A Supplementary material

Theorem 8. *Consider timetabling instances* $I = (\mathcal{N} = (\mathcal{E}, \mathcal{A}), T, L, U, \mathrm{OD})$ *with* $U_a - L_a \leq T - 1$, $a \in \mathcal{A}$, *and* $L_a \geq 1$ *for all* $a \in \mathcal{A}$ *if* $L_a \neq U_a$. *Let* π_I *be an optimal solution to PESP for timetabling instance* I *and* π_I^* *an optimal solution for* (TimPass) *for the same instance. Then we get*

$$\inf_I \frac{\mathcal{R}_{\mathrm{LB}}(\pi_I)}{\mathcal{R}_{\mathrm{SP}}(\pi_I^*)} \geq \inf_I \frac{\mathcal{R}_{\mathrm{SP}}(\pi_I)}{\mathcal{R}_{\mathrm{SP}}(\pi_I^*)} \geq T - 1.$$

Proof. The first inequality is satisfied as $\mathcal{R}_{\mathrm{LB}}(\pi_I) \geq \mathcal{R}_{\mathrm{SP}}(\pi_I)$ due to Lemma 6. To show the second inequality, we consider timetabling instance for the EAN given in Figure 12 with K passenger traveling from station v_1 to station v_2 and one passenger traveling from station v_2 to station v_3. For PESP, we consider the following fixed routes on shortest paths w.r.t the lower bounds L: OD pair (v_1, v_2) uses line l_2, i.e., path $P_1 = ((v_1, l_2, \mathrm{dep}), (v_2, l_2, \mathrm{arr}))$ with length $\mathrm{len}(P_1, L) = 1$, while OD pair (v_2, v_3) uses line l_3, i.e., path $P_2 = ((v_2, l_3, \mathrm{dep}), (v_3, l_3, \mathrm{arr}))$ with length $\mathrm{len}(P_2, L) = 1$. However, for any feasible timetable π we get $\mathrm{len}(P_1, d(\pi)) = T - 1$. For the alternative routes for OD pair (v_1, v_2), i.e., path P_1' using line l_1 and path P_1'' using line l_3 we get $\mathrm{len}(P_1', d(\pi)) = 1$ and $\mathrm{len}(P_1'', d(\pi)) = T - \mathrm{len}(P_1, d(\pi))$. But for any optimal solution π' of PESP where there is positive weight on path P_2 but no weight on the activity of path P_1'', π satisfies $\mathrm{len}(P_2, d(\pi')) = 1$ and $\mathrm{len}(P_1'', d(\pi')) = T - 1$. Thus, any optimal solution π_I of PESP satisfies

$$\mathcal{R}_{\mathrm{SP}}(\pi_I) = (T - 1) \cdot K + 1 \cdot 1.$$

However, in an optimal solution π_I^* of (TimPass), OD pair (v_1, v_2) chooses path P_1'' with length $\mathrm{len}(P_1'', d(\pi_I^*)) = 1$ and OD pair (v_2, v_3) chooses path $P_2' = ((v_2, l_2, \mathrm{dep}), (v_3, l_2, \mathrm{arr}))$ with $\mathrm{len}(P_2', d(\pi_I^*)) = 1$, i.e.,

$$\mathcal{R}_{\mathrm{SP}}(\pi_I^*) = 1 \cdot K + 1 \cdot 1.$$

We therefore get

$$\frac{\mathcal{R}_{\mathrm{SP}}(\pi_I)}{\mathcal{R}_{\mathrm{SP}}(\pi_I^*)} = \frac{(T - 1) \cdot K + 1 \cdot 1}{1 \cdot K + 1 \cdot 1} \overset{k \to \infty}{\to} T - 1.$$

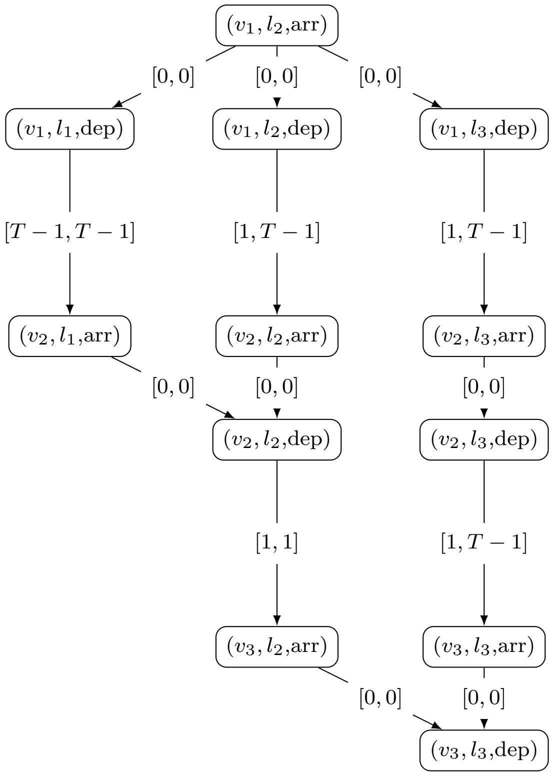


Figure 12: EAN $\mathcal{N}^0 = (\mathcal{E}^0, \mathcal{A}^0)$. Lower and upper bounds on the duration of the activities are given on the arcs.

□

In the following Example 19 we show that although Heuristic UB outperforms Heuristic LB in practice, there are cases where the rerouted travel time $\mathcal{R}_{\mathrm{SP}}(\pi)$ of the solution π found by Heuristic LB is shorter than the one found by Heuristic UB.

Example 19. Consider the event-activity network depicted in Figure 13 with two OD pairs, (A,D) with K_1 passengers and (B,C) with K_2 passengers.

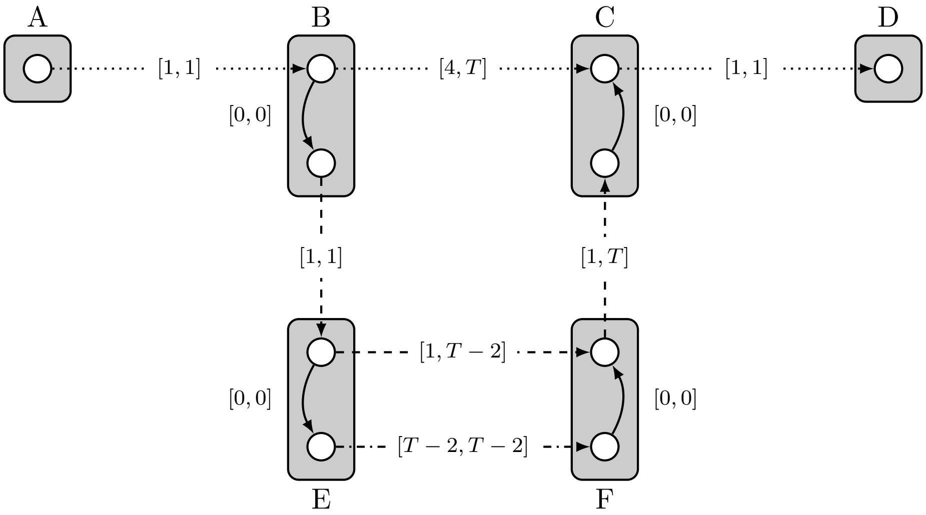
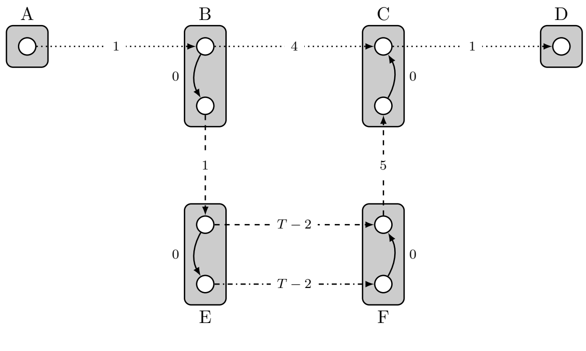
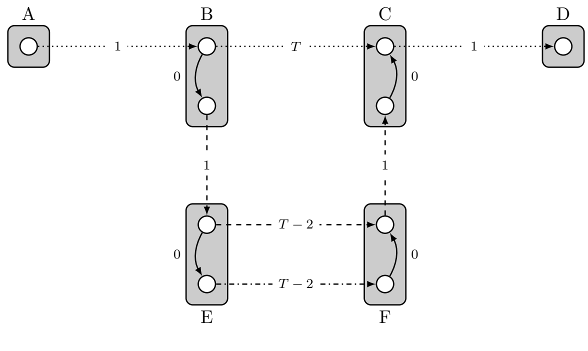


Figure 13: EAN $\mathcal{N}^0 = (\mathcal{E}^0, \mathcal{A}^0)$ for stations A, ..., F and three lines (A,B,C,D), (B,E,F,C) and (E,F). As minimal and maximal waiting times at all stations are fixed to 0, only one node is given for arrival and departure events. Lower and upper bounds on the duration of the activities are given on the arcs.

Note that due to the structure of the EAN, only two decisions can be made, namely the length the drive activity between B and C and the length of the drive activity between F and C. If we assume that the departure/arrival event at station A is fixed to 0, there are only two timetables that could be optimal for the considered OD pairs, namely minimizing the drive time between B and C or minimizing the drive time between F and C. We call the corresponding timetables π^{BC} and π^{FC}, respectively. The corresponding activity durations are given in Figure 14.



(a) Activity durations for timetable π^{BC}, i.e., when the drive time between B and C is minimized.



(b) Activity durations for timetable π^{FC}, i.e., when the drive time between F and C is minimized.

Figure 14: Activity durations for timetables π^{BC} and π^{FC}.

For both OD pairs, there are two paths that could be shortest paths according to activity durations L or $d(\pi)$ for a feasible timetable π, namely either using the dotted line between B and C or using the dashed line from B to C via E and F. The path lengths according to L,

$d(\pi^{BC})$ and $d(\pi^{FC})$ are given in the following Table 5.

	$\text{len}(P, L)$	$\text{len}(P, d(\pi^{BC}))$	$\text{len}(P, d(\pi^{FC}))$
P_1 =(A,B,C,D)	6	$\mathbf{6}$	$\mathbf{T+2}$
P_2 =(A,B,E,F,C,D)	$\mathbf{5}$	$T+6$	$\mathbf{T+2}$
P_3 =(B,C)	4	$\mathbf{4}$	$\mathbf{T}$
P_4 =(B,E,F,C)	$\mathbf{3}$	$T+4$	$\mathbf{T}$

Table 5: Length of the potentially shortest paths according to L, $d(\pi^{BC})$ and $d(\pi^{FC})$. The shortest path lengths for each activity duration are marked in bold.

From Table 5 we get that $\text{SP}_{\text{A,D}}(L) = P_2$ and $\text{SP}_{\text{B,C}}(L) = P_4$.

We compare the classical PESP, Heuristic LB and Heuristic UB for routing $\text{OD}_{\text{route}} = \{(A, D)\}$ and (TimPass) according to the shortest path routing $\mathcal{R}_{\text{SP}}(\pi)$ for period length $T > 4$.

PESP: The objective function of PESP is

$$\mathcal{R}_{\text{LB}}(\pi) = K_1 \cdot \text{len}(\text{SP}_{\text{A,D}}(L), d(\pi)) + K_2 \cdot \text{len}(\text{SP}_{\text{B,C}}(L), d(\pi))$$

Comparing the objective values for both timetables, we get

$$\mathcal{R}_{\text{LB}}(\pi^{BC}) = K_1 \cdot (T+6) + K_2 \cdot (T+4) > K_1 \cdot (T+2) + K_2 \cdot T = \mathcal{R}_{\text{LB}}(\pi^{FC}).$$

We therefore get that π^{FC} is optimal with rerouted travel time $\mathcal{R}_{\text{SP}}(\pi^{FC}) = K_1 \cdot (T+2) + K_2 \cdot T$.

Heuristic LB: For routing $\text{OD}_{\text{route}} = \{(A, D)\}$, the objective of Heuristic LB is

$$\begin{aligned} h(\text{OD}_{\text{route}}, \pi) &= \mathcal{R}_{\text{SP}}(\text{OD}_{\text{route}}, \pi) \\ &= K_1 \cdot \text{len}(\text{SP}_{\text{A,D}}(d(\pi), d(\pi)). \end{aligned}$$

Comparing the objective values for both timetables, we get

$$h(\text{OD}_{\text{route}}, \pi^{BC}) = K_1 \cdot 6 < K_1 \cdot (T+2) = \mathcal{R}_{\text{SP}}(\text{OD}_{\text{route}}, \pi^{FC})$$

We therefore get that π^{BC} is optimal with rerouted travel time $\mathcal{R}_{\text{SP}}(\pi^{BC}) = K_1 \cdot 6 + K_2 \cdot 4$.

Heuristic UB: For routing $\mathrm{OD}_{\mathrm{route}} = \{(A, D)\}$, the objective of Heuristic UB is

$$\begin{aligned} f(\mathrm{OD}_{\mathrm{route}}, \pi) &= \mathcal{R}_{\mathrm{SP}}(\mathrm{OD}_{\mathrm{route}}, \pi) + \mathcal{R}_{\mathrm{LB}}(\mathrm{OD} \setminus \mathrm{OD}_{\mathrm{route}}, \pi) \\ &= K_1 \cdot \mathrm{len}(\mathrm{SP}_{\mathrm{A,D}}(d(\pi), d(\pi)) + K_2 \cdot \mathrm{len}(\mathrm{SP}_{\mathrm{B,C}}(L), d(\pi)). \end{aligned}$$

Comparing the objective values for both timetables, we get

$$\begin{aligned} f(\mathrm{OD}_{\mathrm{route}}, \pi^{BC}) &= K_1 \cdot 6 + K_2 \cdot (T + 4) \\ f(\mathrm{OD}_{\mathrm{route}}, \pi^{FC}) &= K_1 \cdot (T + 2) + K_2 \cdot T. \end{aligned}$$

For $K_2 \cdot 4 > K_1 \cdot (T - 4)$ we get

$$f(\mathrm{OD}_{\mathrm{route}}, \pi^{BC}) > f(\mathrm{OD}_{\mathrm{route}}, \pi^{FC}).$$

We therefore get that π^{FC} is optimal with rerouted travel time $\mathcal{R}_{\mathrm{SP}}(\pi^{FC}) = K_1 \cdot (T+2) + K_2 \cdot T$.

(TimPass): The objective function of (TimPass) is

$$\mathcal{R}_{\mathrm{SP}}(\pi) = K_1 \cdot \mathrm{len}(\mathrm{SP}_{\mathrm{A,D}}(d(\pi)), d(\pi)) + K_2 \cdot \mathrm{len}(\mathrm{SP}_{\mathrm{B,C}}(d(\pi)), d(\pi))$$

Comparing the objective values for both timetables, we get

$$\mathcal{R}_{\mathrm{SP}}(\pi^{BC}) = K_1 \cdot 6 + K_2 \cdot 4 < K_1 \cdot (T + 2) + K_2 \cdot T = \mathcal{R}_{\mathrm{SP}}(\pi^{FC}).$$

We therefore get that π^{BC} is optimal with rerouted travel time $\mathcal{R}_{\mathrm{SP}}(\pi^{BC}) = K_1 \cdot 6 + K_2 \cdot 4$. In case that $K_2 \cdot 4 > K_1 \cdot (T - 4)$ is satisfied, we get that Heuristic LB finds the optimal solution of (TimPass) while Heuristic UB finds the same solution as PESP which has a strictly longer rerouted travel time.