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Online Appendices: Market-oriented Service Network Design When Demand is Sensitive to Congestion

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Appendix A: Summary of Notation

Sets

I: set of customer segments,

J: set of potential facility locations,

J_c: set of competitor facility locations,

S: set of possible service levels in terms of the expected wait time offered at a facility,

K_j: set of possible capacities for a facility $j \in J$.

Parameters

ω_i: potential number of purchases by customers from segment $i \in I$ during the planning period,

r_{ij}: seller's average revenue per purchase at facility $j \in J$ by customers from segment $i \in I$,

c_{ij}: seller's average cost per purchase incurred for customer segment $i \in I$ at facility $j \in J$,

F_{jk}: seller's fixed cost of opening and operating a facility at location $j \in J$ with capacity level $k \in K_j$ (per planning period),

$\hat{A}_{ijs}$: measure of how strongly customer segment $i \in I$ is attracted to facility $j \in J$ if service level $s \in S$ is offered,

A_{i0}: aggregated attraction of segment $i \in I$ to competitor facilities and to the option to not visit a facility at all,

$\hat{W}_s$: upper bound on the expected wait time promised to customers when service level $s \in S$ is offered at a facility,

CV_{jk}: coefficient of variation of the service times at facility $j \in J$ with capacity level $k \in K_j$,

μ_{jk}: service rate at facility j if capacity $k \in K_j$ is chosen by the provider.

Decision Variables

y_{jk}: binary decision variable with $y_{jk} = 1$ if a service facility is opened at potential location $j \in J$ with capacity level $k \in K_j$, or $y_{jk} = 0$ otherwise,

v_{js}: binary decision variable with $v_{js} = 1$ if service level $\hat{W}_s$ is offered at facility $j \in J$, or $v_{js} = 0$ otherwise.

Functions

$x_{ij}(\boldsymbol{v})$: probability that customers from segment $i \in I$ will choose facility $j \in J$,

$W_{jk}(\boldsymbol{x}, \boldsymbol{y})$: expected wait time at facility $j \in J$ with capacity level $k \in K_j$.

Appendix B: Note on FR versus AR Demand Models

In the following, we show that, depending on the parameter values, the attraction-based choice model that we incorporate into the optimization problem can be regarded as either an FR or an AR demand model according to the classification of Berman and Krass (2015). Let λ_i be the total demand from customer segment i aggregated over all established facilities; furthermore, let $\tilde{x}_{ij}$ be the conditional share of demand λ_i allocated to facility $j \in J \cup J_c$ given that the customer visits one of the established facilities, such that $\lambda_i \tilde{x}_{ij}$ is the demand rate for facility j. According to Berman and Krass (2015), the demand model is classified as an AR model if $\tilde{x}_{ij}$ is elastic but λ_i is inelastic. If both $\tilde{x}_{ij}$ and λ_i are elastic, then the demand model is classified as an FR model.

To apply this definition to our demand formulation, we rewrite x_{ij}, the probability with which customers from segment i choose to patronize facility j for a purchase, as follows:

$$x_{ij} = \frac{\sum_{s \in S} \hat{A}_{ijs} v_{js}}{A_{i0} + A_{ic} + \sum_{n \in J} \sum_{s \in S} \hat{A}_{ins} v_{ns}} \qquad \forall\ i \in I,\ j \in J, \tag{37}$$

where A_{ic} is the aggregated attractiveness of visiting competitor facilities for segment i and A_{i0} is the attractiveness of the option to not visit a facility at all. Then, for any constant market potential ω_i, the total demand rate $\omega_i x_{ij}$ from customer segment i attracted to facility j can be rewritten by explicitly differentiating between the market demand λ_i and the conditional share $\tilde{x}_{ij}$:

$$x_{ij} * \omega_i = \underbrace{\frac{\sum_{s \in S} \hat{A}_{ijs} v_{js}}{A_{ic} + \sum_{n \in J} \sum_{s \in S} \hat{A}_{ins} v_{ns}}}_{\tilde{x}_{ij}} * \underbrace{\frac{A_{ic} + \sum_{n \in J} \sum_{s \in S} \hat{A}_{ins} v_{ns}}{A_{i0} + A_{ic} + \sum_{n \in J} \sum_{s \in S} \hat{A}_{ins} v_{ns}} * \omega_i}_{\lambda_i} \qquad \forall\ i \in I,\ j \in J. \tag{38}$$

Obviously, the conditional market share $\tilde{x}_{ij}$ is elastic, and for $A_{i0} \neq 0$, the total market demand λ_i from segment i is also elastic. Thus, we have an FR model for $A_{i0} \neq 0$ and an AR model for $A_{i0} = 0$.

Appendix C: Equivalence of Equations (5) and (6)

To see that (5) and (6) in fact yield the same results, consider the following three cases:

Case 1: $y_{jk} = 1$, $y_{jl} = 0\ \forall \quad l \in K \setminus \{k\}$. Then, (6) reduces to $W_{jk} = \frac{(1+CV_{jk}^2)\sum_{i \in I} \omega_i x_{ij}}{2\mu_{jk}(\mu_{jk} - \sum_{i \in I} \omega_i x_{ij})}$, same as (3).

Case 2: $\exists \quad m \in K,\ m \neq k$: $y_{jm} = 1$, $y_{jl} = 0\ \forall \quad l \in K \setminus \{m\}$. Then, $y_{jk} = 0$, and thus, (6) yields $W_{jk} = \frac{0}{2\mu_{jk}(\mu_{jm} - \sum_{i \in I} \omega_i x_{ij})} = 0$, same as (3).

Case 3: $y_{jl} = 0\ \forall \quad l \in K$. Then, (6) yields $W_{jk} = \frac{0}{1} = 0$, same as (3).

Appendix D: Proof of Lemma 1

For reference we first provide the original Lemma by Wu (1997): A polynomial mixed 0-1 term $w = vz$ with $v \in \{0,1\}$ and $z > 0$ can be represented by the following linear inequalities: (1) $z - w \leq M - Mv$; (2) $w \leq z$; (3) $w \leq Mv$ and (4) $w \geq 0$, where M is a sufficiently large number with $M \geq z$.

Lemma 1 is a generalization of Wu (1997) by allowing for two different BigM numbers M_1 and M_2 for constraints (1) and (3). Through this differentiation, we can generally achieve smaller bigM numbers and tighter constraints. The proof is similar to Wu (1997) where only a single BigM number is considered.

Proof. Part 1: Suppose $w = vz$ and $z = g(v) \leq \begin{cases} M_1, & \text{if } v = 0 \\ M_2, & \text{if } v = 1 \end{cases}$.

Then,

(1) $z - w = z - vz = z(1-v) \overset{?}{\leq} M_1(1-v)$; for $v = 1$ redundant, for $v = 0$ true

(2) $w = vz \overset{?}{\leq} z$; true for $v \in \{0,1\}$

(3) $w = vz \overset{?}{\leq} M_2 v$; for $v = 0$ redundant, for $v = 1$ true

(4) $w = vz \overset{?}{\geq} 0$ true

Part 2: Suppose (1) (4) are true. If $v = 0$, then from conditions (3) and (4), we have $w = 0$. If $v = 1$, we have from conditions (1) and (2) $w = z$. Thus, we can conclude that $w = vz$. □

Appendix E: Linearization of Service Level Constraints

E.1. Service Level Constraint 1

The following linear reformulation concerns the service level constraint (8) with (6) ($\forall\, j \in J,\ k \in K_j$).

$$\frac{y_{jk}(1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij}}{2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})} \leq \sum_{s \in S}\hat{W}_s v_{js}$$

$$\iff (1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij} y_{jk} \leq \left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})\right]\sum_{s \in S}\hat{W}_s v_{js}$$

$$\iff (1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij} y_{jk} \leq 2\mu_{jk}\left(\sum_{l \in K_j}\sum_{s \in S}\hat{W}_s\mu_{jl}y_{jl}v_{js} - \sum_{i \in I}\sum_{s \in S}\hat{W}_s\omega_i x_{ij}v_{js}\right) + \sum_{s \in S}\hat{W}_s v_{js} - \sum_{l \in K}\sum_{s \in S}\hat{W}_s v_{js} y_{jl}$$

E.2. Service Level Constraint 2

Using the auxiliary variables t_{ijk}, u_{ijs} and q_{jks} introduced earlier, (25) can now be linearized as follows ($\forall\, j \in J,\ k \in K_j,\ s = 2, \ldots, |S|$):

$$\frac{y_{jk}(1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij}}{2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})} \geq (\hat{W}_{s-1} + \epsilon)v_{js} - M_{7,s}(1 - y_{jk})$$

$$\iff (1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij} y_{jk} \geq (\hat{W}_{s-1} + \epsilon)v_{js}\left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})\right]$$
$$- M_{7,s}(1 - y_{jk})\left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})\right]$$

$$\iff (1 + CV_{jk}^2)\sum_{i \in I}\omega_i x_{ij} y_{jk} \geq (\hat{W}_{s-1} + \epsilon)\left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl}v_{js} - \sum_{i \in I}\omega_i x_{ij}v_{js}) + (v_{js} - \sum_{l \in K} y_{jl}v_{js})\right]$$
$$- M_{7,s}\left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl} - \sum_{i \in I}\omega_i x_{ij}) + (1 - \sum_{l \in K} y_{jl})\right] + M_{7,s}\left[2\mu_{jk}(\sum_{l \in K_j}\mu_{jl}y_{jl}y_{jk} - \sum_{i \in I}\omega_i x_{ij}y_{jk}) + (y_{jk} - \sum_{l \in K} y_{jl}y_{jk})\right]$$

$$\iff (1+CV_{jk}^2)\sum_{i\in I}\omega_i t_{ijk} \geq (\hat{W}_{s-1}+\epsilon)\Big[2\mu_{jk}(\sum_{l\in K_j}\mu_{jl}q_{jls}-\sum_{i\in I}\omega_i u_{ijs})+(v_{js}-\sum_{l\in K}q_{ljs})\Big]$$
$$-M_{7,s}\Big[2\mu_{jk}(\sum_{l\in K_j}\mu_{jl}y_{jl}-\sum_{i\in I}\omega_i x_{ij})+(1-\sum_{l\in K}y_{jl})\Big]+2M_{7,s}\mu_{jk}(\mu_{jk}y_{jk}-\sum_{i\in I}\omega_i t_{ijk}).$$

Appendix F: Case Study Market Share Results

Facility Location	Customer Segment	Market share attracted by each new facility from customers residing in square x (%)											
		x=1	x=2	x=3	x=4	x=5	x=6	x=7	x=8	x=9	x=10	x=11	x=12
Square 8	x.1	4.89	4.99	4.85	4.88	4.80	4.88	4.97	5.18	4.95	4.97	4.93	5.02
	x.2	4.69	4.89	4.70	4.74	4.54	4.67	4.84	5.13	4.83	4.87	4.79	4.92
	x.3	3.19	4.10	3.96	4.02	2.86	3.10	3.91	3.99	4.04	4.14	3.90	4.06
Square 9	x.1	4.89	4.77	4.85	4.88	5.02	4.88	4.75	4.85	5.29	4.75	4.93	5.02
	x.2	4.69	4.50	4.70	4.74	4.93	4.67	4.45	4.69	5.29	4.48	4.79	4.92
	x.3	3.19	2.82	3.96	4.02	4.16	3.10	2.68	3.87	4.17	2.85	3.90	4.06

Table 7 **Market shares for two new convenience stores in Heidelberg (Germany)**