Appendix.

Details on the Backward Labeling Algorithm

In the backward labeling algorithm, labels are extended from the sink (i.e., $o(v')$) to its predecessors. It is similar to the forward labeling algorithm. To a backward label L_b, we associate the following data:

- η last node of the path;
- t departure time from the node;
- q total load after visiting the last node;
- c accumulated cost;
- $\mathcal{O}$ set of requests that have been finished, but not started (i.e., the request has been delivered either to the delivery node or to a transfer node, but not picked up from its pickup or transfer node);
- $\mathcal{C}$ set of completed requests;
- $\mathcal{O}^T$ set of finished requests at a transfer node, such that $\mathcal{O}^T \subseteq \mathcal{O}$.

Similarly to forward labeling, the resources are t, q, c, $\mathcal{O}$, $\mathcal{O}^T$, and $\mathcal{C}$. These are initialized as follows: $t = u_{o(v)}$, $q = Q_v$, $c = 0$, and $\mathcal{O} \equiv \mathcal{O}^T \equiv \mathcal{C} \equiv \emptyset$.

When extending a backward label L_b along an arc (j, η_{L_b}), the extension is feasible only if:

$$t_{L_b} - s_{\eta_{L_b}} - t_{j,\eta_{L_b}} \geq l_j + s_j \tag{1}$$

$$q_{L_b} + d_j \geq 0. \tag{2}$$

In this case, we ensure the capacity constraints. Similarly to forward labeling, time windows are used to eliminate infeasible extensions. Moreover, L_b and j must also satisfy one of the following five conditions, which ensure that the extension is compatible with the sets $\mathcal{O}$, $\mathcal{O}^T$ and $\mathcal{C}$:

$$j \in \mathcal{D} \wedge j - n \notin \mathcal{O}_{L_b} \wedge j - n \notin \mathcal{C}_{L_b} \tag{3}$$

$$j \in \mathcal{P} \wedge j \in \mathcal{O}_{L_b} \tag{4}$$

$$j \in \mathcal{T} \wedge r(j) \notin \mathcal{O}_{L_b} \wedge r(j) \notin \mathcal{C}_{L_b} \tag{5}$$

$$j \in \mathcal{T} \wedge r(j) \in \mathcal{O}_{L_b} \setminus \mathcal{O}^T_{L_b} \wedge r(j) \notin \mathcal{C}_{L_b} \tag{6}$$

$$j = o(v) \wedge \mathcal{O}_{L_b} \in \emptyset. \tag{7}$$

Condition (3) states that visiting a delivery node $j \in \mathcal{D}$ is feasible if the corresponding request $j - n$ has not been delivered at the delivery node, nor at a transfer node earlier in the partial path, and this specific request is not completed. Expression (4) implies that extending the label to a pickup node $j \in \mathcal{P}$ is feasible if j has already been delivered to its delivery node, or to a transfer node in the partial path. Condition (5) states that visiting a transfer node $j \in \mathcal{T}$ is feasible if the corresponding request $r(j) \in \mathcal{P}$ has not been delivered to its delivery node, or to a transfer node in the partial path, and if this request has not been completed. Expression (6) states that extending to a transfer node $j \in \mathcal{T}$ is feasible if the corresponding request $r(j) \in \mathcal{P}$ has been delivered to its delivery node, and not to a transfer node, and if this request has not been completed in the partial path. Finally, condition (7) implies that a partial path cannot be ended at the origin depot (i.e., $o(v)$), if at least one request is delivered, but not picked up (from the origin, or from a transfer node) in the partial path.

When label η_{L_b} and node j satisfy the conditions (1) and (2), along with one of the following conditions, the corresponding updates on sets $\mathcal{O}$, $\mathcal{O}^T$ and $\mathcal{C}$ are performed as follows:

- expression (3): $\mathcal{O}_{L_b} \leftarrow \mathcal{O}_{L_b} \cup \{j-n\}$;
- expression (4): $\mathcal{C}_{L_b} \leftarrow \mathcal{C}_{L_b} \cup \{j\}$, $\mathcal{O}_{L_b} \leftarrow \mathcal{O}_{L_b} \setminus \{j\}$, and $\mathcal{O}^T_{L_b} \leftarrow \mathcal{O}^T_{L_b} \setminus \{j\}$;
- expression (5): $\mathcal{O}_{L_b} \leftarrow \mathcal{O}_{L_b} \cup \{r(j)\}$ and $\mathcal{O}^T_{L_b} \leftarrow \mathcal{O}^T_{L_b} \cup \{r(j)\}$;
- expression (6): $\mathcal{C}_{L_b} \leftarrow \mathcal{C}_{L_b} \cup \{r(j)\}$ and $\mathcal{O}_{L_b} \leftarrow \mathcal{O}_{L_b} \setminus \{r(j)\}$;
- expression (7).

The dominance condition used in the backward label setting algorithm is the following: a label L_b^1 dominates a label L_b^2 if conditions (1) and (3)–(5) (from Section 4.1.2 of the article) are satisfied, along with the following condition:

$$t_{L_b^1} \geq t_{L_b^2}. \tag{8}$$

The proof is similar to that given for the forward dominance test.

Table 1 reports detailed results concerning the performance of the bi-directional and the mono-directional labeling algorithms. Columns "$\underline{z}$" and "$\overline{z}$" provide the best lower and upper bounds found within the imposed time limit, and "Time" provides the time needed to solve an instance.

Table 1 Comparison between Bi-directional and Mono-directional Algorithms.

Instance	Mono-directional			Bi-directional		
	$\underline{z}$	$\overline{z}$	Time (sec)	$\underline{z}$	$\overline{z}$	Time (sec)
C6_2_4	369.36	369.36	0	369.36	369.36	0
C7_2_4	390.31	390.31	0	390.31	390.31	0
C8_2_4	446.35	446.35	3	446.35	446.35	21
C9_2_4	471.16	471.16	178	471.16	471.16	146
C10_2_4	510.01	510.01	73	510.01	510.01	55
C11_2_4	522.85	522.85	37	522.85	522.85	27
C12_2_4	541.60	541.60	4	541.60	541.60	4
C15_2_6	649.93	649.93	32	649.93	649.93	21
C20_2_8	751.23	751.23	79	751.23	751.23	69
C25_2_8	879.94	879.94	2,627	879.94	879.94	1,215
C40_2_8	–	–	10,800	1,202.13	–	10,800
RC6_2_4	572.76	572.76	0	572.76	572.76	0
RC7_2_4	575.95	575.95	0	575.95	575.95	0
RC8_2_4	585.32	585.32	0	585.32	585.32	1
RC9_2_4	593.70	593.70	1	593.70	593.70	1
RC10_2_4	599.95	599.95	1	599.95	599.95	1
RC11_2_4	624.48	624.48	1	624.48	624.48	1
RC12_2_6	662.03	662.03	1	662.03	662.03	1
RC15_2_6	1,068.16	1,068.16	1	1,068.16	1,068.16	1
RC20_2_8	1,200.36	1,200.36	3	1,200.36	1,200.36	3
RC25_2_8	1,530.43	1,530.43	40	1,530.43	1,530.43	33
RC40_2_12	1,969.45	1,969.45	1,063	1,969.45	1,969.45	682
RC50_2_14	2,200.88	2,200.88	1,199	2,200.88	2,200.88	921
RC60_2_16	2,636.18	–	10,800	2636.22	–	10,800
R6_2_4	416.16	416.16	0	416.16	416.16	0
R7_2_4	473.06	473.06	1	473.06	473.06	1
R8_2_4	558.17	558.17	2	558.17	558.17	2
R9_2_4	632.42	632.42	6	632.42	632.42	1
R10_2_4	636.05	636.05	3	636.05	636.05	3
R11_2_4	748.29	748.29	3	748.29	748.29	4
R12_2_6	913.68	913.68	6	913.68	913.68	7
R15_2_6	1,083.50	1,083.50	14	1,083.50	1,083.50	13
R20_2_8	1,542.93	1,542.93	19	1,542.93	1,542.93	15
R25_2_8	1,636.12	1,636.12	59	1,636.12	1,636.12	24
R40_2_12	1,969.77	1,969.77	606	1,969.77	1,969.77	589
R50_2_14	2,595.14	2,595.14	1,634	2,595.14	2,595.14	543
R60_2_16	2,868.86	–	10,800	2,871.13	–	10,800
Average			226.35			129.56

In Table 2, under columns "$\underline{z}$" and "$\overline{z}$" we indicate the best lower and upper bounds found within the time limit. The column titled "# via SLs" shows the number of demand units shipped on scheduled lines. In addition, "# of vehicles" indicates the number of vehicles used in the solutions. Finally, "Time" and "Savings" show the time needed to obtain a solution and cost savings from using SLs as part of freight journey, respectively.

Table 2 Comparison between PDPTW-SL and PDPTW with Homogeneous Routing Costs.

Instance	PDPTW-SL						PDPTW			
	$\underline{z}$	$\overline{z}$	# via SLs	# of vehicles	Time (sec)	Savings (%)	$\underline{z}$	$\overline{z}$	# of vehicles	Time (sec)
C6_2_4	369.36	369.36	5	3	0	20.00	461.70	461.70	3	0
C7_2_4	390.31	390.31	6	3	0	15.97	464.49	464.49	3	0
C8_2_4	446.35	446.35	5	4	21	7.68	483.50	483.50	3	0
C9_2_4	471.16	471.16	7	4	146	5.43	498.19	498.19	3	0
C10_2_4	510.01	510.01	4	4	55	1.02	515.25	515.25	3	0
C11_2_4	522.85	522.85	4	4	27	0.35	524.69	524.69	3	0
C12_2_4	541.60	541.60	5	4	4	9.51	598.55	598.55	3	0
C15_2_6	649.93	649.93	8	4	21	10.25	724.12	724.12	4	1
C20_2_8	751.23	751.23	2	5	69	5.45	794.53	794.53	5	2
C25_2_8	879.94	879.94	11	5	1,215	18.27	1,076.71	1,076.71	6	20
C40_2_8	1,202.13	–	–	–	10,800	–	1,428.59	1,428.59	8	1004
RC6_2_4	572.76	572.76	1	3	0	13.09	658.99	658.99	4	0
RC7_2_4	575.95	575.95	1	3	0	13.02	662.18	662.18	4	0
RC8_2_4	585.32	585.32	1	3	1	11.74	663.16	663.16	4	0
RC9_2_4	593.70	593.70	3	3	1	15.58	703.30	703.30	4	0
RC10_2_4	599.95	599.95	3	3	1	15.45	709.55	709.55	4	0
RC11_2_4	624.48	624.48	3	3	1	15.90	742.52	742.52	4	0
RC12_2_6	662.03	662.03	3	4	1	16.66	794.42	794.42	5	0
RC15_2_6	1,068.16	1,068.16	0	6	1	0.00	1,068.16	1,068.16	6	0
RC20_2_8	1,200.36	1,200.36	4	7	3	16.48	1,437.13	1,437.13	7	4
RC25_2_8	1,530.43	1,530.43	6	8	33	2.65	1,572.04	1,572.04	7	12
RC40_2_12	1,969.45	1,969.45	2	10	682	3.16	2,033.66	2,033.66	10	73
RC50_2_14	2,200.88	2,200.88	5	10	921	3.08	2,270.83	2,270.83	10	48
RC60_2_16	2,636.22	–	–	–	10,800	–	2,682.66	2,682.66	12	1518
R6_2_4	416.16	416.16	0	3	0	0.00	416.16	416.16	3	0
R7_2_4	473.06	473.06	0	3	1	0.00	473.06	473.06	3	1
R8_2_4	558.17	558.17	0	3	2	0.00	558.17	558.17	3	0
R9_2_4	632.42	632.42	3	4	1	3.17	653.12	653.12	3	3
R10_2_4	636.05	636.05	3	4	3	3.42	658.60	658.60	4	5
R11_2_4	748.29	748.29	3	4	4	0.54	752.38	752.38	4	0
R12_2_6	913.68	913.68	3	5	7	2.41	936.23	936.23	5	3
R15_2_6	1,083.50	1,083.50	0	6	13	0.00	1,083.50	1,083.50	6	1
R20_2_8	1,542.93	1,542.93	0	8	15	0.00	1,542.93	1,542.93	8	1
R25_2_8	1,636.12	1,636.12	3	8	24	4.73	1,717.39	1,717.39	8	24
R40_2_12	1,969.77	1,969.77	0	9	589	0.00	1,969.77	1,969.77	9	62
R50_2_14	2,595.14	2,595.14	2	12	543	0.98	2,620.89	2,620.89	13	120
R60_2_16	2,871.13	–	–	–	10,800	–	2,959.71	2,959.71	14	1158
Average		909.46	3.12	5.06	129.56	6.94		965.88	5.12	11.18

Table 3 illustrates the results obtained after solving the proposed instances with heterogeneous routing costs. The minimum-capacity vehicle is assumed to cost 0.5 per operating time unit. Larger vehicles are assigned a cost that linearly increases with the carrying capacity. The columns are self-explanatory, similar to the previously presented table.

Table 3 Comparison between PDPTW-SL and PDPTW with Heterogeneous Routing Costs.

Instance	PDPTW-SL						PDPTW			
	$\underline{z}$	$\overline{z}$	# via SLs	# of vehicles	Time (sec)	Savings (%)	$\underline{z}$	$\overline{z}$	# of vehicles	Time (sec)
C6_2_4	385.49	385.49	5	3	0	21.07	488.38	488.38	3	0
C7_2_4	410.76	410.76	6	3	0	16.82	493.81	493.81	3	0
C8_2_4	468.30	468.30	5	4	4	8.75	513.19	513.19	3	0
C9_2_4	495.64	495.64	7	4	186	6.25	528.70	528.70	3	0
C10_2_4	535.01	535.01	4	4	100	2.14	546.71	546.71	3	0
C11_2_4	548.55	548.55	4	4	16	1.46	556.67	556.67	3	0
C12_2_4	568.30	568.30	5	4	4	9.46	627.67	627.67	3	1
C15_2_6	980.46	980.46	8	5	67	18.81	1,207.67	1,207.67	4	9
C20_2_8	890.02	890.02	6	4	91	15.49	1,053.12	1,053.12	5	91
C25_2_8	1,330.48	1,330.48	13	6	5,452	20.64	1,676.53	1,676.53	6	20
C40_2_8	1,441.80	–	–	–	10,800	–	1,800.52	1,800.52	8	3,417
RC6_2_4	593.52	593.52	1	3	0	13.66	687.39	687.39	4	0
RC7_2_4	596.89	596.89	1	3	0	13.59	690.76	690.76	4	0
RC8_2_4	606.78	606.78	1	3	0	12.28	691.73	691.73	4	0
RC9_2_4	615.50	615.50	3	3	1	15.90	731.87	731.87	4	0
RC10_2_4	621.75	621.75	3	3	1	15.81	738.47	738.47	4	1
RC11_2_4	646.28	646.28	3	3	1	16.42	773.28	773.28	4	0
RC12_2_6	693.75	693.75	3	4	1	15.72	823.19	823.19	5	0
RC15_2_6	1,586.84	1,586.84	0	6	1	1.44	1,610.10	1,610.10	6	1
RC20_2_8	1,639.68	1,639.68	4	7	7	18.89	2,021.58	2,021.58	7	6
RC25_2_8	2,403.87	2,403.87	6	8	25	4.16	2,508.24	2,508.24	7	13
RC40_2_12	3,145.84	3,145.84	1	10	10,768	3.19	3,249.36	3,249.36	10	59
RC50_2_14	3,442.28	3,442.28	5	10	5,048	6.03	3,663.11	3,663.11	10	43
RC60_2_16	4,024.96	–	–	–	10,800	–	4,077.43	4,077.43	13	1,103
R6_2_4	416.16	416.16	0	3	0	0.00	416.16	416.16	3	0
R7_2_4	473.06	473.06	0	3	1	0.00	473.06	473.06	3	1
R8_2_4	558.17	558.17	0	3	2	0.00	558.17	558.17	3	0
R9_2_4	632.42	632.42	3	4	4	3.17	653.12	653.12	3	1
R10_2_4	636.05	636.05	3	4	5	3.42	658.60	658.60	4	1
R11_2_4	748.29	748.29	3	4	3	0.54	752.38	752.38	4	0
R12_2_6	946.53	946.53	3	5	7	2.46	970.42	970.42	5	5
R15_2_6	1,377.04	1,377.04	0	6	18	0.00	1,377.04	1,377.04	6	3
R20_2_8	1,839.34	1,839.34	1	8	132	0.05	1,840.22	1,840.22	7	1
R25_2_8	2,011.86	2,011.86	1	8	27	5.80	2,135.65	2,135.65	8	35
R40_2_12	2,518.05	2,518.05	0	9	329	1.80	2,564.27	2,564.27	9	108
R50_2_14	3,182.19	3,182.19	2	12	2,684	1.40	3,227.26	3,227.26	13	466
R60_2_16	3,479.13	–	–	–	10,800	–	3,628.98	3,628.98	14	663
Average		1,133.68	3.24	5.09	734.85	8.14		1,220.82	5.09	25.44

Table 4 indicates the results obtained when solving the instances with up to three SLs. The average is computed over all instances that we optimally solved in all SL configurations. As expected, multiple SLs lead to more savings compared to corresponding PDPTW solutions. In particular, 2-SLs and 3-SLs cases led to cost savings of 4.58%, and 8.39% respectively. In is important to note that the cost savings are mainly obtained from using available capacities, thus minimizing the total traveling time of the PD vehicles. In addition, the average number of vehicles used did not significantly change compared to the classical system.

Table 4 The Effect of Multiple Available SLs.

Instance	One SL (sl=2)				Two SLs (sl=4)				Three SLs (sl=6)			
	$\bar{z}$	# of vehicles	# via SLs	Time (sec)	$\bar{z}$	# of vehicles	# via SLs	Time (sec)	$\bar{z}$	# of vehicles	# via SLs	Time (sec)
C6_2_4	369.36	3	5	0	369.36	3	5	1	310.85	3	8	2,437
C7_2_4	390.31	3	6	0	381.21	3	6	104	313.41	3	9	4,890
C8_2_4	446.35	4	5	21	415.84	3	5	229	–	–	–	10,800
C9_2_4	471.16	4	7	146	459.98	4	7	3,731	–	–	–	10,800
RC6_2_4	572.76	3	1	0	572.76	3	1	0	520.11	4	4	0
RC7_2_4	575.95	3	1	0	575.95	3	1	0	539.08	4	4	0
RC8_2_4	585.32	3	1	1	585.32	3	1	1	548.45	4	4	1
RC9_2_4	593.70	3	3	1	593.70	3	3	1	552.71	4	6	1
RC10_2_4	599.95	3	3	1	599.95	3	3	1	558.96	4	6	1
RC11_2_4	624.48	3	3	1	624.48	3	3	2	585.72	4	7	2
RC12_2_6	662.03	4	3	1	662.03	4	3	2	619.59	4	7	2
RC15_2_6	1,068.16	6	0	1	1,059.38	6	1	3	1,059.38	6	1	3
RC20_2_8	1,200.36	7	4	3	1,127.02	8	8	1,128	1,127.02	8	8	1,389
RC25_2_8	1,530.43	8	6	33	1,388.18	7	8	34	1,371.47	7	14	1,641
RC40_2_12	1,969.45	10	2	682	–	–	–	10,800	–	–	–	10,800
R6_2_4	416.16	3	0	0	416.16	3	0	0	416.16	3	0	0
R7_2_4	473.06	3	0	1	473.06	3	0	2	470.15	3	1	7
R8_2_4	558.17	3	0	2	535.27	4	1	3	490.77	3	2	3
R9_2_4	632.42	4	3	1	579.38	4	3	1	579.38	4	3	1
R10_2_4	636.05	4	3	3	584.89	4	3	1	584.89	4	3	2
R11_2_4	748.29	4	3	4	687.36	4	3	1	687.36	4	3	2
R12_2_6	913.68	5	3	7	858.33	5	4	889	822.64	5	5	43
R15_2_6	1,083.50	6	0	13	1,023.99	6	3	28	921.56	6	6	884
R20_2_8	1,542.93	8	0	15	1,425.42	8	3	105	1,378.70	7	8	5,738
R25_2_8	1,636.12	8	3	24	1,495.46	8	9	20	1,493.46	8	7	78
R40_2_12	1,969.77	9	0	589	1,922.00	9	4	346	–	–	–	10,800