Appendix A: Lifting procedure for the GDT branching strategy

This appendix describes the lifting procedure used to find delayed due times yielding the same lower bound in the GDT branching strategy. It consists of an iterative greedy procedure (see Algorithm 1) which is based on a modified version of the MILP (42)–(55).

Let N_i, $i \in \mathcal{O}$, be the maximum number of periods by which the due time of order i can be delayed (with respect to $b^{\hat{t}_i}$). The procedure starts at iteration $k=1$ with the subset of orders $\mathcal{O}^k$ whose due times can be delayed. Then it enters a repeat loop which begins by solving the following MILP that we denote PL^k. For each order $i \in \mathcal{O}^k$, we define N_i binary variables ζ_i^j, $j=1,\dots,N_i$. Variable ζ_i^j takes value 1 if the due time of order i is delayed by j periods and 0 otherwise. For $k=1$, we set $\underline{M}^{ptk} = \underline{M}^{pt}$ for all $p \in \mathcal{P}$ and $t \in \mathcal{T}$. Finally, let ϵ be a sufficiently small positive scalar.

With this notation, MILP PL^k writes as:

$$(PL^k) \qquad \min \quad \sum_{i \in \mathcal{O}^k} \sum_{j=1}^{N_i} j\zeta_i^j \tag{56}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}} \gamma^E n^{SP} w^t + \sum_{p \in \mathcal{P}} \sum_{u \in \mathcal{U}} \gamma^S \beta^{pu} \le \underline{z} - \epsilon, \tag{57}$$

$$\sum_{\substack{u \in \mathcal{U}^p \\ e^u \le e^t}} q^{pu} \ge \underline{M}^{ptk} - \sum_{\substack{i \in \mathcal{O}^k \\ \hat{t}_i = t}} d_i^p \big(\sum_{j=1}^{N_i} \zeta_i^j\big) + \sum_{\substack{i \in \mathcal{O}^k \\ \hat{t}_i < t \le \hat{t}_i + N_i}} d_i^p \zeta_i^{t-\hat{t}_i}, \qquad \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{58}$$

$$\sum_{j=1}^{N_i} \zeta_i^j \le 1, \qquad \forall i \in \mathcal{O}^k \tag{59}$$

$$\zeta_i^j \in \{0,1\}, \qquad \forall i \in \mathcal{O}^k, j \in \{1,2,\dots,N_i\} \tag{60}$$

$$(44) - (55).$$

Objective function (56) aims at minimizing the total number of periods of delay of the orders to obtain production costs that are less than $\underline{z}$ (imposed by constraint (57)). Constraints (58) impose production due times adjusted according to the selected delays. Constraints (59) ensure that at most one delay is chosen for each order and (60) express the binary requirements on the ζ_i^j variables. Finally, constraints (44)-(55) must still be considered.

If PL^k is feasible, then the computed optimal solution indicates for which orders the due times must be delayed and by how many periods to obtain production costs that are less than $\underline{z}$. Given that the number of delayed periods is minimized in PL^k, removing one of these delayed periods yields due times that incur production costs of at least $\underline{z}$. Hence, in Steps 8 and 9, all proposed delays are applied except for one period for one arbitrarily selected order i^* (chosen as the first in $\mathcal{O}^*$ for our tests). Then, the minimum quantities to produce of each product in each period according to the due times are updated in Step 10 before starting a new iteration. The repeat loop stops in two cases: either $\mathcal{O}^*$ becomes empty or PL^k is infeasible in which case the due times of all orders in $\mathcal{O}^k$ can be delayed to their maximum as it is not possible to find a feasible solution yielding a cost less than $\underline{z}$.

Note that this procedure is valid only if PL^k is solved exactly at each iteration. Nevertheless, to avoid long computational times, we limit the number of nodes to explore in the search tree when solving PL^k. When

Algorithm 1 Lifting procedure for the GDT branching

Set $k := 1$, $STOP := false$, $\mathcal{O}^k = \{i \in \mathcal{O} \mid N_i \geq 1\}$
repeat
  Solve MILP PL^k
  if PL^k is feasible **then**
    $\mathcal{O}^* := \{i \in \mathcal{O}^k \mid \sum_{j=1}^{N_i} \zeta_i^j = 1\}$
    $\forall i \in \mathcal{O}^*$, set $j^*(i) := j$ where $j \in \{1, \dots, N_i\}$ is such that $\zeta_i^j = 1$
    Choose $i^* \in \mathcal{O}^*$
    Set $\hat{t}_{i^*} := \hat{t}_{i^*} + j^*(i^*) - 1$
    $\forall i \in \mathcal{O}^* \setminus \{i^*\}$, set $\hat{t}_i := \hat{t}_i + j^*(i)$
    $\forall p \in \mathcal{P}, t \in \mathcal{T}$, set $\underline{M}^{pt,k+1} := \underline{M}^{ptk} - \sum_{\substack{i \in \mathcal{O}^k \\ \hat{t}_i = t}} d_i^p \big(\sum_{j=1}^{N_i} \zeta_i^j \big) + \sum_{\substack{i \in \mathcal{O}^k \\ \hat{t}_i < t \leq \hat{t}_i + N_i}} d_i^p \zeta_i^{t - \hat{t}_i}$
    Set $\mathcal{O}^{k+1} := \mathcal{O}^k \setminus \mathcal{O}^*$
    if $\mathcal{O}^* = \emptyset$ **then**
      Set $STOP := true$
  else
    $\forall i \in \mathcal{O}^k$, set $\hat{t}_i := \hat{t}_i + N_i$
    Set $STOP := true$
  $k := k + 1$
until $STOP = true$

this limit is reached, the procedure is exited immediately without delaying the due times of any additional orders.

Appendix B: Detailed Results for the Exact Algorithms

In this appendix, we present the results obtained by the exact algorithms with the GDT branching, with the GDT branching and the lifting strategy, and without the GDT branching for all 108 tested instances with 15, 20, or 25 customers. The name of an instance takes the form nO-mP-vD-wL-x where nO-mP-vD-wL indicates the instance class as described in Section 6.2 and $x \in \{1, 2, 3\}$ gives the instance number in this class. The results are reported in Tables 10–12. In these tables, beside specifying the total computational time and number of branch-and-bound nodes explored for each instance and algorithm, we provide for each instance the computational time required in the root node including the generation of cuts (Root Time), the lower bound at the root node before adding cuts (LP Bound) and after adding cuts (Root Bound), and the best upper bound computed in all our tests (Best UB). Furthermore, for each instance and each algorithm, we give the lower bound reached at the end of the solution process (End Bound). On the last line of each table, we indicate the average time at the root node, and for each algorithm, the average computational time, the average number of nodes explored and the number of instances solved to optimality. Finally, in each row, we put in bold the best computational time achieved by the algorithms, unless none could solve the instance to optimality. In this case, we highlight the best end bound.

Table 10 Results for instances with 15 customers

					With GDT			With GDT + Lift			Without GDT		
Instance	Root Time (s)	LP Bound	Root Bound	Best UB	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound
15O-2P-oD-1L-1	0.2	446.00	452.50	462.00	4	48	462.00	8	43	462.00	**1**	25	462.00
15O-2P-oD-1L-2	0.3	430.10	464.50	475.75	**1**	16	475.75	4	19	475.75	11	179	475.75
15O-2P-oD-1L-3	0.2	326.62	329.00	350.00	**16**	374	350.00	44	642	350.00	133	2005	350.00
15O-3P-oD-1L-1	0.2	447.00	460.25	469.00	**4**	33	469.00	20	37	469.00	84	1054	469.00
15O-3P-oD-1L-2	0.6	430.38	439.77	478.25	7200	63,424	**477.82**	7200	63,816	**477.82**	7200	58,306	476.75
15O-3P-oD-1L-3	0.1	336.12	338.50	363.00	**245**	1296	363.00	268	1309	363.00	370	1931	363.00
15O-4P-oD-1L-1	0.2	448.00	455.97	469.00	118	1318	469.00	**64**	121	469.00	130	883	469.00
15O-4P-oD-1L-2	0.5	426.87	464.85	-	7200	142,078	**477.75**	7200	130,891	**477.75**	7200	62,916	476.75
15O-4P-oD-1L-3	0.4	337.12	339.82	364.00	1211	5253	364.00	**1198**	5253	364.00	1208	4993	364.00
15O-2P-dD-1L-1	0.2	499.50	518.00	522.00	**2**	26	522.00	6	26	522.00	3	96	522.00
15O-2P-dD-1L-2	0.3	506.11	517.87	-	7200	151,066	**546.25**	7200	116,095	**546.25**	7200	127,126	545.25
15O-2P-dD-1L-3	0.2	365.12	367.82	388.50	**29**	483	388.50	56	651	388.50	59	870	388.50
15O-3P-dD-1L-1	0.4	546.78	572.19	583.75	233	2046	583.75	231	2046	583.75	**207**	2046	583.75
15O-3P-dD-1L-2	0.7	525.13	567.15	597.50	6186	43,767	597.50	**5980**	36,691	597.50	7200	42,538	596.75
15O-3P-dD-1L-3	0.3	381.30	384.09	409.00	346	2016	409.00	**258**	1185	409.00	578	3281	409.00
15O-4P-dD-1L-1	0.4	547.75	566.50	577.75	**30**	288	577.75	102	772	577.75	4259	22,354	577.75
15O-4P-dD-1L-2	0.6	505.51	515.49	-	7200	40,868	**548.25**	7200	57,844	**548.25**	7200	40,684	540.73
15O-4P-dD-1L-3	0.4	381.30	384.11	409.00	**778**	4038	409.00	933	3840	409.00	2316	11,608	409.00
15O-2P-oD-2L-1	0.1	384.25	384.25	384.25	**1**	10	384.25	**1**	10	384.25	**1**	10	384.25
15O-2P-oD-2L-2	0.9	375.69	385.75	399.75	**94**	806	399.75	324	3052	399.75	7200	33,429	398.85
15O-2P-oD-2L-3	0.2	301.52	303.51	326.25	16	242	326.25	11	181	326.25	**7**	131	326.25
15O-3P-oD-2L-1	0.1	393.75	393.75	393.75	**2**	11	393.75	**2**	11	393.75	**2**	11	393.75
15O-3P-oD-2L-2	1.5	385.05	390.62	408.25	816	3764	408.25	**333**	214	408.25	7200	33,537	407.25
15O-3P-oD-2L-3	0.3	310.02	312.05	334.75	266	444	334.75	**46**	298	334.75	66	308	334.75
15O-4P-oD-2L-1	0.2	394.75	394.75	401.25	47	65	401.25	**40**	32	401.25	429	885	401.25
15O-4P-oD-2L-2	0.9	381.96	389.89	408.25	**1242**	4614	408.25	1264	4614	408.25	7200	11,785	400.75
15O-4P-oD-2L-3	0.3	311.02	313.00	335.75	93	237	335.75	**92**	237	335.75	**92**	237	335.75
15O-2P-dD-2L-1	0.3	429.25	429.26	430.25	5	27	430.25	**3**	13	430.25	4	31	430.25
15O-2P-dD-2L-2	1.6	434.78	439.03	483.75	7200	14,184	**477.75**	7200	17,786	476.69	7200	14,190	476.50
15O-2P-dD-2L-3	0.2	332.31	334.59	357.50	182	1139	357.50	**141**	859	357.50	1706	5635	357.50
15O-3P-dD-2L-1	0.2	492.50	492.50	492.50	**1**	10	492.50	**1**	10	492.50	**1**	10	492.50
15O-3P-dD-2L-2	4.3	453.60	461.02	508.50	7200	4360	501.00	7200	8290	507.50	7200	7485	497.31
15O-3P-dD-2L-3	2.3	351.12	354.29	383.00	**6092**	1203	383.00	7200	787	376.43	7200	1903	378.49
15O-4P-dD-2L-1	0.3	486.00	486.00	487.00	33	35	487.00	**23**	50	487.00	362	1161	487.00
15O-4P-dD-2L-2	0.7	446.68	452.20	-	7200	15,366	**485.75**	7200	12,921	**485.75**	7200	22,288	478.83
15O-4P-dD-2L-3	0.4	352.13	354.68	383.25	7200	5222	378.77	7200	11,166	**380.16**	7200	13,746	378.87
Average	0.58				**2103**	14,172	# Opt.: **28**	2118	13,384	# Opt.: 27	2934	14,713	# Opt.: 23

Table 11 Results for instances with 20 customers

					With GDT			With GDT + Lift			Without GDT		
Instance	Root Time (s)	LP Bound	Root Bound	Best UB	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound
20O-2P-oD-1L-1	0.6	412.51	416.60	417.50	**5**	25	417.50	6	25	417.50	6	25	417.50
20O-2P-oD-1L-2	3.6	335.75	338.70	355.25	7200	14,463	351.19	7200	17,473	**351.25**	7200	12,141	351.25
20O-2P-oD-1L-3	0.1	489.50	492.53	508.25	**7**	89	508.25	17	104	508.25	71	402	508.25
20O-3P-oD-1L-1	1.8	420.71	429.51	456.50	4675	6510	456.50	**3910**	5103	456.50	7200	7769	455.25
20O-3P-oD-1L-2	0.4	337.75	337.90	347.75	**242**	1010	347.75	767	2144	347.75	7200	4249	346.86
20O-3P-oD-1L-3	0.5	514.95	520.52	544.75	7200	8204	537.57	7200	2182	**538.50**	7200	1964	537.25
20O-4P-oD-1L-1	1.1	436.29	441.11	457.00	**65**	71	457.00	127	79	457.00	2624	3674	457.00
20O-4P-oD-1L-2	3.9	334.25	338.35	367.50	**3799**	2286	367.50	7200	8256	367.50	7200	4586	366.50
20O-4P-oD-1L-3	0.2	500.84	501.22	527.25	7200	5126	518.09	7200	8260	**519.65**	7200	5849	517.93
20O-2P-dD-1L-1	0.6	459.34	462.55	463.50	**187**	987	463.50	237	1079	463.50	660	3871	463.50
20O-2P-dD-1L-2	0.5	418.35	419.43	436.00	**53**	179	436.00	76	210	436.00	88	351	436.00
20O-2P-dD-1L-3	0.1	513.00	520.88	541.25	7200	112,906	**540.25**	7200	101,655	**540.25**	7200	59,259	539.51
20O-3P-dD-1L-1	0.8	486.68	491.36	498.25	22	62	498.25	**21**	53	498.25	28	68	498.25
20O-3P-dD-1L-2	0.5	433.64	434.93	436.75	**2**	10	436.75	**2**	10	436.75	**2**	10	436.75
20O-3P-dD-1L-3	0.1	558.25	561.29	588.25	7200	12,999	**582.25**	7200	11,459	**582.25**	7200	11,606	581.75
20O-4P-dD-1L-1	1.4	482.31	488.12	503.75	**338**	873	503.75	429	585	503.75	871	1817	503.75
20O-4P-dD-1L-2	2.4	406.14	407.88	440.75	7200	5401	**424.47**	7200	4927	424.31	7200	5857	424.25
20O-4P-dD-1L-3	0.3	543.52	547.33	569.25	7200	17,152	**564.01**	7200	12,315	564.00	7200	16,055	563.10
20O-2P-oD-2L-1	0.6	390.62	394.73	404.75	**136**	249	404.75	347	193	404.75	7200	12,808	403.53
20O-2P-oD-2L-2	6.0	312.35	314.23	317.75	220	93	317.75	339	105	317.75	**196**	53	317.75
20O-2P-oD-2L-3	0.1	460.50	460.62	479.75	7200	17,437	464.50	7200	25,653	**464.75**	7200	27,307	463.75
20O-3P-oD-2L-1	5.2	395.30	398.33	411.75	**6950**	3915	411.75	7200	3802	411.15	7200	3998	410.00
20O-3P-oD-2L-2	4.6	315.99	318.24	328.00	686	129	328.00	1588	442	328.00	**431**	89	328.00
20O-3P-oD-2L-3	0.3	476.62	476.75	489.25	1703	360	489.25	**628**	103	489.25	829	195	489.25
20O-4P-oD-2L-1	1.9	399.14	403.23	443.50	7200	3493	**416.25**	7200	2044	416.00	7200	3708	408.29
20O-4P-oD-2L-2	7.7	319.66	321.88	336.50	4934	479	336.50	**3207**	595	336.50	7200	598	335.25
20O-4P-oD-2L-3	1.8	458.04	461.14	510.75	7200	1377	480.75	7200	1970	**481.00**	7200	850	478.91
20O-2P-dD-2L-1	0.7	442.40	444.61	453.25	**592**	1262	453.25	3174	8926	453.25	7200	12,153	446.82
20O-2P-dD-2L-2	0.8	382.52	383.05	412.00	7200	13,840	411.93	**2387**	4916	412.00	7200	13,621	401.28
20O-2P-dD-2L-3	0.4	478.01	481.99	509.25	**1808**	3485	509.25	2727	3392	509.25	7200	9647	508.75
20O-3P-dD-2L-1	6.2	463.06	466.94	489.00	7200	1981	485.36	7200	2572	**486.00**	7200	2517	482.16
20O-3P-dD-2L-2	1.0	406.52	408.40	428.00	7200	1644	421.62	7200	1316	**421.65**	7200	3302	419.46
20O-3P-dD-2L-3	0.2	530.12	530.25	538.50	7200	7818	**537.50**	7200	5202	**537.50**	7200	5531	**537.50**
20O-4P-dD-2L-1	2.4	449.60	453.44	477.75	7200	1065	**467.21**	7200	866	467.06	7200	2894	463.50
20O-4P-dD-2L-2	3.5	382.55	384.40	409.75	7200	1994	**399.00**	7200	1209	**399.00**	7200	3168	398.50
20O-4P-dD-2L-3	0.9	508.97	514.92	540.25	7200	678	**532.23**	7200	726	531.57	7200	1997	531.75
Average	1.65				**4134**	6935	# opt.: **19**	4155	6665	# opt.: 18	5161	6778	# opt.: 11

Table 12 Results for instances with 25 customers

					With GDT			With GDT + Lift			Without GDT		
Instance	Root Time (s)	LP Bound	Root Bound	Best UB	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound	Time (s)	# Nodes	End Bound
25O-2P-oD-1L-1	2.4	525.12	532.90	555.75	6162	14,627	555.75	**5632**	9378	555.75	7200	22,126	552.50
25O-2P-oD-1L-2	0.5	546.69	551.30	575.00	7200	39,182	569.86	7200	32,373	**570.74**	7200	32,173	568.03
25O-2P-oD-1L-3	1.6	567.17	570.16	595.25	7200	12,828	594.12	7200	11,694	**594.25**	7200	19,043	594.05
25O-3P-oD-1L-1	3.7	527.17	539.40	557.75	7200	12,179	557.75	7200	12,786	557.75	**195**	308	557.75
25O-3P-oD-1L-2	0.9	548.69	558.31	586.50	7200	18,993	**575.43**	7200	19,169	571.10	7200	21,504	571.05
25O-3P-oD-1L-3	2.2	569.17	576.24	597.00	7200	14,224	**596.92**	7200	22,731	596.75	7200	16,058	596.75
25O-4P-oD-1L-1	3.8	529.12	532.23	597.00	7200	8136	554.48	7200	8024	**556.33**	7200	12,358	551.75
25O-4P-oD-1L-2	1.0	544.19	553.44	578.00	7200	19,336	**571.17**	7200	16,655	570.40	7200	19,363	569.00
25O-4P-oD-1L-3	2.8	572.17	574.22	604.25	7200	14,499	594.42	7200	14,441	**599.62**	7200	10,983	596.75
25O-2P-dD-1L-1	3.0	586.12	599.66	610.25	193	268	610.25	188	268	610.25	**185**	268	610.25
25O-2P-dD-1L-2	0.6	600.19	612.50	628.50	2861	11,040	628.50	2760	11,040	628.50	**2714**	11,040	628.50
25O-2P-dD-1L-3	2.9	617.17	623.66	657.25	7200	11,831	645.85	7200	10,975	**646.03**	7200	11,686	643.39
25O-3P-dD-1L-1	6.1	590.12	609.53	622.00	6834	6053	622.00	**6782**	6053	622.00	7200	6959	621.75
25O-3P-dD-1L-2	1.0	618.43	624.88	-	7200	14,535	**639.64**	7200	8769	639.03	7200	12,644	636.53
25O-3P-dD-1L-3	3.3	631.17	634.20	659.75	7200	11,386	**656.50**	7200	11,008	656.25	7200	14,984	**656.50**
25O-4P-dD-1L-1	6.7	589.41	599.89	-	7200	4830	**616.13**	7200	5132	615.93	7200	5992	615.07
25O-4P-dD-1L-2	0.9	609.15	613.73	-	7200	16,355	**631.50**	7200	12574	**631.50**	7200	15,206	624.40
25O-4P-dD-1L-3	1.6	626.67	628.36	-	7200	10,851	647.82	7200	8361	648.30	7200	8714	**650.25**
25O-2P-oD-2L-1	3.1	472.88	473.67	500.00	7200	5013	497.75	**1338**	829	500.00	7200	3029	497.25
25O-2P-oD-2L-2	1.2	489.46	494.81	513.75	7200	10,171	**508.00**	7200	9190	507.61	7200	10,027	506.53
25O-2P-oD-2L-3	4.8	522.50	526.68	549.75	7200	8941	547.17	**6390**	7288	549.75	7200	10,038	547.26
25O-3P-oD-2L-1	6.0	484.47	485.27	514.25	**6488**	2995	514.25	7200	1691	512.25	7200	1034	510.50
25O-3P-oD-2L-2	20.0	497.69	502.34	536.25	7200	4346	522.22	7200	4465	**522.93**	7200	4061	515.99
25O-3P-oD-2L-3	11.4	530.95	533.80	555.50	7200	4234	549.41	7200	3341	549.50	7200	5220	**555.48**
25O-4P-oD-2L-1	2.4	484.38	485.17	607.00	7200	2611	509.25	7200	1113	**512.50**	7200	2827	508.81
25O-4P-oD-2L-2	4.0	496.77	500.40	-	7200	2998	522.00	7200	4576	**522.25**	7200	4844	510.92
25O-4P-oD-2L-3	6.1	531.43	533.77	581.75	7200	3294	547.22	7200	2463	548.66	7200	6446	**556.73**
25O-2P-dD-2L-1	2.5	526.38	527.17	546.00	1307	566	546.00	635	362	546.00	**222**	170	546.00
25O-2P-dD-2L-2	3.0	541.27	544.83	-	7200	8707	**559.13**	7200	6110	558.75	7200	8025	554.68
25O-2P-dD-2L-3	4.7	572.57	575.26	595.00	3677	3434	595.00	3287	2793	595.00	**1822**	1745	595.00
25O-3P-dD-2L-1	3.7	542.67	543.47	563.00	1658	518	563.00	3217	623	563.00	**350**	156	563.00
25O-3P-dD-2L-2	0.7	567.88	571.48	581.25	7200	3932	580.25	7200	2701	**581.22**	7200	5540	580.25
25O-3P-dD-2L-3	2.4	581.10	582.85	621.50	7200	4931	602.88	7200	5037	602.72	7200	2879	**603.25**
25O-4P-dD-2L-1	3.2	536.88	537.67	557.50	7030	2120	557.50	**3853**	771	557.50	7200	1861	557.50
25O-4P-dD-2L-2	1.4	553.88	557.48	580.25	7200	674	**567.23**	7200	784	**567.23**	7200	3692	566.25
25O-4P-dD-2L-3	2.6	581.10	582.86	616.00	7200	1262	590.36	7200	864	590.36	7200	4091	**602.75**
Average	3.53				6408	8654	# opt.: 9	**6147**	7679	# opt.: **10**	6153	8780	# opt.: 6

Appendix C: Statistics on the Optimal Solutions

In Tables 13 to 17 (one table per number of orders), we report some statistics on the computed optimal solutions for the instances solved to optimality. When more than one optimal solution is available for an instance, we give the statistics of the solution obtained by the algorithm with the GDT branching. The reported statistics are the numbers of drivers, vehicles, production employees, workstations, and setups. In each table, the last row provides averages over all solved instances.

Table 13 Statistics on the optimal solutions for instances with 15 customers

Instance	# Drivers	# Vehicles	# Employees	# Workstations	# Setups
15O-2P-oD-1L-1	4	3	3	2	8
15O-2P-oD-1L-2	4	3	4	3	7
15O-2P-oD-1L-3	4	4	1	1	4
15O-3P-oD-1L-1	4	3	3	2	9
15O-3P-oD-1L-3	4	4	1	1	5
15O-4P-oD-1L-1	4	3	3	2	9
15O-4P-oD-1L-3	4	4	1	1	6
15O-2P-dD-1L-1	4	3	4	3	8
15O-2P-dD-1L-3	4	4	2	2	5
15O-3P-dD-1L-1	5	3	4	3	10
15O-3P-dD-1L-2	6	5	4	3	9
15O-3P-dD-1L-3	4	4	2	2	6
15O-4P-dD-1L-1	5	3	4	3	10
15O-4P-dD-1L-3	4	4	2	2	6
15O-2P-oD-2L-1	4	3	2	1	5
15O-2P-oD-2L-2	4	3	2	2	6
15O-2P-oD-2L-3	4	4	1	1	3
15O-3P-oD-2L-1	4	3	2	1	7
15O-3P-oD-2L-2	5	3	3	2	7
15O-3P-oD-2L-3	4	4	1	1	4
15O-4P-oD-2L-1	4	3	2	1	7
15O-4P-oD-2L-2	4	3	2	2	7
15O-4P-oD-2L-3	4	4	1	1	5
15O-2P-dD-2L-1	4	3	3	2	6
15O-2P-dD-2L-3	4	4	1	1	4
15O-3P-dD-2L-1	5	3	3	2	7
15O-3P-dD-2L-3	4	4	2	2	5
15O-4P-dD-2L-1	5	3	3	2	9
Average	4.25	3.46	2.36	1.82	6.57

Table 14 Statistics on the optimal solutions for instances with 20 customers

Instance	# Drivers	# Vehicles	# Employees	# Workstations	# Setups
20O-2P-oD-1L-1	4	3	2	1	5
20O-2P-oD-1L-3	5	4	2	1	6
20O-3P-oD-1L-1	4	3	3	2	7
20O-3P-oD-1L-2	4	3	2	1	8
20O-4P-oD-1L-1	4	3	3	2	8
20O-4P-oD-1L-2	3	2	3	2	8
20O-2P-dD-1L-1	4	3	3	2	6
20O-2P-dD-1L-2	5	4	3	2	6
20O-3P-dD-1L-1	5	4	3	2	8
20O-3P-dD-1L-2	5	4	3	2	6
20O-4P-dD-1L-1	4	3	4	3	9
20O-2P-oD-2L-1	4	3	2	1	5
20O-2P-oD-2L-2	3	2	2	1	4
20O-3P-oD-2L-1	4	3	2	1	7
20O-3P-oD-2L-2	4	3	2	1	7
20O-4P-oD-2L-2	3	2	2	1	7
20O-3P-oD-2L-3	5	4	2	1	7
20O-2P-dD-2L-1	4	3	3	2	5
20O-2P-dD-2L-2	5	4	3	2	5
20O-2P-dD-2L-3	5	4	2	1	5
Average	4.25	3.20	2.55	1.55	6.45

Table 15 Statistics on the optimal solutions for instances with 25 customers

Instance	# Drivers	# Vehicles	# Employees	# Workstations	# Setups
25O-2P-oD-1L-1	6	5	3	2	8
25O-3P-oD-1L-1	6	5	3	2	10
25O-2P-dD-1L-1	6	5	4	3	9
25O-2P-dD-1L-2	6	5	4	3	9
25O-3P-dD-1L-1	6	5	4	3	11
25O-2P-oD-2L-1	6	5	2	1	5
25O-2P-oD-2L-3	7	6	2	1	6
25O-3P-oD-2L-1	6	5	2	1	7
25O-2P-dD-2L-1	6	5	3	2	6
25O-2P-dD-2L-3	7	6	2	2	6
25O-3P-dD-2L-1	6	5	3	2	8
25O-4P-dD-2L-1	6	5	3	2	10
Average	6.17	5.17	2.92	2.00	7.92

Table 16 Statistics on the optimal solutions for instances with 30 customers

Instance	# Drivers	# Vehicles	# Employees	# Workstations	# Setups
30O-3P-dD-1L-1	9	7	5	4	12
30O-3P-dD-2L-1	8	7	3	3	5
Average	9.50	7.00	4.00	3.50	8.50

Table 17 Statistics on the optimal solutions for instances with 40 customers

Instance	# Drivers	# Vehicles	# Employees	# Workstations	# Setups
40O-2P-oD-2L-3	8	7	2	2	7
40O-2P-dD-2L-2	7	7	4	4	5
Average	7.50	7.00	3.00	3.00	6.00

Appendix D: Detailed Results for the Matheuristics

In this appendix, we report the results obtained by the three matheuristics SEQ_H, E_SEQ_H, and INT_H for each of the 108 tested instances with 30, 40 or 50 customers. These results are given in Tables 18–20. In these tables, we indicate for each instance the lower bounds obtained by the exact branch-price-and-cut algorithm at the root node before adding cuts (LP) and at the end of the solution process (End). Optimality was reached only for two instances with 30 customers and two instances with 40 customers: the end lower bounds of these instances are highlighted in bold. For all other instances, the solution process stopped at the 14,400-second time limit. Then for each matheuristic, we report the total time in seconds (split by phase for the sequential heuristics, where VR stands for vehicle routing phase and Prod for production phase), the number of extra workstations/employees required (xWE), the cost of the best solution found (UB), and the gap in percentage between this cost and the end lower bound yielded by the exact algorithm. At the bottom of each table, a row indicates averages as well as the total number of instances solved to optimality by the exact algorithm.

Table 18 Results for instances with 30 customers

	Exact		SEQ_H					E_SEQ_H					INT_H			
	Lower bound		Time (s)					Time (s)								
Instance	LP	End	VR	Prod	xWE	UB	Gap (%)	VR	Prod	xWE	UB	Gap (%)	Time (s)	xWE	UB	Gap (%)
30O-2P-oD-1L-1	594.76	631.27	27	1	0	665.25	5.38	41	42	0	648.25	2.69	660	0	655.50	3.84
30O-2P-oD-1L-2	607.38	634.75	72	2	0	670.50	5.63	31	17	0	660.00	3.98	53	0	664.75	4.73
30O-2P-oD-1L-3	599.06	651.50	1	19	0	701.75	7.71	51	21	0	673.25	3.34	133	0	670.75	2.95
30O-3P-oD-1L-1	624.59	657.75	58	1852	1	746.00	13.42	15	123	0	687.00	4.45	434	0	680.25	3.42
30O-3P-oD-1L-2	650.88	691.57	89	291	0	754.75	9.14	55	104	0	721.50	4.33	268	0	716.75	3.64
30O-3P-oD-1L-3	615.52	663.77	1	343	1	720.75	8.58	29	177	0	727.25	9.56	296	0	687.75	3.61
30O-4P-oD-1L-1	620.38	655.77	11	1986	1	719.25	9.68	117	296	0	709.25	8.16	661	0	689.50	5.14
30O-4P-oD-1L-2	644.78	680.85	75	308	0	749.75	10.12	115	530	0	733.25	7.70	6803	0	741.50	8.91
30O-4P-oD-1L-3	607.75	653.49	1	1071	1	754.75	15.50	41	227	0	722.75	10.60	3741	0	717.00	9.72
30O-2P-dD-1L-1	788.47	810.75	10	4	2	861.00	6.20	41	10	1	826.00	1.88	47	0	821.25	1.30
30O-2P-dD-1L-2	749.76	779.75	1	3	2	824.75	5.77	22	47	1	805.25	3.27	84	0	829.25	6.35
30O-2P-dD-1L-3	728.92	778.00	37	52	3	913.25	17.38	33	10	0	820.25	5.43	83	0	792.25	1.83
30O-3P-dD-1L-1	849.75	**864.50**	15	308	1	934.75	8.13	50	9	0	864.50	0.00	53	0	864.50	0.00
30O-3P-dD-1L-2	859.56	913.27	4	495	3	970.00	6.21	154	129	1	948.50	3.86	211	0	948.00	3.80
30O-3P-dD-1L-3	766.55	800.27	34	191	1	915.75	14.43	76	202	0	839.00	4.84	223	0	813.00	1.59
30O-4P-dD-1L-1	868.54	883.41	380	71	0	908.00	2.78	2682	29	0	891.25	0.89	543	0	906.25	2.59
30O-4P-dD-1L-2	849.03	888.82	23	1109	1	916.00	3.06	13	117	1	901.00	1.37	1703	0	912.25	2.64
30O-4P-dD-1L-3	758.80	794.60	6	57	1	865.50	8.92	59	67	1	815.50	2.63	582	0	823.50	3.64
30O-2P-oD-2L-1	555.12	582.88	83	50	0	594.25	1.95	155	12	0	583.75	0.15	277	0	583.75	0.15
30O-2P-oD-2L-2	565.75	589.88	25	56	0	606.50	2.82	318	83	0	620.50	5.19	205	0	605.25	2.61
30O-2P-oD-2L-3	549.60	584.25	1	170	0	601.00	2.87	6	86	0	617.25	5.65	259	0	592.75	1.45
30O-3P-oD-2L-1	581.13	601.61	16	375	0	630.75	4.84	47	605	0	628.75	4.51	2252	0	625.25	3.93
30O-3P-oD-2L-2	604.44	631.31	206	251	1	717.75	13.69	617	728	0	669.50	6.05	1541	0	666.25	5.53
30O-3P-oD-2L-3	567.00	601.00	1	285	0	609.50	1.41	9	1003	0	624.75	3.95	2286	0	626.50	4.24
30O-4P-oD-2L-1	578.52	601.70	22	588	0	627.25	4.25	78	350	0	626.25	4.08	1751	0	628.25	4.41
30O-4P-oD-2L-2	604.44	625.29	128	1666	0	671.25	7.35	1691	1070	0	667.50	6.75	1375	0	662.00	5.87
30O-4P-oD-2L-3	562.77	598.33	1	936	0	634.00	5.96	14	671	0	624.75	4.42	1930	0	639.25	6.84
30O-2P-dD-2L-1	737.01	748.50	9	20	0	758.50	1.34	83	14	0	753.00	0.60	161	0	755.50	0.94
30O-2P-dD-2L-2	691.35	712.54	54	4	0	722.75	1.43	176	9	0	728.25	2.20	584	0	720.50	1.12
30O-2P-dD-2L-3	670.54	707.39	78	11	0	733.75	3.73	242	52	0	723.75	2.31	1000	0	736.75	4.15
30O-3P-dD-2L-1	811.60	**827.25**	57	695	0	842.25	1.81	321	2	0	827.50	0.03	905	0	835.75	1.03
30O-3P-dD-2L-2	792.38	802.43	13	3666	0	842.50	4.99	102	1421	0	833.25	3.84	2359	0	821.25	2.35
30O-3P-dD-2L-3	715.76	741.40	27	864	0	768.50	3.66	342	1664	0	777.50	4.87	6840	0	768.00	3.59
30O-4P-dD-2L-1	824.65	845.05	367	336	0	856.00	1.30	5132	1	0	846.75	0.20	9660	0	855.50	1.24
30O-4P-dD-2L-2	802.99	814.38	46	1272	0	855.25	5.02	580	1292	0	855.25	5.02	3375	0	859.50	5.54
30O-4P-dD-2L-3	709.16	739.22	26	633	0	790.00	6.87	419	1229	0	795.75	7.65	1425	0	770.25	4.20
Average		# opt.: 2	55.7	556.7	0.53	762.6	6.5	387.7	345.8	0.14	744.38	4.1	1521.2	0.00	741.28	3.6

Table 19 Results for instances with 40 customers

	Exact		SEQ_H					E_SEQ_H					INT_H			
	Lower bound		Time (s)					Time (s)								
Instance	LP	End	VR	Prod	xWE	UB	Gap (%)	VR	Prod	xWE	UB	Gap (%)	Time (s)	xWE	UB	Gap (%)
40O-2P-oD-1L-1	770.30	806.00	116	867	2	942.00	16.87	316	38	0	857.00	6.33	255	0	821.25	1.89
40O-2P-oD-1L-2	746.81	766.62	156	30	1	820.75	7.06	29	901	1	823.25	7.39	398	0	775.75	1.19
40O-2P-oD-1L-3	758.65	819.11	31	121	2	903.00	10.24	112	8	0	828.00	1.09	115	0	829.25	1.24
40O-3P-oD-1L-1	799.83	826.12	55	191	2	955.50	15.66	203	816	1	917.50	11.06	473	0	887.75	7.46
40O-3P-oD-1L-2	787.57	815.29	65	718	1	927.75	13.79	49	773	1	869.75	6.68	405	1	861.00	5.61
40O-3P-oD-1L-3	777.10	832.89	17	1095	2	910.00	9.26	248	340	1	875.00	5.06	603	1	871.00	4.58
40O-4P-oD-1L-1	784.15	817.70	121	1083	1	933.50	14.16	456	918	0	918.75	12.36	1078	1	883.50	8.05
40O-4P-oD-1L-2	777.41	805.67	66	661	1	879.75	9.19	170	6039	1	892.50	10.78	279	0	1005.00	24.74
40O-4P-oD-1L-3	764.08	818.60	10	3630	1	931.50	13.79	442	276	1	883.50	7.93	298	0	908.25	10.95
40O-2P-dD-1L-1	1022.12	1036.84	33	1	8	1395.75	34.62	44	6	0	1069.75	3.17	145	0	1053.00	1.56
40O-2P-dD-1L-2	915.15	943.47	3	12	2	1009.00	6.95	209	50	1	982.75	4.16	261	1	984.00	4.30
40O-2P-dD-1L-3	971.28	999.27	126	16	4	1162.00	16.28	98	99	1	1033.75	3.45	183	1	1024.00	2.47
40O-3P-dD-1L-1	1096.49	1115.88	65	29	1	1215.00	8.88	336	39	0	1146.75	2.77	485	0	1176.00	5.39
40O-3P-dD-1L-2	1042.41	1090.62	59	295	2	1141.25	4.64	443	977	1	1144.50	4.94	764	2	1136.25	4.18
40O-3P-dD-1L-3	1021.32	1044.30	117	6583	2	1240.50	18.79	78	539	0	1097.75	5.12	382	0	1071.50	2.60
40O-4P-dD-1L-1	1114.14	1138.17	69	167	2	1320.75	16.04	921	87	0	1241.50	9.08	869	0	1199.25	5.37
40O-4P-dD-1L-2	1029.34	1055.28	52	3582	2	1195.50	13.29	144	1968	1	1135.75	7.63	1266	1	1127.75	6.87
40O-4P-dD-1L-3	764.08	818.38	10	14400	0	932.50	13.94	288	3716	0	922.00	12.66	5414	0	880.50	7.59
40O-2P-oD-1L-1	713.20	731.00	309	55	0	787.25	7.69	906	43	0	764.00	4.51	174	0	760.00	3.97
40O-2P-oD-1L-2	695.96	714.69	772	42	0	738.00	3.26	923	112	0	748.50	4.73	944	0	745.75	4.35
40O-2P-oD-1L-3	700.90	**747.50**	22	27	0	763.00	2.07	31	122	0	770.00	3.01	177	0	762.50	2.01
40O-3P-oD-1L-1	742.26	760.06	338	156	1	836.00	9.99	755	39	0	789.75	3.91	3421	0	828.00	8.94
40O-3P-oD-1L-2	739.98	759.84	92	830	0	790.25	4.00	235	686	0	787.25	3.61	1057	0	780.50	2.72
40O-3P-oD-1L-3	719.66	767.48	29	136	0	802.50	4.56	98	1252	0	807.50	5.21	1337	0	821.50	7.04
40O-4P-oD-1L-1	735.32	752.87	972	353	0	792.25	5.23	1172	6382	0	866.25	15.06	3293	0	837.75	11.27
40O-4P-oD-1L-2	733.48	752.50	88	3011	0	792.75	5.35	478	782	0	795.75	5.75	1529	0	830.50	10.37
40O-4P-oD-1L-3	713.94	757.50	28	933	0	831.00	9.70	224	837	0	797.25	5.25	1402	0	827.25	9.21
40O-2P-dD-1L-1	966.58	979.50	83	59	0	991.00	1.17	128	5	0	980.50	0.10	542	0	999.50	2.04
40O-2P-dD-1L-2	856.14	**859.00**	16	93	0	881.50	2.62	35	18	0	867.50	0.99	348	0	879.00	2.33
40O-2P-dD-1L-3	916.67	936.76	305	70	0	953.50	1.79	668	66	0	951.00	1.52	384	0	938.25	0.16
40O-3P-dD-1L-1	1055.81	1068.71	242	194	0	1080.00	1.06	656	8	0	1071.50	0.26	1170	0	1083.75	1.41
40O-3P-dD-1L-2	977.36	988.49	418	719	0	1031.25	4.33	1806	632	0	1062.25	7.46	3370	0	1033.75	4.58
40O-3P-dD-1L-3	974.52	988.60	140	124	0	1028.50	4.04	483	600	0	1034.50	4.64	2842	0	1132.75	14.58
40O-4P-dD-1L-1	1080.50	1096.69	302	426	0	1154.25	5.25	1273	2218	0	1153.50	5.18	1800	0	1182.25	7.80
40O-4P-dD-1L-2	979.90	987.31	47	623	0	1050.75	6.43	234	2468	0	1035.00	4.83	2727	0	1071.25	8.50
40O-4P-dD-1L-3	713.94	754.28	50	1378	0	810.00	7.39	181	1599	0	795.75	5.50	807	0	795.75	5.50
Average		# opt.: 2	150.7	1186.4	1.03	970.27	9.1	413.1	985.0	0.28	936.60	5.6	1138.8	0.22	939.03	5.9

Table 20 Results for instances with 50 customers

	Exact		SEQ_H					E_SEQ_H					INT_H			
	Lower bound		Time (s)					Time (s)								
Instance	LP	End	VR	Prod	xWE	UB	Gap (%)	VR	Prod	xWE	UB	Gap (%)	Time (s)	xWE	UB	Gap (%)
50O-2P-oD-1L-1	921.72	957.60	3	2	2	1044.25	9.05	796	1	0	978.50	2.18	425	0	1003.50	4.79
50O-2P-oD-1L-2	859.14	891.82	721	31	2	999.50	12.07	2233	31	0	930.00	4.28	2092	0	940.50	5.46
50O-2P-oD-1L-3	905.39	946.53	54	36	2	1036.75	9.53	685	85	0	994.75	5.09	255	0	996.00	5.23
50O-3P-oD-1L-1	936.19	966.87	13	7	2	1080.75	11.78	900	286	0	1043.50	7.93	571	0	1015.00	4.98
50O-3P-oD-1L-2	900.84	928.55	486	1818	1	1036.50	11.63	391	6478	1	1000.25	7.72	682	1	1017.50	9.58
50O-3P-oD-1L-3	930.80	968.50	202	13	1	1039.00	7.28	566	222	1	1025.75	5.91	802	0	1028.25	6.17
50O-4P-oD-1L-1	923.51	953.53	2	1112	2	1077.25	12.97	1419	677	0	1037.00	8.75	1085	0	1031.00	8.12
50O-4P-oD-1L-2	891.91	914.06	444	13055	1	1027.00	12.36	243	1358	0	1049.75	14.84	11013	1	1021.00	11.70
50O-4P-oD-1L-3	922.30	955.72	186	2842	1	1093.50	14.42	550	568	0	1038.75	8.69	2389	0	1045.75	9.42
50O-2P-dD-1L-1	1197.18	1219.31	312	4	5	1470.25	20.58	985	96	1	1268.50	4.03	539	0	1294.25	6.15
50O-2P-dD-1L-2	1069.47	1093.82	131	540	5	1296.50	18.53	594	62	1	1129.75	3.28	426	0	1131.00	3.40
50O-2P-dD-1L-3	1123.12	1185.19	471	328	5	1376.50	16.14	1736	35	0	1269.75	7.13	2295	1	1429.75	20.63
50O-3P-dD-1L-1	1277.81	1311.71	200	835	3	1435.75	9.46	143	65	1	1398.25	6.60	1507	1	1335.25	1.79
50O-3P-dD-1L-2	1206.15	1247.01	348	789	5	1429.75	14.65	411	2615	1	1296.25	3.95	807	1	1321.25	5.95
50O-3P-dD-1L-3	1208.99	1238.06	131	12497	4	1449.25	17.06	817	4823	1	1345.50	8.68	1188	1	1462.00	18.09
50O-4P-dD-1L-1	1291.80	1311.39	288	1651	1	1475.50	12.51	779	6560	0	1464.00	11.64	1473	1	1393.00	6.22
50O-4P-dD-1L-2	1193.36	1211.22	181	5612	3	1357.50	12.08	704	10412	0	1327.00	9.56	3348	1	1322.25	9.17
50O-4P-dD-1L-3	1200.85	1227.65	23	14400	3	1388.00	13.06	1674	4575	1	1308.00	6.55	2162	1	1313.50	6.99
50O-2P-oD-2L-1	865.78	875.50	48	160	0	904.75	3.34	123	14	0	898.25	2.60	460	0	893.25	2.03
50O-2P-oD-2L-2	806.46	822.00	22	47	0	839.75	2.16	882	12	0	843.00	2.55	337	0	841.50	2.37
50O-2P-oD-2L-3	825.25	853.83	32	201	0	892.25	4.50	105	138	0	901.75	5.61	477	0	896.75	5.03
50O-3P-oD-2L-1	883.78	886.81	23	1088	0	947.25	6.82	124	661	0	938.75	5.86	2336	0	935.25	5.46
50O-3P-oD-2L-2	858.08	865.25	43	656	0	897.50	3.73	48	667	0	920.00	6.33	1103	0	882.25	1.96
50O-3P-oD-2L-3	860.94	891.77	84	960	0	955.75	7.17	365	1801	0	955.75	7.17	2347	0	948.75	6.39
50O-4P-oD-2L-1	877.43	886.60	26	2142	0	965.25	8.87	150	2599	0	976.25	10.11	2635	0	936.25	5.60
50O-4P-oD-2L-2	852.08	860.00	43	1101	0	905.00	5.23	57	362	0	916.50	6.57	1979	0	912.50	6.10
50O-4P-oD-2L-3	845.94	872.75	125	4191	0	984.75	12.83	355	10703	0	948.25	8.65	2855	0	951.75	9.05
50O-2P-dD-2L-1	1142.94	1154.12	851	72	0	1189.25	3.04	1141	42	0	1175.25	1.83	1170	0	1170.00	1.38
50O-2P-dD-2L-2	1011.76	1021.52	153	24	0	1044.50	2.25	364	55	0	1038.50	1.66	636	0	1065.00	4.26
50O-2P-dD-2L-3	1065.33	1091.95	854	198	0	1125.25	3.05	1896	89	0	1111.75	1.81	1821	0	1169.75	7.12
50O-3P-dD-2L-1	1220.72	1226.75	288	1018	0	1247.25	1.67	1307	422	0	1242.00	1.24	2041	0	1241.50	1.20
50O-3P-dD-2L-2	1150.83	1161.39	578	932	0	1221.25	5.15	775	638	0	1195.75	2.96	9734	0	1221.25	5.15
50O-3P-dD-2L-3	1158.47	1183.21	308	2903	0	1216.50	2.81	1018	263	0	1205.75	1.90	14311	0	1274.00	7.67
50O-4P-dD-2L-1	1255.67	1264.09	816	6406	0	1327.00	4.98	2504	2761	0	1327.00	4.98	2499	0	1297.00	2.60
50O-4P-dD-2L-2	1146.48	1159.93	662	666	0	1216.25	4.86	2285	533	0	1222.25	5.37	14400	0	1234.75	6.45
50O-4P-dD-2L-3	1150.38	1174.65	360	3985	0	1241.50	5.69	901	2038	0	1207.50	2.80	4075	0	1276.25	8.65
Average		# opt.: 0	264.2	2286.7	1.33	1145.4	9.0	834.1	1743.0	0.22	1109.2	5.7	2729.9	0.25	1118.0	6.5