

Appendix A.

Assuming all travelers choose the mode with minimal travel costs, then based on the expected time and non-time costs of each mode in Table 1 and Table 2, we can obtain the intended mode choices of travelers with different VOT as listed in Table 3, where β_{rc} , β_{rd} , β_{cd} and β_{rb} in Table 3 are the critical VOT that respectively satisfy $C_r^1(\beta_{rc}) = C_c(\beta_{rc})$, $C_r^1(\beta_{rd}) = C_d(\beta_{rd})$, $C_d(\beta_{cd}) = C_c(\beta_{cd})$ and $C_r^0(\beta_{rb}) = C_b(\beta_{rb})$, or equivalently,

$$\beta_{rc} = \frac{c_v - \tau_{RS} - \Delta}{t_r} - \frac{\theta_r}{p_r t_r} \quad (39)$$

$$\beta_{rd} = \frac{p_d(c_o t_d + \Delta - \tau_{RS}) + p_r(c_v - \tau_{RS} - \Delta) + \theta_d - \theta_r}{t_r p_r - p_d t_d} \quad (40)$$

$$\beta_{cd} = \frac{p_d(\tau_{RS} - \Delta - c_o t_d) - \theta_d}{p_d t_d} \quad (41)$$

$$\beta_{rb} = \frac{p_r(\tau_{RS} + \Delta - \tau_b) + \theta_r}{p_r(t_b - t_c - t_r)} \quad (42)$$

Based on Table 3, we readily have:

$$N_s = N_c - N_r^1 - N_d$$

$$N_d = \begin{cases} N \max \left\{ 0, \int_{\max\{\beta_{rd}, \beta_0\}}^{\beta_{cd}} f(\beta) d\beta \right\}, & \text{if } p_r t_r > p_d t_d \\ N \max \left\{ 0, \int_{\beta_0}^{\min\{\beta_{cd}, \beta_{rd}\}} f(\beta) d\beta \right\}, & \text{if } p_r t_r < p_d t_d \\ 0, & \text{if } p_r t_r = p_d t_d \text{ and } \beta_{rc} > \beta_{cd} \\ N \max \left\{ 0, \int_{\beta_0}^{\beta_{cd}} f(\beta) d\beta \right\}, & \text{if } p_r t_r = p_d t_d \text{ and } \beta_{rc} \leq \beta_{cd} \end{cases}$$

$$N_r^1 = \begin{cases} N \max \left\{ 0, \int_{\beta_0}^{\min\{\beta_{rc}, \beta_{rd}\}} f(\beta) d\beta \right\}, & \text{if } p_r t_r > p_d t_d \\ N \max \left\{ 0, \int_{\max\{\beta_{rd}, \beta_0\}}^{\beta_{rc}} f(\beta) d\beta \right\}, & \text{if } p_r t_r < p_d t_d \\ N \max \left\{ 0, \int_{\beta_0}^{\beta_{rc}} f(\beta) d\beta \right\}, & \text{if } p_r t_r = p_d t_d \text{ and } \beta_{rc} > \beta_{cd} \\ 0, & \text{if } p_r t_r = p_d t_d \text{ and } \beta_{rc} \leq \beta_{cd} \end{cases}$$

$$N_r^0 = \begin{cases} \max \left\{ 0, N \int_{\max\{\beta_{rb}, 0\}}^{\beta_0} f(\beta) d\beta \right\}, & \text{if } t_b > t_c + t_r \\ \max \left\{ 0, N \int_0^{\min\{\beta_0, \beta_{rb}\}} f(\beta) d\beta \right\}, & \text{if } t_b < t_c + t_r \\ N \int_0^{\beta_0} f(\beta) d\beta, & \text{if } t_b = t_c + t_r \text{ and } p_r(\tau_{RS} + \Delta - \tau_b) + \theta_r < 0 \\ 0, & \text{if } t_b = t_c + t_r \text{ and } p_r(\tau_{RS} + \Delta - \tau_b) + \theta_r \geq 0 \end{cases}$$

$$N_b = N - N_c - N_r^0$$

Appendix B.

From Eq.(14), since $a \geq 1$, $b \geq 1$, $c > 0$ and $0 < A \leq 1$, $m < N_d$ and $m < N_r$.

Taking the partial derivative of $M(N_r, N_d)$, $p_r(N_r, N_d)$ and $p_d(N_r, N_d)$ in Eqs.

(14)-(16) with respect to N_r and N_d respectively, we have

$$\frac{\partial M(N_r, N_d)}{\partial N_d} = \frac{AN_r(bN_r + c)}{(aN_d + bN_r + c)^2} > 0$$

$$\frac{\partial M(N_r, N_d)}{\partial N_r} = \frac{AN_d(aN_d + c)}{(aN_d + bN_r + c)^2} > 0$$

$$\frac{\partial p_r(N_r, N_d)}{\partial N_r} = -\frac{AbN_d}{(aN_d + bN_r + c)^2} < 0$$

$$\frac{\partial p_r(N_r, N_d)}{\partial N_d} = \frac{A(bN_r + c)}{(aN_d + bN_r + c)^2} > 0$$

Similarly, $\partial p_d(N_r, N_d)/\partial N_d < 0$ and $\partial p_d(N_r, N_d)/\partial N_r > 0$. Thus all of the desirable properties mentioned in subsection 2.3 hold.

Appendix C.

Proposition 1. *The existence of equilibria is guaranteed under any cost-sharing strategy.*

Proof. Let $\mathbf{z} = (\mathbf{x}, \mathbf{y})^T$, $\mathbf{Z} = (\mathbf{X}, \mathbf{Y})^T$, $\mathbf{F}_z = (\mathbf{F}_x, \mathbf{F}_y)^T$, $\mathbf{G}_z = (\mathbf{G}_x, \mathbf{G}_y)^T$, and

$\Omega_z = \Omega_x \times \Omega_y$. We first show that the solutions of IDVI-1 in Eqs. (27) and (28) are identical to the solutions of the following IDVI problem:

IDVI-2: find $\mathbf{z}^* \in \Omega_z$, such that

$$\langle \beta \mathbf{F}_z(\mathbf{Z}^*) + \mathbf{G}_z(\mathbf{Z}^*), \mathbf{z} - \mathbf{z}^* \rangle \geq 0, \forall \mathbf{z} \in \Omega_z \quad (43)$$

First of all, we note that

$$\langle \beta \mathbf{F}_z(\mathbf{Z}^*) + \mathbf{G}_z(\mathbf{Z}^*), \mathbf{z} - \mathbf{z}^* \rangle = \langle \beta \mathbf{F}_x(\mathbf{Z}^*) + \mathbf{G}_x(\mathbf{Z}^*), \mathbf{x} - \mathbf{x}^* \rangle + \langle \beta \mathbf{F}_y(\mathbf{Z}^*) + \mathbf{G}_y(\mathbf{Z}^*), \mathbf{y} - \mathbf{y}^* \rangle$$

and thus if $(\mathbf{x}^*, \mathbf{y}^*)$ solves IDVI-1, we can readily conclude that $\mathbf{z}^* = (\mathbf{x}^*, \mathbf{y}^*)^T$

solves IDVI-2. Now suppose $\mathbf{z}^* = (\mathbf{x}^*, \mathbf{y}^*)^T$ is a solution to IDVI-2. From Eq. (43),

the following inequality then holds for any $\mathbf{z} = (\mathbf{x}, \mathbf{y}^*)^T$ with $\mathbf{x} \in \Omega_x$

$$\langle \beta \mathbf{F}_z(\mathbf{Z}^*) + \mathbf{G}_z(\mathbf{Z}^*), \mathbf{z} - \mathbf{z}^* \rangle = \langle \beta \mathbf{F}_x(\mathbf{Z}^*) + \mathbf{G}_x(\mathbf{Z}^*), \mathbf{x} - \mathbf{x}^* \rangle \geq 0$$

And similarly, for any $\mathbf{z} = (\mathbf{x}^*, \mathbf{y})^T$ with $\mathbf{y} \in \Omega_y$, the following inequality holds:

$$\langle \beta \mathbf{F}_z(\mathbf{Z}^*) + \mathbf{G}_z(\mathbf{Z}^*), \mathbf{z} - \mathbf{z}^* \rangle = \langle \beta \mathbf{F}_y(\mathbf{Z}^*) + \mathbf{G}_y(\mathbf{Z}^*), \mathbf{y} - \mathbf{y}^* \rangle \geq 0$$

Thus $\mathbf{z}^* = (\mathbf{x}^*, \mathbf{y}^*)^T$ must be a solution to IDVI-1 as well.

Let

$$\hat{\Omega}_x = \left\{ \mathbf{X} \in R^3 \mid \sum_{r \in A^c} X_r = N_c, X_r \geq 0, r \in A^c \right\}$$

$$\hat{\Omega}_y = \left\{ \mathbf{Y} \in R^2 \mid \sum_{r \in A^{nc}} Y_r = N - N_c, Y_r \geq 0, r \in A^{nc} \right\}$$

and let $\hat{\Omega}_z = \hat{\Omega}_x \times \hat{\Omega}_y$. According to Theorem 1 in Marcotte and Zhu (1997), if the

real functions $\mathbf{F}_z(\mathbf{Z})$ and $\mathbf{G}_z(\mathbf{Z})$ are continuous, and the finite dimensional set

$\hat{\Omega}_z$ is compact, then the existence of solutions to IDVI-2 is guaranteed. Since

$p_r(\mathbf{Z})$, $p_d(\mathbf{Z})$, $t_r(\mathbf{Z})$ and $t_d(\mathbf{Z})$ are all continuous functions, the continuity of

$F_z(\mathbf{Z})$ is apparent. Additionally, from the definition of $\hat{\Omega}_z$, the compactness of $\hat{\Omega}_z$

is also obvious; hence the existence of solutions for IDVI-2 is guaranteed. Since the

solutions of IDVI-1 and IDVI-1 are identical, the existence of solutions for IDVI-1

can also be concluded. ■

Appendix D.

In this appendix, we revisit Proposition 2 under the assumption that taking public transit is faster but more expensive than driving alone, i.e., $c_v + c_f < \tau_b$ and $t_c > t_b$. In this case, without a ridesharing program, travelers with $\beta > \beta_0$ commute by driving alone, travelers with $\beta < \beta_0$ commute by public transit, and travelers with $\beta = \beta_0$ are indifferent between the two modes, where β_0 is defined by Eq. (3). Hence from Table 1, if using ridesharing instead of driving alone, a car owner ($\beta < \beta_0$) suffers a time loss of $p_r t_r$ while saving a non-time cost of $p_r(c_v - \tau_{RS} - \Delta) - \theta_r$, and thus her/his generalized cost savings are

$$S_r(\beta) = C_c(\beta) - C_r^l(\beta) = p_r(c_v - \Delta - \tau_{RS}) - \beta p_r t_r - \theta_r \quad (\beta < \beta_0) \quad (44)$$

From Table 2, if using ridesharing instead of public transit, a non-car owner ($\beta \geq \beta_0$) suffers a time loss of $p_r(t_r + t_c - t_b)$ while saving a non-time cost of $P_r(\tau_b - \tau_{RS} - \Delta) - \theta_r$, so her/his cost savings are

$$S_r(\beta) = C_b(\beta) - C_r^0(\beta) = \beta p_r(t_b - t_r - t_c) - p_r(\tau_{RS} + \Delta - \tau_b) - \theta_r \quad (\beta \geq \beta_0) \quad (45)$$

If $p_r = 0$, we have $S_r(\beta) = -\theta_r < 0$ for all $\beta \in \mathcal{R}^+$. If $p_r > 0$, since $t_b < t_c < t_r + t_c$, Eqs. (44) and (45) imply that $S_r(\beta)$ decreases with β on both $[0, \beta_0)$ and $[\beta_0, +\infty)$. Furthermore, from Eqs. (44) and (45), we have

$$\begin{aligned} \lim_{\beta \rightarrow \beta_0^+} S_r(\beta) &= p_r[\beta_0(t_b - t_r - t_c) - (\tau_{RS} + \Delta - \tau_b)] - \theta_r \\ &= p_r[c_f + c_v - \tau_b - \beta_0 t_r - (\tau_{RS} + \Delta - \tau_b)] - \theta_r \\ &= p_r[c_f + c_v - \tau_{RS} - \Delta - \beta_0 t_r] - \theta_r \\ &> p_r(c_v - \tau_{RS} - \Delta - \beta_0 t_r) - \theta_r = \lim_{\beta \rightarrow \beta_0^-} S_r(\beta) \end{aligned} \quad (46)$$

where the second equality holds because $\beta_0(t_b - t_c) = c_f + c_v - \tau_b$ from Eq. (3). Thus due to the fixed car ownership cost $c_f > 0$, when $p_r > 0$, $S_r(\beta)$ is not continuous at $\beta = \beta_0$, and $\lim_{\beta \rightarrow \beta_0^+} S_r(\beta) > \lim_{\beta \rightarrow \beta_0^-} S_r(\beta)$. A typical cost savings curve in this case is thus depicted in Figure 8. Since $t_b < t_c$, the slope of the right segment (i.e., $[0, \beta_0)$) is less steeper than that of the left segment (i.e., $[\beta_0, +\infty)$). When τ_{RS} varies, the entire

$S_r(\beta)$ curve moves upward or downward, and the slope of both segments changes due to the change in rider waiting time at different values of τ_{RS} . Nevertheless, from Eqs. (44) and (45), under any τ_{RS} , the $S_r(\beta)$ curve decreases with β on the two segments $[0, \beta_0)$ and $[\beta_0, +\infty)$, and at the breaking point β_0 , we always have $\lim_{\beta \rightarrow \beta_0^+} S_r(\beta) > \lim_{\beta \rightarrow \beta_0^-} S_r(\beta)$.

In view of the properties of the $S_r(\beta)$ curve, we readily have the following Proposition.

Proposition 2-B. *Suppose public transit is faster but more expensive than driving alone, i.e., $t_b < t_c$ and $c_f + c_v < \tau_b$. If the fixed car ownership cost is higher than $\beta_0 \bar{t}_r$, i.e., $c_f > \beta_0 \bar{t}_r$, then there exists no cost-sharing strategy that can prevent transit users from switching modes while sustaining an active ridesharing program; otherwise, if the fixed car ownership cost is lower than $\beta_0 \bar{t}_r$, i.e., $c_f < \beta_0 \bar{t}_r$, then a ridesharing program can prevent transit users from switching modes while successfully encouraging private car drivers to give up driving solo, i.e., $N_r^0 = 0$ and $N_r^1 > 0$, only if the cost-sharing strategy τ_{RS} satisfies*

$$\max\{\theta_d/p_d + c_o t_d + \Delta, c_f + c_v - \beta_0 \bar{t}_r - \Delta - \theta_r/p_r\} < \tau_{RS} < c_v - \Delta - \theta_r/p_r \quad (47)$$

Proof. From the definition of $S_r(\beta)$ in Eqs. (44) and (45), a ridesharing program can prevent transit users from switching modes while successfully encouraging private car drivers to give up driving solo only if $S_r(\beta) \leq 0$ holds for $\forall \beta \in [\beta_0, +\infty)$ and $S_r(\beta) > 0$ for $\beta \in [0, \beta_0)$. If $p_r = 0$, then $S_r(\beta) = -\theta_r$ for all $\beta \in R^+$, and thus we only need to focus on the case when $p_r > 0$. When $p_r > 0$, $S_r(\beta)$ decreases on the two segments $[0, \beta_0)$ and $[\beta_0, +\infty)$, and $\lim_{\beta \rightarrow \beta_0^+} S_r(\beta) > \lim_{\beta \rightarrow \beta_0^-} S_r(\beta)$ at the breaking point β_0 , and thus the two conditions are equivalent to

$\lim_{\beta \rightarrow \beta_0^+} S_r(\beta) \leq 0$ and $S_r(0) > 0$. From Eq.(45), $\lim_{\beta \rightarrow \beta_0^+} S_r(\beta) \leq 0$ implies

$$\tau_{RS} \geq \beta_0(t_b - t_r - t_c) - \theta_r/p_r - \Delta + \tau_b$$

In view of Eq. (3), the above inequality can be further written as

$$\tau_{RS} \geq c_f + c_v - \beta_0 t_r - \Delta - \frac{\theta_r}{p_r} \quad (48)$$

And from Eq. (44), the condition $S_r(0) > 0$ is equivalent to

$$\tau_{RS} < c_v - \Delta - \frac{\theta_r}{p_r} \quad (49)$$

Hence a cost-sharing strategy τ_{RS} can prevent transit users from switching modes while successfully encouraging private car drivers to give up driving solo only if $c_f + c_v - \beta_0 t_r - \Delta - \theta_r/p_r \leq \tau_{RS} < c_v - \Delta - \theta_r/p_r$. If $c_f > \beta_0 \bar{t}_r$, we have $c_f - \beta_0 t_r > 0$ for any $t_x \leq t_r \leq \bar{t}_r$, such that $\tau_{RS} \in [c_f + c_v - \beta_0 t_r - \Delta - \theta_r/p_r, c_v - \Delta - \theta_r/p_r) = \emptyset$. In this case, no cost-sharing strategies exist to reduce total vehicle usage. Otherwise, if $c_f < \beta_0 \bar{t}_r$, then $[c_f + c_v - \beta_0 \bar{t}_r - \Delta - \theta_r/p_r, c_v - \Delta - \theta_r/p_r) \neq \emptyset$. In this case we may have $N_r^0 = 0$ and $N_r^1 > 0$ at equilibrium only if $p_r > 0$, and

$$c_f + c_v - \beta_0 t_r - \Delta - \theta_r/p_r < \tau_{RS} < c_v - \Delta - \theta_r/p_r \quad (50)$$

Meanwhile, from Eq. (15), the condition $p_r > 0$ is equivalent to $N_d > 0$. So we further derive the conditions under which $N_d > 0$. From Eqs. (5) and (1), the cost savings to car owners of being a RS driver is equivalent to

$$S_d(\beta) = C_c(\beta) - C_d(\beta) = -\beta p_d t_d + p_d (\tau_{RS} - c_o t_d - \Delta) - \theta_d \quad (\beta < \beta_0) \quad (51)$$

$S_d(\beta)$ decreases with β , and thus $N_d > 0$ is equivalent to $S_d(0) > 0$, which holds only if $p_d > 0$, and

$$\tau_{RS} > \theta_d/p_d + c_o t_d + \Delta \quad (52)$$

Combining Eqs. (50) and (52), we readily have the conclusion established when $c_f < \beta_0 \bar{t}_r$. ■

The impacts of the condition $c_f < \beta_0 \bar{t}_r$ on the existence of a desirable τ_{RS} in Proposition 2-B is illustrated in Figure 8. If the fixed car ownership cost c_f is higher than $\beta_0 \bar{t}_r$ such that $c_f + c_v - \beta_0 \bar{t}_r - \Delta - \theta_r/p_r > c_v - \Delta - \theta_r/p_r$, then the cost savings curve $S_r(\beta)$ under any cost-sharing strategy must be as depicted in Figure 8(b) with

$S_r(0) < \lim_{\beta \rightarrow \beta_0^+} S_r(\beta)$. Hence no cost-sharing strategy can prevent transit users from switching modes while also encouraging some car owners to give up driving solo in this case. The cost savings curve as in Figure 8 (a) appears only when $c_f < \beta_0 \bar{t}_r$ and τ_{RS} satisfies Eq. (47). Thus a ridesharing program can prevent transit users from switching modes while successfully encouraging private car drivers to give up driving solo only if $c_f < \beta_0 \bar{t}_r$ and τ_{RS} satisfies (47).

Appendix E.

Proposition 5. *When the additional costs of participating in a ridesharing program are non-negative to both riders and drivers, i.e., $\theta_d \geq 0$ and $\theta_r \geq 0$, a state with no ridesharing participants, i.e., $N_r^1 = N_d = N_r^0 = 0$, $N_s = N_c$, and $N_b = N - N_c$, is one of several possible equilibria under any cost-sharing strategy.*

Proof. We first note that if $(\mathbf{x}^*, \mathbf{y}^*)$ with

$$\mathbf{x}^*(\beta) = \begin{pmatrix} x_1(\beta) \\ x_2(\beta) \\ x_3(\beta) \end{pmatrix} = \begin{pmatrix} Nf(\beta) \\ 0 \\ 0 \end{pmatrix}, \forall \beta > \beta_0, \quad \mathbf{x}^*(\beta) = \begin{pmatrix} x_1(\beta) \\ x_2(\beta) \\ x_3(\beta) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \forall \beta \leq \beta_0 \quad (53)$$

and

$$\mathbf{y}^*(\beta) = \begin{pmatrix} y_1(\beta) \\ y_2(\beta) \end{pmatrix} = \begin{pmatrix} 0 \\ Nf(\beta) \end{pmatrix}, \forall \beta \leq \beta_0, \quad \mathbf{y}^*(\beta) = \begin{pmatrix} y_1(\beta) \\ y_2(\beta) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \forall \beta > \beta_0 \quad (54)$$

solves IDVI-1 under any cost-sharing strategy when $\theta_r, \theta_d > 0$, then the state with $N_r^1 = N_d = N_r^0 = 0$, $N_s = N_c$, and $N_b = N - N_c$ is one of several equilibria under any cost-sharing strategy.

For $(\mathbf{x}^*, \mathbf{y}^*)$ satisfying Eqs. (53) and (54), we have $\mathbf{X}^* = (N_c, 0, 0)^T$ and $\mathbf{Y}^* = (0, N - N_c)^T$. Inserting $(\mathbf{X}^*, \mathbf{Y}^*)$ into Eqs. (15) yields

$$p_r(\mathbf{X}^*, \mathbf{Y}^*) = 0, \quad p_d(\mathbf{X}^*, \mathbf{Y}^*) = 0, \quad t_r(\mathbf{X}^*, \mathbf{Y}^*) = \bar{t}_r \quad \text{and} \quad t_d(\mathbf{X}^*, \mathbf{Y}^*) = \bar{t}_d$$

Hence if $(\mathbf{x}^*, \mathbf{y}^*)$ is a solution to IDVI-1, it must satisfy: for any $\mathbf{x}(\beta) \in \Omega_x$ and $\mathbf{y}(\beta) \in \Omega_y$,

$$\int_0^{\beta_0} \left[(\beta t_c + c_f + c_v) x_1(\beta) + (\beta t_c + c_f + c_v + \theta_d) x_2(\beta) \right] d\beta \\ + \int_{\beta_0}^{+\infty} \left[(\beta t_c + c_f + c_v) (x_1(\beta) - Nf(\beta)) + (\beta t_c + c_f + c_v + \theta_d) x_2(\beta) \right] d\beta \geq 0 \\ + \int_{\beta_0}^{+\infty} \left[(\beta t_c + c_f + c_v + \theta_r) x_3(\beta) \right] d\beta \geq 0$$

and

$$\int_0^{\beta_0} \left[(\beta t_b + \tau_b + \theta_r) y_1(\beta) + (\beta t_b + \tau_b) (y_2(\beta) - Nf(\beta)) \right] d\beta + \\ \int_{\beta_0}^{+\infty} \left[(\beta t_b + \tau_b + \theta_r) y_1(\beta) + (\beta t_b + \tau_b) y_2(\beta) \right] d\beta \geq 0$$

which can be simplified into

$$\int_{\beta_0}^{+\infty} \left[\theta_d x_2(\beta) + \theta_r x_3(\beta) \right] d\beta \geq 0$$

and

$$\int_0^{\beta_0} \theta_r y_1(\beta) d\beta \geq 0$$

The above conditions hold for any $\mathbf{x} \in \Omega_x$ and $\mathbf{y} \in \Omega_y$ if $\theta_r, \theta_d \geq 0$. Hence

$(\mathbf{x}^*, \mathbf{y}^*)$ is an equilibrium state under any cost-sharing strategy if $\theta_r, \theta_d \geq 0$. ■