

## **Electronic Companion.**

**“An Exact Algorithm for the Pickup and Delivery Problem with Time Windows and LIFO Loading” by Ali Mehsin Alyasiry, Michael Forbes, and Michael Bulmer.**

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## Appendix A: Computational Results setting $\delta = 1$

To evaluate the impact of the parameter  $\delta$  we provide computational results setting  $\delta = 1$  and time limit to 3,600 seconds. The columns of tables EC.1, EC.2, EC.3 and EC.4 represent the following: *Name*: the instances name;  $Z_R$ : the lower bound at the root node (i.e., the LP relaxation of PDPTWL-R augmented with the integer lower bound on the number of vehicles); *Time*: the total computing time in seconds;  $Z^*$ : the optimal solution obtained, otherwise we report, in bold, the best upper bound (feasible solution) obtained or left blank if no feasible solution found within the time limit;  $Z_{LB}$ : the best lower bound (LB relaxation) at a child node if the instance was not solved to optimality within the time limit, left blank otherwise;  $F_N$ : the total number of original fragments;  $TF_N$ : the total number of undominated timed fragments; *Nodes*: the number of nodes that have been explored in the branch and bound tree;  $LC$ : the total number of lazy constraints used in any solution.

Table EC.1: Computational results ( $\delta = 1$ )

<i>Name</i>	$F_N$	$TF_N$	$Z_R$	$Z_{LB}$	<i>Nodes</i>	$LC$	<i>Time</i>	$Z^*$
AA30	176	1,782	31,129.13		0	0	1.3	31,129.13
AA35	294	2,690	31,284.77		0	0	1.2	31,293.74
AA40	368	3,159	41,348.80		0	0	1.0	41,348.80
AA45	623	4,115	41,520.87		0	0	1.5	41,520.87
AA50	768	5,112	41,643.10		0	0	2.1	41,643.10
AA55	935	6,160	51,742.52		0	0	2.6	51,742.52
AA60	1,015	6,759	51,927.76		0	0	6.8	51,949.04
AA65	1,118	7,464	52,059.30		0	1	6.9	52,076.67
AA70	1,469	9,151	52,210.63		0	0	7.2	52,218.44
AA75	2,498	13,337	52,326.92		0	0	11.4	52,329.30
BB30	145	1,158	31,074.09		0	1	1.2	31,077.12
BB35	267	1,936	31,310.77		0	1	1.0	31,311.97
BB40	342	2,564	41,397.36		0	0	2.4	41,403.49
BB45	540	3,780	41,519.72		90	0	6.8	41,536.99
BB50	706	4,925	41,774.53		0	1	3.5	41,790.49
BB55	799	5,565	51,894.39		178	1	11.2	51,911.05
BB60	742	5,143	62,280.82		0	3	4.0	62,304.78
BB65	801	5,606	62,514.38		13	2	9.8	62,563.85
BB70	997	6,855	72,528.06		0	0	15.4	72,534.42
BB75	1,446	8,805	72,644.69		34	0	22.2	72,655.87
CC30	241	9,814	31,086.67		0	0	14.2	31,088.20
CC35	438	14,833	31,232.69		0	0	24.9	31,237.04
CC40	1,021	28,693	31,335.62		82	0	93.8	31,339.79
CC45	2,359	53,865	31,487.28	31,525.87	23,735	13	3,600.0	<b>31,532.09</b>
CC50	3,813	76,858	41,662.26		1,172	1	1,113.6	41,673.05
CC55	5,294	98,541	41,776.14		2,056	4	1,267.4	41,792.84
CC60	6,589	120,878	41,934.61		2,163	0	1,280.6	41,946.63
CC65	9,243	160,111	42,056.52	42,058.98	2,749	0		
CC70	13,888	221,355	42,243.43	42,244.32	1,433	0		
CC75	19,480	302,256	52,283.07	52,294.88	1,951	5	3,600.0	<b>52,316.09</b>
DD30	1,437	25,383	21,079.16		2,464	1	406.7	21,102.82
DD35	3,052	49,033	21,340.89		31	0	123.3	31,127.43
DD40	5,196	79,021	31,242.14		0	0	23.9	31,244.79
DD45	8,940	124,040	31,344.15		0	0	50.1	31,349.87
DD50	16,064	211,385	31,438.30		119	2	194.4	31,449.70
DD55	25,271	316,750	31,590.32	31,596.03	1,440	11		
DD60	39,687	466,993	31,768.22	31,778.29	1,031	4	3,600.0	<b>31,814.13</b>
DD65	25,579	306,298	41,967.35		446	0	1,381.1	41,984.09
DD70	40,855	451,227	42,067.83		1,744	2	2,387.4	42,092.28
DD75	62,468	640,506	42,190.34	42,190.34	172	2		

Table EC.2: Computational results ( $\delta = 1$ )

<i>Name</i>	$F_N$	$TF_N$	$Z_R$	$Z_{LB}$	<i>Nodes</i>	<i>LC</i>	<i>Time</i>	$Z^*$
AA*30	271	287	31,051.08		0	0	1.2	31,051.20
AA*35	415	448	31,231.30		0	0	0.9	31,244.37
AA*40	569	604	41,318.38		0	0	0.9	41,330.99
AA*45	882	922	41,505.21		0	0	2.1	41,514.90
AA*50	1,073	1,124	41,612.97		25	0	2.3	41,636.97
AA*55	1,283	1,342	41,842.35		0	0	3.9	41,879.62
AA*60	1,381	1,447	51,807.57		0	0	2.3	51,807.57
AA*65	1,522	1,591	51,952.25		0	1	3.9	51,961.24
AA*70	1,939	2,027	52,137.83		268	3	10.0	52,170.77
AA*75	3,116	3,217	52,265.70		311	6	14.5	52,298.84
BB*30	169	180	41,105.43		0	0	0.6	41,110.65
BB*35	307	327	41,329.09		11	1	1.0	41,332.47
BB*40	381	404	41,463.74		0	0	1.4	41,476.65
BB*45	596	629	41,645.10		21	2	2.3	41,698.98
BB*50	773	814	51,681.97		133	0	3.4	51,718.58
BB*55	926	971	51,937.74		1,395	5	16.8	52,033.90
BB*60	1,025	1,070	72,168.09		0	2	4.0	72,183.56
BB*65	1,111	1,158	72,346.54		174	3	5.0	72,393.77
BB*70	1,383	1,444	72,555.38		175	2	7.4	72,604.07
BB*75	1,930	1,997	72,675.02		180	6	10.6	72,746.85

Table EC.3: Computational results ( $\delta = 1$ )

<i>Name</i>	$F_N$	$TF_N$	$Z_R$	$Z_{LB}$	<i>Nodes</i>	<i>LC</i>	<i>Time</i>	$Z^*$
AA80	1,826	11,260	62,520.77		134	3	27.4	62,541.33
AA85	2,508	15,060	72,696.41		31	0	24.9	72,703.23
AA90	2,492	17,136	63,154.47		1,422	9	192.3	72,924.18
AA95	3,117	19,874	63,147.64		7,061	37	1,171.0	63,260.65
AA100	2,747	17,529	73,384.93		1,436	10	153.2	73,424.04
AA105	4,194	23,189	83,495.18		1,323	0	128.5	83,522.13
AA110	3,561	20,724	83,253.67		436	3	140.1	83,268.80
AA115	4,729	26,380	93,664.54		354	0	89.8	93,690.44
AA120	3,790	24,368	103,951.74		232	5	61.2	103,963.13
AA125	4,829	29,041	93,827.53		441	3	116.2	93,840.61
BB80	2,356	14,143	62,709.36		1,210	7	240.5	62,743.40
BB85	2,372	14,328	62,651.09		1,396	2	159.1	62,688.02
BB90	2,773	16,251	82,969.78		341	1	36.7	82,985.83
BB95	3,016	19,509	63,237.19		1,153	3	266.8	73,005.02
BB100	3,334	21,613	73,026.31		761	2	112.7	73,059.83
BB105	8,852	47,421	72,979.73		3,669	20	858.4	73,032.72
BB110	5,087	27,332	73,463.63		2,970	5	722.3	73,543.43
BB115	4,811	28,277	83,750.81		3,255	25	1,086.8	83,812.37
BB120	9,500	48,836	83,650.85		2,631	14	1,335.4	83,700.24
BB125	6,480	36,196	93,825.12		2,258	3	606.4	93,868.50

Table EC.4: Computational results ( $\delta = 1$ )

<i>Name</i>	$F_N$	$TF_N$	$Z_R$	$Z_{LB}$	<i>Nodes</i>	<i>LC</i>	<i>Time</i>	$Z^*$
AA*80	2,531	2,617	62,549.18		616	10	33.9	62,588.19
AA*85	3,565	3,700	72,589.79		2,385	3	40.1	72,633.39
AA*90	3,413	3,681	72,869.41		1,237	23	124.9	72,923.07
AA*95	4,447	4,673	72,923.12		2,072	5	175.7	72,996.92
AA*100	4,076	4,234	83,240.77		30	0	17.9	83,254.93
AA*105	5,638	5,874	83,270.80		1,351	12	101.9	83,349.25
AA*110	5,315	5,575	83,173.48		1,436	7	103.0	83,232.45
AA*115	7,171	7,445	93,511.66		889	9	124.1	93,577.39
AA*120	5,668	5,927	93,857.13		1,173	4	120.2	93,899.67
AA*125	7,273	7,530	83,738.36		7,254	13	926.8	83,844.43
BB*80	2,803	2,967	62,803.09		466	4	36.8	72,556.05
BB*85	2,769	2,892	62,633.73		1,332	3	52.6	62,718.69
BB*90	3,439	3,596	72,970.80		806	3	68.5	73,075.64
BB*95	3,856	4,081	72,972.30		511	0	30.1	73,001.11
BB*100	4,385	4,621	73,121.12		500	4	64.4	73,210.37
BB*105	10,206	10,537	73,014.55		5,450	21	838.2	73,132.54
BB*110	6,110	6,381	83,421.67		6,940	35	737.9	83,540.28
BB*115	6,699	6,944	93,581.56		1,135	0	107.5	93,619.79
BB*120	11,417	11,984	83,564.85	83,686.01	14,864	14	3,600.0	<b>83,714.11</b>
BB*125	8,334	8,682	93,783.97		3,946	8	473.6	93,883.89