Appendices

A Estimation of the weight parameter $\tilde{w}$

This section describes the procedure followed to formulate, and to fit the coefficients of, the weight parameter of Equation (40). Recall that the goal of the mixture model is to accurately approximate the upstream and the downstream boundary conditions of the link. In other words, it should yield an accurate approximation of the distribution of UQ and of DQ. We consider a single isolated link and conduct a total of 180 experiments with varied combinations of the space capacity ($\ell \in \{5, 10, 15, \ldots, 100\}$), the traffic intensity ($\rho = \lambda/\mu \in \{0.25, 0.5, 0.75\}$) and the service rate (or downstream flow capacity) ($\mu \in \{0.2, 0.4, 0.6\}$). Each experiment considers a time period of duration 250 seconds. For each experiment we compare the approximation of the UQ and of the DQ distributions, over time T, to the distributions estimated via stochastic simulation with a discrete-event simulator of the stochastic link transmissions model. Based on the results of these experiments, we first observed that the parameters that most impact the quality of the approximation are ℓ, μ and k^{fwd}. This lead us to formulate the following expression for the weight parameter:

$$\tilde{w}(\ell, \mu, k^{\text{fwd}}; \beta) = e^{-\frac{\ell^2}{\beta \mu k^{\text{fwd}}}}, \tag{1}$$

where β is a scalar coefficient. The coefficient is fit such as to minimize, over all 180 experiments, the following error function:

$$\frac{1}{2}\left[\frac{1}{250}\sum_{\mathsf{T}=1}^{250} \text{JSD}(P_1^{\mathsf{UQ}}(\mathsf{T}) \parallel P_2^{\mathsf{UQ}}(\mathsf{T}))\right] + \frac{1}{2}\left[\frac{1}{250}\sum_{\mathsf{T}=1}^{250} \text{JSD}(P_1^{\mathsf{DQ}}(\mathsf{T}) \parallel P_2^{\mathsf{DQ}}(\mathsf{T}))\right], \tag{2}$$

where $P_1^{\mathsf{UQ}}(\mathsf{T})$ (resp. $P_1^{\mathsf{DQ}}(\mathsf{T})$) is the UQ (resp. DQ) distribution obtained from the mixture model at time T and $P_2^{\mathsf{UQ}}(\mathsf{T})$ (resp. $P_2^{\mathsf{DQ}}(\mathsf{T})$) is the UQ (resp. DQ) distribution estimated via stochastic simulation at time T. The two summations of (2) consider the error in the UQ distributions and in the DQ distributions, respectively. This leads to $\beta = 70$. This results in the final weight parameter expression defined in Equation (40).

B Tables of time-average JSD metric

Tables 1 and 2 display, respectively, the time-average JSD metric of the UQ and DQ distributions.

Experiment		Time-average JSD of the $\cup Q$ distribution				
$\lambda(k)$	ℓ	Mixture	Multivariate	DetDet	DetExp	ExpDet
0.1	10	0.0010	0.0000	0.3539	0.2600	0.0003
	20	0.0012	0.0000	0.3988	0.3177	0.0001
	30	0.0013	0.0000	0.4248	0.3543	0.0001
	40	0.0014	NaN	0.4402	0.3779	0.0000
	60	0.0013	NaN	0.4590	0.4111	0.0000
	80	0.0011	NaN	0.4692	0.4320	0.0000
	100	0.0010	NaN	0.4753	0.4489	0.0000
0.2	10	0.0054	0.0000	0.4261	0.2434	0.0020
	20	0.0068	0.0000	0.4644	0.3080	0.0013
	30	0.0070	0.0000	0.4839	0.3476	0.0008
	40	0.0070	NaN	0.4961	0.3767	0.0005
	60	0.0062	NaN	0.5105	0.4189	0.0003
	80	0.0054	NaN	0.5181	0.4476	0.0001
	100	0.0045	NaN	0.5223	0.4713	0.0001
0.3	10	0.0081	0.0000	0.4654	0.1615	0.0036
	20	0.0206	0.0000	0.5071	0.2387	0.0045
	30	0.0237	0.0000	0.5214	0.2863	0.0041
	40	0.0223	NaN	0.5294	0.3215	0.0032
	60	0.0182	NaN	0.5384	0.3772	0.0016
	80	0.0145	NaN	0.5434	0.4215	0.0008
	100	0.0115	NaN	0.5458	0.4602	0.0004

Table 1: Time-average JSD metric of the $\cup Q$ distribution. The value NaN denotes cases where the evaluation of the multivariate model exceeded the limit of 40 hours.

	Experiment	Time-average JSD of the DQ distribution				
$\lambda(k)$	ℓ	Mixture	Multivariate	DetDet	DetExp	ExpDet
0.1	10	0.0007	0.0000	0.1550	0.0486	0.0028
	20	0.0002	0.0000	0.1530	0.0476	0.0028
	30	0.0000	0.0000	0.1486	0.0468	0.0027
	40	0.0000	NaN	0.1466	0.0460	0.0026
	60	0.0000	NaN	0.1401	0.0441	0.0025
	80	0.0000	NaN	0.1335	0.0422	0.0024
	100	0.0000	NaN	0.1270	0.0402	0.0022
0.2	10	0.0030	0.0000	0.2679	0.0527	0.0115
	20	0.0008	0.0000	0.2640	0.0538	0.0112
	30	0.0002	0.0000	0.2584	0.0538	0.0107
	40	0.0000	NaN	0.2528	0.0518	0.0105
	60	0.0000	NaN	0.2414	0.0497	0.0099
	80	0.0000	NaN	0.2301	0.0476	0.0095
	100	0.0000	NaN	0.2188	0.0456	0.0089
0.3	10	0.0077	0.0000	0.3677	0.0347	0.0262
	20	0.0033	0.0000	0.3811	0.0507	0.0217
	30	0.0007	0.0000	0.3760	0.0544	0.0202
	40	0.0002	NaN	0.3680	0.0539	0.0196
	60	0.0000	NaN	0.3512	0.0516	0.0184
	80	0.0000	NaN	0.3343	0.0492	0.0174
	100	0.0000	NaN	0.3173	0.0467	0.0164

Table 2: Time-average JSD metric of the DQ distribution. The value NaN denotes cases where the evaluation of the multivariate model exceeded the limit of 40 hours.