can be derived as $z_{\text{AP}} = (n-1) \cdot \bar{r} \cdot \bar{c} - z_{\text{DC}} - n \text{Cont}$, where $(n-1) \cdot \bar{r} \cdot \bar{c}$ is the total (constant) number of containers stowed upon leaving the first $n-1$ ports. Constraints (EC.7) stipulate that all transports originated from port 1 have to be satisfied. Constraints (EC.8) stipulate that, upon leaving port $i \in P_2^{n-1}$, the number of j -containers ($j \in P_{i+1}^n$) on board has to be equal to the number of j -containers on board upon leaving port $i-1$ plus the transport t_{ij} . Constraints (EC.9) ensure that exactly one container is stowed in tier $r \in R$ of stack $c \in C$ upon leaving port $i \in P_1^{n-1}$. Constraints (EC.10) guarantee that no more than one container is stowed in tier $r \in R$ of stack $c \in C$ before loading operations take place in port $i \in P_2^{n-1}$. Constraints (EC.11) stipulate that all containers on-board before any loading operations in port $i \in P_2^{n-1}$ have to be stowed at the bottom of the stacks. Constraints (EC.12) and (EC.13) link variables μ and ϕ by

stating that if, at port $i \in P_2^{n-1}$, there is a j -container ($j \in P_{i+1}^n$) stowed in tier $r \in R$ of stack $c \in C$ before any loading operations, then even upon leaving port i there has to be a j -container in tier r of stack c , and that there can be a j -container ($j \in P_{i+2}^n$) stowed in tier $r \in R$ of stack $c \in C$ before loading operations start at port $i + 1$ if and only if there was a j -container in tier r of stack c upon leaving port $i \in P_1^{n-2}$. Integrality on the decision variables is imposed with constraints (EC.14)-(EC.15).

Detailed Description of the Lower Bounding Procedure

Algorithm 1 Lower Bounding Procedure

```

1: procedure LOWERBOUNDINGPROCEDURE
2:    $LB_0 \leftarrow \text{CombinatorialLB}()$  ▷ See §5.2
3:    $\text{solveY} \leftarrow \text{false}$ 
4:   if  $LB_0 > 0$  then
5:      $\text{callHeuX2} \leftarrow \text{true}$ 
6:   else
7:      $\text{callHeuX2} \leftarrow \text{false}$ 
8:    $\text{InitializeRMP}()$  ▷ See §5.3
9:   while  $\text{true}$  do
10:     $z_{\text{RMP}} = \text{SolveRMP}()$ 
11:    if  $z_{\text{RMP}} \leq LB_0$  then
12:      break
13:     $(\mathbf{u}, \mathbf{v}) = \text{GetDualRMPSolution}()$ 
14:     $\text{cardS} \leftarrow |\hat{\mathcal{S}}|$ 
15:     $\text{HeuX1}(\mathbf{v})$  ▷ See §5.4.1
16:    if  $\text{callHeuX2}$  then
17:       $\text{HeuX2}(\mathbf{v})$  ▷ See §5.4.2
18:    if  $\text{cardS} = |\hat{\mathcal{S}}|$  then
19:       $\text{solveY} \leftarrow \text{true}$ 
20:    if  $\text{solveY}$  then
21:       $\text{cardL} \leftarrow \sum_{i=1}^{n-1} |\hat{\mathcal{L}}_i|$ 
22:      for  $i = 1, \dots, n-2$  do
23:         $\text{HeuY}(i, \mathbf{u}, \mathbf{v})$  ▷ See §5.4.3
24:      if  $\text{cardL} = \sum_{i=1}^{n-1} |\hat{\mathcal{L}}_i|$  then
25:        for  $i = 1, \dots, n-2$  do
26:           $\text{ExactY}(i, \mathbf{u}, \mathbf{v})$  ▷ See §4.2
27:        if  $\text{cardS} = |\hat{\mathcal{S}}| \wedge \text{cardL} = \sum_{i=1}^{n-1} |\hat{\mathcal{L}}_i|$  then
28:           $\text{ExactX}(\mathbf{u})$  ▷ See §4.1
29:        if  $\text{cardS} = |\hat{\mathcal{S}}| \wedge \text{cardL} = \sum_{i=1}^{n-1} |\hat{\mathcal{L}}_i|$  then
30:          break
31:     $LB = \max\{LB_0, \lceil z_{\text{RMP}} \rceil\}$ 
32:  return  $LB$ 

```

A pseudo-code of the lower bounding procedure outlined in §5 is provided in Algorithm 1.

First, the combinatorial lower bound LB_0 described in §5.2 is computed (see Line 2). Then, the two parameters solveY and callHeuX2 are initialized (see Lines 3-7); solveY indicates whether or not the \mathbf{y} variables are priced out, and callHeuX2 indicates if Procedure HeuX2 (see §5.4.2) will be used.

Procedure $\text{InitializeRMP}()$ (see Line 8) initializes the subsets $\hat{\mathcal{S}}$ and $\hat{\mathcal{L}}_i$ with the variables of a solution that is obtained via a greedy algorithm that, for the sake of conciseness, is not described; furthermore, additional columns are added to RMP in order to stabilize the column generation procedure (see §5.3).

The main loop of the bounding procedure is given in Lines 9-30. The optimal value of RMP, z_{RMP} , is computed with a general purpose linear programming solver. If z_{RMP} is less than or equal to LB_0 , the main loop terminates. Otherwise, the optimal RMP dual solution (\mathbf{u}, \mathbf{v}) is retrieved and used

to generate negative reduced cost columns. The cardinality, $cardS$, of the current set \hat{S} is stored. Negative reduced cost stack plans are generated and added to RMP by mean of Procedures *HeuX1* and *HeuX2* (see Lines 15-17 and §§5.4.1-5.4.2). Parameter *solveY* is set to *true* (see Lines 18-19) at the very first iteration when both *HeuX1* and *HeuX2* do not generate any negative reduced cost stack plans. Lines 20-26 correspond to the port layouts pricing phase. The cardinality, $cardL$, of the current sets $\hat{\mathcal{L}}_i$ is stored, and negative reduced cost port layouts are generated via Procedure *HeuY* (see §5.4.3) and by solving the exact pricing MIP (see §4.2). Whenever no negative reduced cost columns have been found at the current iteration, the exact MIP described in §4.1 is called to solve the stack plans pricing problem to optimality (see Lines 27-28). The main loop terminates if no negative reduced cost column exists (see Lines 29-30). The final lower bound returned by the bounding procedure is the best between LB_0 and $\lceil z_{RMP} \rceil$.

Detailed Computational Results

We report detailed computational results achieved by the compact formulation of Avriel and Penn (1993) (hereafter called AP93), the compact formulation of Ding and Chou (2015) (hereafter DC15), and the solution method proposed in this paper. We summarized these results in §7 of the main manuscript. We report three tables for each of the five types of instances: one table for the six-port instances, one for the eight-port instances, and another one for the ten-port instances. Tables EC.1-EC.3 regard **Long** instances, Tables EC.4-EC.6 **Mixed** instances, Tables EC.7-EC.9 **Short** instances, Tables EC.10-EC.12 **Authentic** instances, and Tables EC.13-EC.15 **Required** instances.

For each of the 405 test instances, the following information is reported:

General features

- number of tiers per stack (\bar{r})
- number of stacks (\bar{c})
- instance number (no, which can be 1, 2 or 3)
- total number of containers transported (nCont)

Formulation of Avriel and Penn (1993)

- number of constraints (Rows) and of variables (Cols)
- final lower (LB) and upper (UB) bound upon termination
- computing time to achieve upper bound UB (T_{ub})
- total computing time (T_{tot})

Formulation of Ding and Chou (2015)

- number of constraints (Rows) and of variables (Cols)
- final lower (LB) and upper (UB) bound upon termination
- computing time to achieve upper bound UB (T_{ub})
- total computing time (T_{tot})

Solution method of this paper

- combinatorial lower bound described in §5.2 (LB_0) - the constant number nCont is subtracted
- value of the solution found by the greedy heuristic to initial the RMP (UB_0)
- total number of inequalities (20), (21), (22) added to stabilized column generation (nDOI)
- number of variables in the final RMP (Vars)
- number of column generation iterations (Iter)
- final lower bound (LB) - the constant number nCont is subtracted
- computing time of the bounding procedure (T_{lb})
- number of constraints (Rows) and variables (Cols) in the final MIP to find an optimal solution
- final upper bound (UB) - the constant number nCont is subtracted - a dash indicates that no feasible solution was found
- total computing time (T_{tot})

