The Commute Trip Sharing Problem (Online Supplement)

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Appendix. Computational Results

Results of the REA are presented in Table 1. The first three columns show the cluster size, vehicle capacity, and cluster ID which characterize each problem instance. The next column lists the number of columns generated by the algorithm, while the following three show the results of solving the MP as a MIP. They show the vehicle count, the total distance of selected routes, and the optimality gap of the MIP solution. Finally, the remaining two columns display computation times of the route enumeration phase and of the entire algorithm including MIP solve times.

Table 2 shows results of the BPA. Similar to Table 1, the first three columns list the cluster size, vehicle capacity, and cluster ID for each problem instance. The next two columns present the total number of unique feasible edges from all inbound and outbound graphs to further characterize the size of the problem instances, while the following two show the total number of tree nodes explored and columns generated by the algorithm. The next two display the results in terms of the vehicle count and the total distance of selected routes. The following two columns list optimality gaps of the MIP solution at the root node and of the best feasible solution, while the next lists the integrality gap of the best feasible solution, which is the relative gap between its objective value and z^*. Finally, the remaining four columns show computation times for the RMP to converge, for finding the MIP solution at the root node, for arriving at the best feasible solution, and for executing the entire algorithm.

Results for the instances testing scalability are summarized in Tables 3 and 4 for the REA and the BPA respectively. Both tables list the same quantities as those listed in Tables 1 and 2 respectively.

The results for the root-node heuristic are summarized in Table 5. The first two columns show the cluster size and ID of each problem instance. The next set of five columns list the results of the heuristic which enforces forbidden paths. They show the number of columns generated, the resulting vehicle count, its optimality gap, and the times spent on solving the RMP and its MIP. The next set of six columns display the same information for the heuristic which relaxes forbidden paths, with an additional column showing the number of infeasible columns it generated.

Table 1 Results of REA Scalability with Increasing Vehicle Capacity ($\Delta = 10$ mins, $R = 0.5$).

Cluster size	Vehicle capacity	Cluster ID	Column #	Vehicle #	Total distance (m)	Optimality gap (%)	Wall time (s)	
							Route enumeration	Total
75	4	C0-75	508	47	356296	0.00	11	11
		C2-75	1946	40	643541	0.00	9	9
		C3-75	2068	38	487061	0.00	11	11
		C4-75	1483	43	398265	0.00	10	10
		C5-75	274	61	230460	0.00	9	9
		C6-75	3071	36	451262	0.00	11	12
		C7-75	690	46	437743	0.00	10	10
		C8-75	1350	37	487561	0.00	10	10
		C9-75	3926	31	328525	0.00	10	37
		C10-75	4137	32	527544	0.00	13	14
		C11-75	866	47	443770	0.00	12	12
		C12-75	592	46	298162	0.00	11	11
		C13-75	2143	37	481869	0.00	12	12
		C14-75	863	47	301108	0.00	10	10
		C15-75	1512	36	460087	0.00	10	10
		C17-75	457	50	348581	0.00	9	9
		C19-75	496	57	245827	0.00	10	10
		C20-75	752	46	420825	0.00	10	10
		C22-75	2288	40	472247	0.00	10	11
		C23-75	1574	36	385434	0.00	9	10
		C24-75	1926	37	425807	0.00	11	12
		C26-75	468	52	333692	0.00	9	9
		C28-75	750	44	293838	0.00	10	10
		C29-75	2541	32	548779	0.00	10	11
	5	C0-75	509	47	356296	0.00	306	306
		C2-75	2014	40	643540	0.00	316	316
		C3-75	2117	38	487061	0.00	341	342
		C4-75	1559	43	398216	0.00	326	326
		C5-75	274	61	230460	0.00	264	264
		C6-75	3331	35	450414	0.00	355	356
		C7-75	690	46	437743	0.00	316	316
		C8-75	1352	37	487245	0.00	335	335
		C9-75	4110	31	327993	0.00	346	378
		C10-75	5589	32	526188	0.00	352	354
		C11-75	870	47	443770	0.00	300	300
		C12-75	592	46	298162	0.00	308	308
		C13-75	2207	37	479166	0.00	351	351
		C14-75	867	47	301108	0.00	304	304
		C15-75	1522	36	460087	0.00	335	335
		C17-75	457	50	348581	0.00	293	293
		C19-75	496	57	245827	0.00	279	279
		C20-75	752	46	420825	0.00	309	309
		C22-75	2344	40	472040	0.00	347	348
		C23-75	1628	35	379234	0.00	306	307
		C24-75	1996	37	425806	0.00	339	339
		C26-75	468	52	333692	0.00	292	292
		C28-75	751	44	293838	0.00	315	315
		C29-75	2594	32	543850	0.00	350	351
	6	C0-75	509	47	356296	0.00	4761	4761
		C2-75	2016	40	643540	0.00	5109	5110
		C3-75	2120	38	487061	0.00	6009	6009
		C4-75	1562	43	398216	0.00	5375	5375
		C5-75	274	61	230460	0.00	3220	3220
		C6-75	3406	35	450414	0.00	6341	6341
		C7-75	690	46	437743	0.00	6135	6135
		C8-75	1352	37	487245	0.00	7172	7172
		C9-75	4116	31	327993	0.00	7893	7916
		C10-75	6571	32	525115	0.00	7480	7482
		C11-75	870	47	443770	0.00	5413	5413
		C12-75	592	46	298162	0.00	5206	5206
		C13-75	2209	37	479166	0.00	6164	6164
		C14-75	867	47	301108	0.00	4658	4658
		C15-75	1522	36	460087	0.00	5668	5668
		C17-75	457	50	348581	0.00	4147	4147
		C19-75	496	57	245827	0.00	4000	4000
		C20-75	752	46	420825	0.00	4983	4983
		C22-75	2345	40	472040	0.00	5961	5961
		C23-75	1629	35	379234	0.00	5008	5008
		C24-75	2001	37	425806	0.00	5843	5844
		C26-75	468	52	333692	0.00	4254	4254
		C28-75	751	44	293838	0.00	4935	4935
		C29-75	2594	32	543850	0.00	6071	6071

Cluster size	Vehicle capacity	Cluster ID	Column #	Vehicle #	Total distance (m)	Optimality gap (%)	Wall time (s)	
							Route enumeration	Total
100	4	C0-100	854	63	488119	0.00	34	34
		C1-100	456	75	331497	0.00	31	31
		C2-100	3802	46	596824	0.00	36	36
		C3-100	4510	46	586434	0.00	35	36
		C4-100	4232	44	558323	0.00	35	41
		C5-100	732	70	366565	0.00	31	31
		C6-100	2579	47	724844	0.00	35	35
		C7-100	2020	53	661648	0.00	34	34
		C8-100	1803	51	609120	0.00	34	34
		C9-100	12034	40	527217	0.00	36	38
		C10-100	1095	58	426386	0.00	34	34
		C11-100	2374	51	427829	0.00	34	35
		C12-100	989	62	413012	0.00	33	33
		C13-100	909	61	483087	0.00	34	34
		C14-100	4306	40	693825	0.00	35	40
		C15-100	4605	48	790005	0.00	34	37
		C16-100	1578	55	627423	0.00	33	33
		C17-100	1151	60	640945	0.00	33	33
		C18-100	952	58	681240	0.00	34	34
		C19-100	2226	51	503252	0.00	34	35
		C20-100	4254	46	569724	0.00	35	36
		C21-100	667	78	328216	0.00	32	32
	5	C0-100	854	63	488119	0.00	1185	1186
		C1-100	456	75	331497	0.00	1072	1072
		C2-100	3910	46	596658	0.00	1356	1357
		C3-100	4814	46	584077	0.00	1357	1359
		C4-100	4487	44	552862	0.00	1298	1342
		C5-100	735	70	366565	0.00	1034	1034
		C6-100	2596	47	724844	0.00	1362	1362
		C7-100	2043	53	661528	0.00	1326	1326
		C8-100	1842	51	609038	0.00	1317	1317
		C9-100	16749	40	519564	0.00	1419	1451
		C10-100	1096	58	426386	0.00	1254	1254
		C11-100	2418	51	427829	0.00	1320	1321
		C12-100	990	62	413012	0.00	1208	1208
		C13-100	912	61	483087	0.00	1221	1221
		C14-100	4448	40	689319	0.00	1383	1457
		C15-100	5985	48	788340	0.00	1287	1294
		C16-100	1599	55	626787	0.00	1226	1227
		C17-100	1155	60	640945	0.00	1218	1218
		C18-100	953	58	681240	0.00	1292	1293
		C19-100	2291	51	503252	0.00	1352	1352
		C20-100	4761	46	568797	0.00	1352	1354
		C21-100	667	78	328216	0.00	1080	1080
	6	C0-100	854	63	488119	0.00	25088	25088
		C1-100	456	75	331497	0.00	20117	20117
		C2-100	3917	46	596658	0.00	33455	33456
		C3-100	4872	46	584048	0.00	33801	33802
		C4-100	4510	44	552862	0.00	29993	30180
		C5-100	735	70	366565	0.00	18613	18613
		C6-100	2598	47	724844	0.00	43135	43136
		C7-100	2043	53	661528	0.00	38791	38792
		C8-100	1848	51	609038	0.00	39437	39437
		C9-100	18758	40	517624	0.00	48359	48403
		C10-100	1096	58	426386	0.00	36941	36941
		C11-100	2419	51	427829	0.00	39761	39763
		C12-100	990	62	413012	0.00	31038	31038
		C13-100	912	61	483087	0.00	33881	33881
		C14-100	4456	40	689319	0.00	41455	41492
		C15-100	6939	48	788340	0.00	36980	36990
		C16-100	1600	55	626787	0.00	32031	32031
		C17-100	1155	60	640945	0.00	34810	34810
		C18-100	953	58	681240	0.00	38501	38501
		C19-100	2294	51	503252	0.00	42573	42574
		C20-100	4898	46	568797	0.00	43900	43903
		C21-100	667	78	328216	0.00	29507	29507

Table 2 Results of BPA Scalability with Increasing Vehicle Capacity ($\Delta = 10$ mins, $R = 0.5$).

Cluster size	Vehicle capacity	Cluster ID	Inbound edge #	Outbound edge #	Tree node #	Column #	Vehicle #	Total distance (m)	Optimality gap (%)		Integrality gap (%)	Wall time (s)			
									Root node soln.	Best feasible soln.		RMP conv.	Root node soln.	Best feasible soln.	Total
75	4	C0-75	1661	1565	15	462	47	356296	0.00	0.00	0.00	1	1	1	1
		C2-75	1906	1405	57	1043	40	643541	0.01	0.00	0.01	2	2	7	12
		C3-75	3099	1809	31	1247	38	487061	0.00	0.00	0.00	2	2	12	13
		C4-75	2052	1851	79	915	43	398265	0.01	0.00	0.01	1	1	8	12
		C5-75	823	810	1	267	61	230460	0.00	0.00	0.00	1	1	1	1
		C6-75	3427	2012	17	1420	36	451262	2.77	0.00	2.77	4	5	5	16
		C7-75	2458	1649	25	615	46	437743	2.17	0.00	2.17	1	1	1	2
		C8-75	3002	2183	27	1028	37	487561	2.69	0.00	2.69	2	2	11	15
		C9-75	3550	2341	428	2613	31	328525	3.22	0.00	3.21	7	8	493	528
		C10-75	2856	1580	151	2085	32	527544	3.11	0.00	3.11	11	11	168	232
		C11-75	1955	1193	23	639	47	443770	0.00	0.00	0.00	1	1	1	2
		C12-75	1801	1347	5	501	46	298162	0.00	0.00	0.00	1	1	1	1
		C13-75	2699	1302	141	1392	37	481869	2.70	0.00	2.70	2	2	17	37
		C14-75	2045	1332	17	626	47	301108	0.00	0.00	0.00	1	1	1	2
		C15-75	3007	1443	23	1098	36	460087	0.01	0.00	0.01	2	2	2	11
		C17-75	1492	835	3	404	50	348581	0.00	0.00	0.00	1	1	1	1
		C19-75	1497	664	1	390	57	245827	0.00	0.00	0.00	1	1	1	1
		C20-75	2323	1254	17	683	46	420825	0.00	0.00	0.00	1	1	2	2
		C22-75	3113	1524	119	1373	40	472247	2.49	0.00	2.49	2	2	17	45
		C23-75	2069	1548	169	1105	36	385434	5.53	0.00	5.53	2	2	28	45
		C24-75	2446	1694	305	1479	37	425807	2.70	0.00	2.69	2	3	72	94
		C26-75	1957	1039	17	432	52	333692	0.00	0.00	0.00	1	1	1	1
		C28-75	1781	1452	5	611	44	293838	0.00	0.00	0.00	1	1	1	1
		C29-75	2495	2238	1242	2217	32	548779	3.03	0.00	0.02	4	19	623	805
	5	C0-75	1661	1565	15	462	47	356296	0.00	0.00	0.00	1	1	1	1
		C2-75	1906	1405	115	1085	40	643540	0.02	0.00	0.01	2	2	33	42
		C3-75	3099	1809	37	1274	38	487061	0.00	0.00	0.00	3	3	22	24
		C4-75	2052	1851	75	946	43	398216	2.27	0.00	0.01	1	2	14	19
		C5-75	823	810	1	267	61	230460	0.00	0.00	0.00	1	1	1	1
		C6-75	3427	2012	39	1519	35	450414	2.85	0.00	2.85	48	48	107	243
		C7-75	2458	1649	21	619	46	437743	2.17	0.00	2.17	1	1	1	2
		C8-75	3002	2183	41	1049	37	487245	2.69	0.00	2.69	2	2	26	27
		C9-75	3550	2341	399	2597	31	327993	3.22	0.00	3.22	14	15	721	781
		C10-75	2856	1580	197	2350	32	526188	3.12	0.00	3.12	130	131	382	1854
		C11-75	1955	1193	23	645	47	443770	0.00	0.00	0.00	1	1	1	3
		C12-75	1801	1347	5	506	46	298162	0.00	0.00	0.00	1	1	1	1
		C13-75	2699	1302	161	1367	37	479166	2.70	0.00	2.70	3	4	59	67
		C14-75	2045	1333	21	626	47	301108	0.00	0.00	0.00	1	1	2	3
		C15-75	3003	1443	35	1117	36	460087	0.01	0.00	0.01	3	3	3	23
		C17-75	1492	835	3	404	50	348581	0.00	0.00	0.00	1	1	1	1
		C19-75	1497	664	1	390	57	245827	0.00	0.00	0.00	1	1	1	1
		C20-75	2323	1254	17	683	46	420825	0.00	0.00	0.00	1	1	2	2
		C22-75	3113	1524	199	1399	40	472040	2.50	0.00	2.49	3	3	19	105
		C23-75	2069	1548	25	983	35	379234	2.84	0.00	2.84	3	3	13	18
		C24-75	2446	1694	705	1620	37	425806	5.25	0.00	2.70	3	6	109	252
		C26-75	1957	1039	17	430	52	333692	0.00	0.00	0.00	1	1	1	1
		C28-75	1781	1452	5	628	44	293838	0.00	0.00	0.00	1	1	1	1
		C29-75	2495	2238	619	2097	32	543850	0.02	0.00	0.02	8	8	454	578
	6	C0-75	1661	1565	15	462	47	356296	0.00	0.00	0.00	1	1	1	1
		C2-75	1906	1405	87	1072	40	643540	0.02	0.00	0.01	2	3	27	39
		C3-75	3099	1809	35	1281	38	487061	0.00	0.00	0.00	4	4	30	33
		C4-75	2052	1851	75	929	43	398216	2.27	0.00	0.01	2	2	16	20
		C5-75	823	810	1	267	61	230460	0.00	0.00	0.00	1	1	1	1
		C6-75	3427	2012	53	1557	35	450414	2.85	0.00	2.85	42	42	224	336
		C7-75	2458	1649	29	617	46	437743	2.17	0.00	2.17	1	1	1	2
		C8-75	3002	2183	19	1019	37	487245	2.69	0.00	2.69	2	2	10	14
		C9-75	3550	2341	103	2127	31	327993	3.22	0.00	3.22	17	18	246	339
		C10-75	2856	1580	73	2029	32	525115	3.11	0.00	3.11	2763	2764	2764	17162
		C11-75	1955	1193	23	648	47	443770	0.00	0.00	0.00	1	1	1	3
		C12-75	1801	1347	5	505	46	298162	0.00	0.00	0.00	1	1	1	1
		C13-75	2699	1302	369	1484	37	479166	2.70	0.00	2.70	5	5	88	151
		C14-75	2045	1333	15	620	47	301108	0.00	0.00	0.00	1	1	1	2
		C15-75	3007	1443	19	1061	36	460087	0.01	0.00	0.01	3	3	3	13
		C17-75	1492	835	5	411	50	348581	0.00	0.00	0.00	1	1	1	1
		C19-75	1497	664	1	390	57	245827	0.00	0.00	0.00	1	1	1	1
		C20-75	2323	1254	17	671	46	420825	0.00	0.00	0.00	1	1	2	2
		C22-75	3113	1524	141	1358	40	472040	2.50	0.00	2.49	3	4	53	84
		C23-75	2069	1548	35	981	35	379234	2.84	0.00	2.84	3	3	16	26
		C24-75	2446	1694	613	1572	37	425806	5.25	0.00	2.70	4	9	94	256
		C26-75	1957	1039	17	430	52	333692	0.00	0.00	0.00	1	1	1	1
		C28-75	1781	1452	5	622	44	293838	0.00	0.00	0.00	1	1	1	1
		C29-75	2495	2238	555	2050	32	543850	0.02	0.00	0.02	9	9	9	667

Cluster size	Vehicle capacity	Cluster ID	Inbound edge #	Outbound edge #	Tree node #	Column #	Vehicle #	Total distance (m)	Optimality gap (%)		Integrality gap (%)	Wall time (s)			
									Root node soln.	Best feasible soln.		RMP conv.	Root node soln.	Best feasible soln.	Total
75	7	C0-75	1661	1565	15	463	47	356296	0.00	0.00	0.00	1	1	1	1
		C2-75	1906	1405	79	1072	40	643540	0.02	0.00	0.01	3	3	30	39
		C3-75	3099	1809	21	1238	38	487061	0.00	0.00	0.00	4	4	4	20
		C4-75	2052	1851	139	962	43	398216	2.27	0.00	0.01	1	2	33	33
		C5-75	823	810	1	267	61	230460	0.00	0.00	0.00	1	1	1	1
		C6-75	3427	2012	33	1515	35	450414	2.85	0.00	2.85	48	48	186	247
		C7-75	2458	1649	31	616	46	437743	2.17	0.00	2.17	1	1	1	3
		C8-75	3002	2183	23	1025	37	487245	2.69	0.00	2.69	2	2	9	18
		C9-75	3550	2341	189	2347	31	327993	3.22	0.00	3.22	20	20	482	594
		C10-75	2856	1580	2	1459	32	525115	3.12	3.12	3.12	39153	39154	39154	43200
		C11-75	1955	1193	24	650	47	443770	0.00	0.00	0.00	1	1	1	3
		C12-75	1801	1347	5	502	46	298162	0.00	0.00	0.00	1	1	1	1
		C13-75	2699	1302	337	1470	37	479166	2.70	0.00	2.70	2	3	78	105
		C14-75	2045	1332	19	619	47	301108	0.00	0.00	0.00	1	1	2	2
		C15-75	3007	1443	25	1089	36	460087	0.01	0.00	0.01	2	3	3	17
		C17-75	1492	835	7	411	50	348581	0.00	0.00	0.00	1	1	1	1
		C19-75	1497	664	1	390	57	245827	0.00	0.00	0.00	1	1	1	1
		C20-75	2323	1254	11	672	46	420825	0.00	0.00	0.00	1	1	2	2
		C22-75	3113	1524	161	1382	40	472040	2.50	0.00	2.49	3	3	52	79
		C23-75	2069	1548	23	940	35	379234	2.84	0.00	2.84	3	3	12	19
		C24-75	2446	1694	745	1666	37	425806	5.25	0.00	2.70	3	6	148	329
		C26-75	1957	1039	17	430	52	333692	0.00	0.00	0.00	1	1	1	1
		C28-75	1781	1452	5	621	44	293838	0.00	0.00	0.00	1	1	1	1
		C29-75	2495	2238	523	2052	32	543850	0.02	0.00	0.02	7	8	8	619
	8	C0-75	1661	1565	15	461	47	356296	0.00	0.00	0.00	1	1	1	1
		C2-75	1906	1405	75	1047	40	643540	0.02	0.00	0.01	2	2	26	36
		C3-75	3099	1809	35	1268	38	487061	0.00	0.00	0.00	4	4	30	33
		C4-75	2052	1851	101	928	43	398216	0.01	0.00	0.01	2	2	27	28
		C5-75	823	810	1	267	61	230460	0.00	0.00	0.00	1	1	1	1
		C6-75	3427	2012	62	1584	35	450414	5.54	0.00	2.85	39	40	266	362
		C7-75	2458	1649	31	618	46	437743	2.17	0.00	2.17	1	1	1	2
		C8-75	3002	2183	49	1064	37	487245	2.69	0.00	2.69	3	4	9	81
		C9-75	3550	2341	105	2199	31	327993	3.22	0.00	3.22	26	27	300	398
		C10-75	2856	1580	1	1425	32	525207	3.12	3.12	3.12	84928	84928	84928	84928
		C11-75	1955	1193	24	641	47	443770	0.00	0.00	0.00	1	1	1	3
		C12-75	1801	1347	5	508	46	298162	0.00	0.00	0.00	1	1	1	1
		C13-75	2699	1302	427	1513	37	479166	2.70	0.00	2.70	4	4	77	144
		C14-75	2045	1332	17	624	47	301108	0.00	0.00	0.00	1	1	1	2
		C15-75	3005	1443	21	1093	36	460087	0.01	0.00	0.01	3	3	3	16
		C17-75	1492	835	7	410	50	348581	0.00	0.00	0.00	1	1	1	1
		C19-75	1497	664	1	390	57	245827	0.00	0.00	0.00	1	1	1	1
		C20-75	2323	1254	11	671	46	420825	0.00	0.00	0.00	1	1	2	2
		C22-75	3113	1524	161	1357	40	472040	2.50	0.00	2.49	3	4	65	103
		C23-75	2069	1548	25	959	35	379234	2.84	0.00	2.84	4	4	16	22
		C24-75	2446	1694	677	1567	37	425806	2.70	0.00	2.70	3	3	83	265
		C26-75	1957	1039	15	430	52	333692	0.00	0.00	0.00	1	1	1	1
		C28-75	1781	1452	5	620	44	293838	0.00	0.00	0.00	1	1	1	1
		C29-75	2495	2238	573	2163	32	543850	0.02	0.00	0.02	9	9	353	625
100	4	C0-100	3488	1930	45	777	63	488119	0.00	0.00	0.00	2	2	2	5
		C1-100	1825	1642	5	429	75	331497	0.00	0.00	0.00	2	2	2	3
		C2-100	5540	3106	1445	2652	46	596824	2.17	0.00	2.17	6	7	995	1397
		C3-100	5383	3127	823	2794	46	586434	2.17	0.00	2.16	8	9	558	695
		C4-100	4211	2787	50653	6120	44	558323	2.27	0.00	2.26	8	9	730	31745
		C5-100	1960	1452	39	575	70	366565	0.00	0.00	0.00	2	2	2	4
		C6-100	4708	3393	33	1805	47	724844	0.00	0.00	0.00	8	8	39	44
		C7-100	4120	2205	23	1227	53	661648	0.00	0.00	0.00	3	3	3	10
		C8-100	4952	2656	3	1285	51	609120	0.00	0.00	0.00	3	3	3	4
		C9-100	5664	3572	2316	6235	40	527217	2.50	0.00	2.49	57	59	7410	14795
		C10-100	2995	2611	147	980	58	426386	1.72	0.00	1.72	2	2	14	16
		C11-100	4606	2964	61	1506	51	427829	0.00	0.00	0.00	3	4	14	32
		C12-100	2863	2459	1	794	62	413012	0.00	0.00	0.00	2	2	2	2
		C13-100	3232	2441	17	755	61	483087	0.00	0.00	0.00	2	2	3	3
		C14-100	4335	3403	2695	3757	40	693825	4.85	0.00	2.49	15	1297	3093	6348
		C15-100	3711	2642	26493	3217	48	790005	4.16	0.00	4.16	7	8	2068	26100
		C16-100	3278	2302	21	1085	55	627423	1.81	0.00	1.81	3	3	5	8
		C17-100	3413	1603	7	881	60	640945	0.00	0.00	0.00	2	2	2	3
		C18-100	3778	2518	81	872	58	681240	1.72	0.00	1.72	2	2	2	10
		C19-100	3722	3405	353	1656	51	503252	1.96	0.00	1.96	4	4	64	192
		C20-100	4377	2653	1863	2708	46	569724	2.17	0.00	2.17	7	7	1135	1144
		C21-100	2597	1223	1	524	78	328216	0.00	0.00	0.00	2	2	2	2

Cluster size	Vehicle capacity	Cluster ID	Inbound edge #	Outbound edge #	Tree node #	Column #	Vehicle #	Total distance (m)	Optimality gap (%)		Integrality gap (%)	Wall time (s)			
									Root node soln.	Best feasible soln.		RMP conv.	Root node soln.	Best feasible soln.	Total
100	5	C0-100	3488	1930	43	772	63	488119	0.00	0.00	0.00	2	2	2	5
		C1-100	1825	1642	5	429	75	331497	0.00	0.00	0.00	2	2	2	2
		C2-100	5540	3106	1513	2637	46	596658	2.17	0.00	2.17	9	10	1269	1942
		C3-100	5383	3127	191	2449	46	584077	2.17	0.00	2.16	60	61	860	950
		C4-100	4209	2787	67	2111	44	552862	2.26	0.00	2.26	18	27	141	183
		C5-100	1964	1452	47	586	70	366565	0.00	0.00	0.00	2	2	2	4
		C6-100	4708	3393	29	1807	47	724844	0.00	0.00	0.00	8	8	26	54
		C7-100	4120	2205	27	1234	53	661528	0.00	0.00	0.00	4	4	4	13
		C8-100	4952	2656	3	1287	51	609038	0.00	0.00	0.00	3	3	3	5
		C9-100	5559	3570	459	4860	40	521285	2.49	2.49	2.49	798	799	29422	43304
		C10-100	2995	2611	169	971	58	426386	1.72	0.00	1.72	2	2	2	17
		C11-100	4606	2964	53	1462	51	427829	0.00	0.00	0.00	4	5	18	38
		C12-100	2863	2459	1	787	62	413012	0.00	0.00	0.00	2	2	2	2
		C13-100	3232	2441	17	758	61	483087	0.00	0.00	0.00	2	2	2	3
		C14-100	4335	3403	1067	3293	40	690964	2.49	2.49	2.49	70	72	39918	43218
		C15-100	3711	2642	11238	4146	48	788340	4.16	4.16	4.16	22	22	4167	43205
		C16-100	3278	2302	65	1151	55	626787	1.81	0.00	1.81	2	2	14	24
		C17-100	3413	1603	19	902	60	640945	0.00	0.00	0.00	2	2	2	5
		C18-100	3778	2518	65	868	58	681240	1.72	0.00	1.72	2	3	3	9
		C19-100	3722	3405	633	1695	51	503252	1.96	0.00	1.96	4	4	263	359
		C20-100	4377	2653	1785	2948	46	568797	2.17	0.00	2.17	12	13	1765	1815
		C21-100	2597	1223	1	525	78	328216	0.00	0.00	0.00	2	2	2	2
	6	C0-100	3488	1931	59	782	63	488119	0.00	0.00	0.00	2	3	3	7
		C1-100	1825	1642	5	429	75	331497	0.00	0.00	0.00	2	2	2	2
		C2-100	5540	3106	1439	2614	46	596658	2.17	0.00	2.17	10	11	1618	2309
		C3-100	5383	3127	415	2690	46	584048	2.17	0.00	2.16	76	77	1791	1841
		C4-100	4211	2787	309	2656	44	552862	2.26	0.00	2.26	21	24	178	773
		C5-100	1959	1452	43	583	70	366565	0.00	0.00	0.00	2	2	2	4
		C6-100	4708	3393	27	1821	47	724844	0.00	0.00	0.00	11	11	30	85
		C7-100	4120	2205	29	1243	53	661528	0.00	0.00	0.00	4	4	4	15
		C8-100	4952	2656	3	1277	51	609038	0.00	0.00	0.00	3	3	3	4
		C9-100	5664	3572	35	2997	40	518571	2.49	2.49	2.49	10273	10275	10275	43211
		C10-100	2995	2611	149	982	58	426386	1.72	0.00	1.72	2	2	16	17
		C11-100	4606	2964	45	1453	51	427829	0.00	0.00	0.00	4	4	18	40
		C12-100	2863	2459	1	790	62	413012	0.00	0.00	0.00	2	2	2	2
		C13-100	3232	2441	17	760	61	483087	0.00	0.00	0.00	2	2	2	3
		C14-100	4335	3403	532	3161	40	693512	2.50	2.50	2.50	144	148	21052	43203
		C15-100	3711	2642	3810	3876	48	788340	4.16	4.16	4.16	142	142	15775	43217
		C16-100	3278	2302	69	1151	55	626787	1.81	0.00	1.81	3	3	3	41
		C17-100	3411	1603	9	881	60	640945	0.00	0.00	0.00	2	2	2	3
		C18-100	3778	2518	59	867	58	681240	1.72	0.00	1.72	2	2	2	8
		C19-100	3722	3405	1364	1761	51	503252	1.96	0.00	1.96	5	5	696	977
		C20-100	4377	2653	2993	3190	46	568797	2.17	0.00	2.17	21	22	3242	3252
		C21-100	2597	1223	1	527	78	328216	0.00	0.00	0.00	2	2	2	2
	7	C0-100	3488	1931	65	783	63	488119	0.00	0.00	0.00	2	2	2	6
		C1-100	1825	1642	5	429	75	331497	0.00	0.00	0.00	2	2	2	2
		C2-100	5540	3106	1613	2710	46	596658	2.17	0.00	2.17	11	12	1811	2822
		C3-100	5383	3127	247	2572	46	584048	2.17	0.00	2.16	68	69	1250	1297
		C4-100	4209	2787	87	2212	44	552334	2.26	0.00	2.26	13	15	220	240
		C5-100	1959	1452	35	583	70	366565	0.00	0.00	0.00	2	2	2	4
		C6-100	4708	3393	27	1807	47	724844	0.00	0.00	0.00	11	11	29	82
		C7-100	4120	2205	27	1209	53	661528	0.00	0.00	0.00	3	4	4	13
		C8-100	4952	2656	3	1270	51	609038	0.00	0.00	0.00	3	3	3	4
		C9-100	5664	3572	1	2482	40	520597	2.50	2.50	2.50	43846	43847	43847	43847
		C10-100	2995	2611	155	981	58	426386	1.72	0.00	1.72	2	2	16	17
		C11-100	4606	2964	57	1456	51	427829	0.00	0.00	0.00	5	5	29	49
		C12-100	2863	2459	1	793	62	413012	0.00	0.00	0.00	2	2	2	2
		C13-100	3232	2441	17	757	61	483087	0.00	0.00	0.00	2	2	2	3
		C14-100	4335	3403	369	2927	40	692956	2.49	2.49	2.49	240	242	242	43244
		C15-100	3711	2642	993	2445	48	789091	4.16	4.16	4.16	942	942	942	43215
		C16-100	3278	2302	63	1131	55	626787	1.81	0.00	1.81	4	4	28	53
		C17-100	3413	1603	9	875	60	640945	0.00	0.00	0.00	2	2	2	3
		C18-100	3778	2518	81	872	58	681240	1.72	0.00	1.72	2	2	2	10
		C19-100	3722	3405	639	1714	51	503252	1.96	0.00	1.96	5	6	306	407
		C20-100	4377	2653	1669	2890	46	568797	2.17	0.00	2.17	16	17	1180	2009
		C21-100	2597	1223	1	525	78	328216	0.00	0.00	0.00	2	2	2	2
	8	C0-100	3488	1931	65	784	63	488119	0.00	0.00	0.00	2	2	2	7
		C1-100	1825	1642	5	429	75	331497	0.00	0.00	0.00	1	1	1	2
		C2-100	5540	3106	1477	2619	46	596658	2.17	0.00	2.17	13	13	1790	2605
		C3-100	5383	3127	71	2209	46	584048	2.17	0.00	2.16	90	91	389	518
		C4-100	4210	2787	71	2176	44	552334	2.26	0.00	2.26	13	15	253	271
		C5-100	1964	1452	53	575	70	366565	0.00	0.00	0.00	2	2	2	5
		C6-100	4708	3393	27	1822	47	724844	0.00	0.00	0.00	11	12	29	87
		C7-100	4120	2205	29	1250	53	661528	0.00	0.00	0.00	3	4	4	15
		C8-100	4952	2656	3	1273	51	609038	0.00	0.00	0.00	3	3	3	5
		C9-100	5664	3572	4	2577	40	517627	2.49	2.49	2.49	34816	34818	34818	43200
		C10-100	2995	2611	159	976	58	426386	1.72	0.00	1.72	2	2	16	18
		C11-100	4606	2964	53	1457	51	427829	0.00	0.00	0.00	4	5	19	45
		C12-100	2863	2459	1	789	62	413012	0.00	0.00	0.00	2	2	2	2
		C13-100	3232	2441	13	760	61	483087	0.00	0.00	0.00	2	2	2	3
		C14-100	4060	3403	460	3150	40	693850	2.50	2.50	2.50	196	198	33298	43354
		C15-100	3711	2642	321	2353	48	789091	4.16	4.16	4.16	2597	2598	2598	44157
		C16-100	3278	2302	29	1116	55	626787	1.81	0.00	1.81	4	4	13	32
		C17-100	3413	1603	9	873	60	640945	0.00	0.00	0.00	2	2	2	3
		C18-100	3778	2518	71	871	58	681240	1.72	0.00	1.72	2	2	2	9
		C19-100	3722	3405	719	1709	51	503252	1.96	0.00	1.96	5	5	394	487
		C20-100	4377	2653	1760	2899	46	568797	2.17	0.00	2.17	21	22	1737	2192
		C21-100	2597	1223	1	524	78	328216	0.00	0.00	0.00	1	2	2	2

Table 3 **Results of REA Scalability with Increasing Cluster Size ($K = 4$, $\Delta = 10$ mins, $R = 0.5$).**

Cluster size	Cluster ID	Column #	Vehicle #	Total distance (m)	Optimality gap (%)	Wall time (s)	
						Route enumeration	Total
200	C0-200	2286	129	702670	0.00	618	618
	C1-200	10955	84	1122469	0.00	653	654
	C2-200	15149	85	1067503	0.00	646	2361
	C3-200	3101	113	824730	0.00	629	630
	C4-200	2627	114	684981	0.00	632	878
	C5-200	22061	80	937514	0.00	660	744
	C6-200	11844	86	1136870	0.00	673	680
	C7-200	16235	76	1365475	0.00	668	1165
	C8-200	27913	78	923290	0.49	684	43211
	C9-200	5289	99	1055393	0.00	655	702
	C10-200	7790	87	1249784	0.00	676	8731
300	C0-300	33262	118	1652972	0.16	3301	43206
	C1-300	32536	119	1928887	0.44	2796	43203
	C2-300	7994	150	1096341	0.00	3249	4367
	C3-300	36568	113	1511806	0.00	3499	3650
	C4-300	15394	134	1477823	0.13	3198	43205
	C5-300	22072	124	1773852	0.00	3342	4325
	C6-300	33541	119	1927882	0.35	3191	43205
	C7-300	6554	157	1105570	0.00	3152	3158
	C8-300	53120	114	1418494	0.20	3129	43204
	C9-300	30568	114	1911085	0.17	2258	43203
	C10-300	6370	156	1085481	0.00	2736	2776
	C11-300	34630	118	1903034	0.37	3037	43204
	C12-300	7137	153	1067479	0.00	2339	2632
400	C0-400	28968	180	1617748	0.00	8897	35048
	C1-400	52194	145	1901974	0.19	6972	43203
	C2-400	28025	173	1627074	0.00	10060	18906
	C3-400	41012	152	2114911	0.20	7842	43203
	C4-400	26314	182	1687079	0.00	10447	10531
	C5-400	53948	151	1988361	0.00	7043	12685
	C6-400	26109	173	1640438	0.00	10716	10986
	C7-400	68190	145	1887086	0.32	7117	43202

Table 4 Results of BPA Scalability with Increasing Cluster Size ($K = 4$, $\Delta = 10$ mins, $R = 0.5$).

Cluster size	Cluster ID	Inbound edge #	Outbound edge #	Tree node #	Column #	Vehicle #	Total distance (m)	Optimality gap (%)		Integrality gap (%)	Wall time (s)			
								Root node soln.	Best feasible soln.		RMP conv.	Root node soln.	Best feasible soln.	Total
200	C0-200	10281	5900	113	1747	129	702670	1.55	0.00	1.55	13	13	35	40
	C1-200	17096	10087	5870	6121	84	1122544	3.50	2.37	2.37	59	96	6951	43201
	C2-200	13922	9341	7479	7506	85	1067640	2.35	2.34	2.34	55	74	6263	43203
	C3-200	10551	7643	99	2272	113	824730	0.00	0.00	0.00	19	20	20	78
	C4-200	9806	8163	5901	2616	114	684981	1.75	0.00	1.75	17	31	2896	4149
	C5-200	18385	10756	3695	9115	80	937514	1.26	1.25	1.25	111	121	14244	43203
	C6-200	16047	9533	13563	9572	86	1136870	2.29	1.16	1.16	45	202	14218	43201
	C7-200	15672	11696	2279	8993	76	1367209	3.88	2.62	2.62	140	6148	23012	43213
	C8-200	16483	12874	1	6027	79	928401	2.53	2.53	2.53	319	43209	43209	43209
	C9-200	14587	9841	15783	4425	99	1055393	1.01	0.00	1.00	25	37	6671	20722
	C10-200	15459	11600	4244	6328	87	1249965	2.29	2.29	2.29	45	150	9471	43202
300	C0-300	38471	31409	242	11624	118	1665487	0.85	0.85	0.85	900	1018	1018	43265
	C1-300	36931	22949	496	12533	119	1938007	2.52	2.52	2.52	944	4151	4151	43229
	C2-300	26167	20901	11208	6629	150	1096341	1.33	1.33	1.33	81	124	7967	43200
	C3-300	35614	23695	762	13192	114	1510444	1.75	1.75	1.75	426	12864	12864	43212
	C4-300	30651	26297	1091	8757	134	1478291	1.49	1.49	1.49	278	519	39914	43243
	C5-300	32453	25677	400	9653	125	1772096	0.80	0.80	0.80	622	14460	14460	43259
	C6-300	35954	20702	483	12056	119	1934456	2.52	2.52	2.52	734	1914	1914	43218
	C7-300	24094	19880	3185	5592	157	1105570	1.26	0.00	0.64	70	1027	3841	10016
	C8-300	38059	22799	255	12441	115	1423615	2.60	2.60	2.60	480	29027	29027	43201
	C9-300	33512	23617	779	13090	114	1934405	2.64	2.64	2.64	515	666	666	43252
	C10-300	22339	16909	23037	5367	156	1085482	0.64	0.64	0.64	63	70	7449	43202
	C11-300	34544	21130	698	12376	118	1910713	3.38	3.38	3.38	568	988	988	43227
	C12-300	24520	17238	12001	6091	153	1067479	0.65	0.65	0.65	88	124	4043	43200
400	C0-400	49218	39758	1	10357	181	1613436	1.65	1.65	1.65	528	43205	43205	43205
	C1-400	53747	35507	1	13136	147	1926300	2.72	2.72	2.72	751	43207	43207	43207
	C2-400	54300	35489	675	12751	173	1631283	1.16	1.16	1.16	406	540	540	43223
	C3-400	55542	42004	1	12834	153	2124836	1.96	1.96	1.96	1596	43204	43204	43204
	C4-400	49195	31033	3523	11881	182	1688282	1.10	1.10	1.10	222	239	25641	43201
	C5-400	57633	34496	1	13557	152	1983300	1.97	1.97	1.97	684	43204	43204	43204
	C6-400	46155	30320	3708	12494	173	1640472	1.16	1.15	1.15	240	289	11566	43210
	C7-400	61393	38346	1	14833	146	1886534	2.73	2.73	2.73	960	43206	43206	43206

Table 5 Results of Root-Node Heuristics with $t_{\mathsf{RMP}} = 8$ mins and $t_{\mathsf{MIP}} = 2$ mins ($K = 4$, $\Delta = 10$ mins, $R = 0.5$).

Cluster size	Cluster ID	Enforce forbidden paths					Relax forbidden paths					
		Column #	Vehicle #	Optimality gap (%)	Wall time (s)		Column #	Infeasible column #	Vehicle #	Optimality gap (%)	Wall time (s)	
					RMP	MIP					RMP	MIP
100	C0-100	720	63	0.00	2	0	720	0	63	0.00	2	0
	C1-100	422	75	0.00	2	0	423	0	75	0.00	2	0
	C2-100	1828	46	2.17	5	0	1792	6	46	4.35	4	0
	C3-100	1735	46	2.17	4	0	1729	6	46	2.17	4	0
	C4-100	1653	44	2.27	6	5	1624	4	44	2.27	5	1
	C5-100	537	70	0.00	1	0	541	0	70	0.00	2	0
	C6-100	1637	47	0.00	5	0	1628	3	47	0.00	4	0
	C7-100	1130	53	0.00	2	0	1123	5	53	0.00	2	0
	C8-100	1198	52	1.92	2	0	1184	3	51	0.00	2	0
	C9-100	2360	41	4.88	29	10	2467	10	41	4.88	37	2
	C10-100	891	58	1.72	2	0	895	1	58	1.72	2	0
	C11-100	1272	51	0.00	3	0	1252	0	51	0.00	2	0
	C12-100	767	62	0.00	2	0	776	2	62	0.00	2	0
	C13-100	708	61	0.00	2	0	708	0	61	0.00	1	0
	C14-100	2003	41	4.88	11	2	1994	12	41	4.88	8	2
	C15-100	1548	48	4.17	3	0	1580	9	48	6.25	4	0
	C16-100	992	55	1.82	2	0	988	2	55	1.82	2	0
	C17-100	817	60	0.00	2	0	810	3	60	0.00	2	0
	C18-100	808	58	1.72	2	0	808	0	58	1.72	1	0
	C19-100	1337	51	1.96	3	0	1334	9	52	3.85	3	0
	C20-100	1672	46	2.17	4	1	1643	2	46	2.17	4	1
	C21-100	500	78	0.00	2	0	503	0	78	0.00	1	0
200	C0-200	1537	129	1.55	12	0	1539	2	129	1.55	12	0
	C1-200	4216	85	3.53	35	1	4146	19	84	2.38	35	2
	C2-200	4019	85	2.35	34	2	4018	16	85	3.53	33	3
	C3-200	2029	113	0.00	13	0	2022	4	113	0.00	13	0
	C4-200	2017	114	1.75	14	0	2039	4	114	1.75	13	0
	C5-200	5161	80	1.25	82	9	5010	29	81	2.47	48	35
	C6-200	4294	87	2.30	28	1	4315	18	87	2.30	25	3
	C7-200	5516	76	2.63	81	67	5563	38	77	3.90	63	12
	C8-200	5925	79	2.53	245	120	5986	29	79	2.53	209	120
	C9-200	3142	99	1.01	16	2	3165	8	99	1.01	17	1
	C10-200	4118	87	2.30	38	5	4150	10	87	2.30	30	3
300	C0-300	9312	119	1.68	449	120	9366	58	119	1.68	235	120
	C1-300	9928	119	2.52	376	120	9795	44	120	4.17	171	114
	C2-300	4879	150	1.33	63	5	4894	3	150	1.33	62	6
	C3-300	9465	114	1.75	286	62	9530	43	116	3.45	264	120
	C4-300	7002	134	1.49	192	8	7021	24	134	1.49	113	14
	C5-300	7920	125	0.80	296	120	7832	30	125	1.60	142	19
	C6-300	9495	119	2.52	433	28	9531	70	119	3.36	157	14
	C7-300	4311	158	1.27	63	4	4298	3	158	1.27	55	1
	C8-300	9726	116	3.45	243	120	9756	31	115	2.61	219	120
	C9-300	9796	115	3.48	401	120	9835	55	115	3.48	304	120
	C10-300	3884	156	0.64	50	2	3915	8	156	1.28	49	1
	C11-300	9354	119	4.20	303	120	9685	51	118	4.24	196	26
	C12-300	4207	153	0.65	59	1	4285	6	153	0.65	53	1
400	C0-400	10289	181	1.66	356	120	10355	27	182	2.20	318	120
	C1-400	12853	152	5.92	417	120	12965	42	151	5.30	419	120
	C2-400	9989	173	1.16	185	42	10112	19	173	1.16	203	21
	C3-400	12615	155	11.61	486	120	12714	44	157	4.46	472	120
	C4-400	8872	182	1.10	153	18	8948	22	183	2.19	146	26
	C5-400	13162	154	3.25	415	120	13248	45	153	2.61	443	120
	C6-400	8971	173	1.16	155	70	9130	28	173	1.73	159	25
	C7-400	14322	150	6.67	504	120	14412	64	152	6.58	461	120