A Bucket Graph Based Labeling Algorithm with Application to Vehicle Routing: online appendix

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Appendix. Detailed Results

In the Appendix, we present the detailed results for our BCP algorithm on all tested instances.

In the following tables, we show the instance name, n — the number of customers, m — the number of different vehicle types or the number of depots, when applicable, the initial upper bound, the "robust" dual bound (before adding non-robust cuts), the dual bound obtained in the root node, the root node time, the number of R1Cs at the end of the root node, the number of nodes, the total time, and the best solution value obtained by the algorithm. For instances solved before by another algorithm in the literature, we give the corresponding time (which takes into account the difference between the speed of the computers used), and the improvement factor (ratio of the running times of the literature algorithm and ours). In Tables 1–7, "Time PCDU" refers to the running time of the algorithm by Pecin et al. (2017). In Tables 14–15, "Time CM" refers to the running time of the algorithm by Contardo and Martinelli (2014). In the tables, * near the best solution value indicates that its optimality was proven for the first time. If the best solution value is underlined, it improves on the best known solution value. All obtained solutions were proven to be optimal except when the total time is given with the ">" sign.

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
C203	588.8	588.7	588.7	27s	0	1	27s	588.7	6m05s	13.3
C204	588.2	588.1	588.1	40s	0	1	40s	588.1	27m28s	40.9
R202	1029.7	1022.2	1029.6	2m25s	72	1	2m25s	1029.6	5m16s	2.2
R203	870.9	866.9	870.8	1m16s	17	1	1m16s	870.8	9m25s	7.4
R204	731.4	724.9	731.3	3m04s	56	1	3m04s	731.3	24m55s	8.1
R206	876.0	866.8	875.9	2m30s	51	1	2m30s	875.9	9m06s	3.6
R207	794.1	790.1	794.0	1m43s	27	1	1m43s	794.0	12m08s	7.0
R208	701.1	691.6	697.7	14m57s	138	9	1h14m	701.0	17h48m	14.3
R209	854.9	841.4	854.8	3m45s	105	1	3m45s	854.8	9m59s	2.7
R210	900.6	889.4	900.5	4m34s	107	1	4m34s	900.5	12m09s	2.7
R211	746.8	734.6	746.7	3m42s	110	1	3m42s	746.7	22m00s	5.9
RC204	783.6	779.3	783.5	2m11s	15	1	2m11s	783.5	19m27s	8.9
RC207	963.0	947.3	962.9	3m21s	91	1	3m21s	962.9	5m47s	1.7
RC208	776.2	766.6	776.1	2m02s	36	1	2m02s	776.1	9m59s	4.9

Table 1: Detailed results for the hardest 100-customers Solomon VRPTW instances

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
C1_2_1	2698.7	2698.6	2698.6	8s	0	1	8s	2698.6	20s	2.2
C1_2_2	2694.4	2694.3	2694.3	20s	0	1	20s	2694.3	44s	2.1
C1_2_3	2675.9	2659.7	2675.8	4m39s	66	1	4m39s	2675.8	10m26s	2.2
C1_2_4	2625.7	2619.8	2625.6	6m51s	42	1	6m51s	2625.6	5m40s	0.8
C1_2_5	2695.0	2694.9	2694.9	11s	0	1	11s	2694.9	24s	2.1
C1_2_6	2695.0	2694.9	2694.9	15s	0	1	15s	2694.9	26s	1.7
C1_2_7	2695.0	2694.9	2694.9	18s	0	1	18s	2694.9	22s	1.2
C1_2_8	2684.1	2683.7	2684.0	19s	0	1	19s	2684.0	28s	1.4
C1_2_9	2639.7	2637.2	2639.6	54s	0	1	54s	2639.6	1m24s	1.5
C1_2_10	2624.8	2620.2	2624.7	2m40s	34	1	2m40s	2624.7	1m53s	0.7

Table 2: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class C1

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
C2_2_1	1922.2	1915.0	1922.1	3m06s	70	1	3m06s	1922.1	14m20s	4.6
C2_2_2	1851.5	1839.4	1851.4	6m57s	80	1	6m57s	1851.4	1h29m	12.9
C2_2_3	1763.5	1737.0	1752.1	32m16s	287	37	11h10m	1763.4*	>96h	>8.6
C2_2_4	1695.1	1652.3	1663.2	28m59s	187	355	>60h	1695.1	>96h	–
C2_2_5	1869.7	1858.7	1869.6	3m57s	52	1	3m57s	1869.6	23m48s	6.0
C2_2_6	1844.9	1836.6	1844.8	3m22s	23	1	3m22s	1844.8	34m03s	10.1
C2_2_7	1842.3	1834.9	1842.2	3m03s	21	1	3m03s	1842.2	44m33s	14.6
C2_2_8	1813.8	1801.3	1813.7	3m11s	12	1	3m11s	1813.7	42m22s	13.3
C2_2_9	1815.1	1795.0	1815.0	9m17s	101	1	9m17s	1815.0	2h01m	13.1
C2_2_10	1791.3	1765.7	1791.2	13m04s	96	1	13m04s	1791.2	2h56m	13.5

Table 3: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class C2

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
R1_2_1	4667.3	4655.0	4667.2	14s	22	1	14s	4667.2	59s	4.0
R1_2_2	3920.0	3907.2	3919.9	37s	75	1	37s	3919.9	2m42s	4.3
R1_2_3	3374.0	3320.3	3352.5	13m27s	502	693	17h01m	3373.9	25h56m	1.5
R1_2_4	3047.7	3001.5	3035.3	23m03s	635	267	10h45m	3047.6	5h00m	0.5
R1_2_5	4053.3	4007.6	4044.8	3m21s	290	9	10m51s	4053.2	7m25s	0.7
R1_2_6	3559.3	3493.4	3545.3	11m16s	559	71	2h00m	3559.1	2h00m	1.0
R1_2_7	3142.0	3099.2	3141.9	9m27s	335	1	9m27s	3141.9	16m42s	1.8
R1_2_8	2938.5	2902.2	2933.9	19m02s	436	5	27m38s	2938.4	41m39s	1.5
R1_2_9	3734.8	3678.3	3726.4	6m23s	443	7	14m43s	3734.7	12m51s	0.9
R1_2_10	3293.2	3244.9	3278.2	8m35s	489	49	1h47m	3293.1	1h23m	0.8

Table 4: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class R1

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
R2_2_1	3468.1	3462.5	3468.0	2m20s	19	1	2m20s	3468.0	34m43s	14.9
R2_2_2	3008.3	3000.9	3008.2	7m33s	69	1	7m33s	3008.2	1h07m	9.0
R2_2_3	2537.6	2516.3	2537.5	33m26s	181	1	33m26s	2537.5	5h50m	10.5
R2_2_4	1928.6	1918.3	1924.9	33m39s	94	33	3h59m	1928.5*	>96h	>24.0
R2_2_5	3061.2	3029.4	3061.1	21m41s	123	1	21m41s	3061.1	1h45m	4.9
R2_2_6	2675.5	2644.9	2675.2	37m20s	249	3	50m39s	2675.4	4h04m	4.8
R2_2_7	2304.8	2277.8	2304.7	1h48m	335	1	1h48m	2304.7	12h16m	6.8
R2_2_8	1842.5	1808.9	1832.1	1h41m	393	19	11h47m	1842.4*	>96h	>8.1
R2_2_9	2843.4	2824.3	2843.3	12m37s	110	1	12m37s	2843.3	1h23m	6.6
R2_2_10	2549.5	2528.7	2545.4	15m33s	202	3	24m04s	2549.4	3h14m	8.1

Table 5: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class R2

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor
RC1_2_1	3517.0	3459.6	3498.7	4m26s	338	43	40m29s	3516.9	42m10s	1.0
RC1_2_2	3221.7	3177.8	3206.1	11m52s	522	17	33m38s	3221.6	3h10m	5.7
RC1_2_3	3001.5	2952.1	2982.9	22m40s	630	187	14h41m	3001.4	18h15m	1.2
RC1_2_4	2845.3	2814.4	2831.0	26m52s	361	41	4h58m	2845.2	6h23m	1.3
RC1_2_5	3325.7	3272.9	3312.7	49m59s	678	15	1h22m	3325.6	2h23m	1.7
RC1_2_6	3300.8	3255.3	3295.1	12m30s	468	5	16m35s	3300.7	26m46s	1.6
RC1_2_7	3177.9	3114.7	3151.8	17m14s	554	109	10h46m	3177.8	52h20m	4.9
RC1_2_8	3060.1	3022.6	3047.4	19m03s	614	5	49m56s	3060.0	2h50m	3.4
RC1_2_9	3074.9	3006.6	3035.6	16m42s	464	493	>60h	3074.9	>96h	–
RC1_2_10	2990.6	2938.7	2966.4	24m40s	583	683	>60h	2990.6	>96h	–

Table 6: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class RC1

Instance	Initial	Robust	Root			Nodes	Total	Final	Time	Improv.
	UB	bound	bound	time	#R1C	number	time	UB	PCDU	factor

RC2_2_1	2797.5	2789.4	2797.4	4m11s	38	1	4m11s	2797.4	35m49s	8.6
RC2_2_2	2481.7	2469.5	2477.0	19m21s	183	3	39m25s	2481.6	4h50m	7.4
RC2_2_3	2227.8	2204.7	2227.7	1h54m	308	1	1h54m	2227.7	13h32m	7.1
RC2_2_4	1854.9	1839.7	1848.3	1h33m	196	31	10h52m	1854.8*	>96h	>8.8
RC2_2_5	2493.6	2479.0	2491.4	13m36s	142	1	13m36s	2491.4	1h25m	6.3
RC2_2_6	2496.4	2475.6	2495.1	14m09s	83	1	14m09s	2495.1	1h33m	6.6
RC2_2_7	2287.8	2271.2	2279.0	17m22s	98	11	59m07s	2287.7	8h11m	8.3
RC2_2_8	2151.3	2140.7	2151.2	29m09s	92	1	29m09s	2151.2	23h05m	47.5
RC2_2_9	2086.7	2056.5	2071.1	1h54m	240	109	59h29m	2086.6*	>96h	>1.6
RC2_2_10	1989.3	1953.2	1955.8	51m23s	12	201	>60h	1989.2	>96h	–

Table 7: Detailed results for 200-customer Gehring and Homberger VRPTW instances of class RC2

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB
C1_4_1	7138.9	7138.8	7138.8	1m34s	0	1	1m34s	7138.8[c]
C1_4_2	7113.2	7095.0	7102.2	6m13s	79	3	7m17s	7113.2*
C1_4_3	6930.0	6919.8	6929.9	12m51s	81	1	12m51s	6929.9*
C1_4_4	6777.8	6769.6	6777.7	45m22s	118	1	45m22s	6777.7*
C1_4_5	7138.9	7138.8	7138.8	1m58s	0	1	1m58s	7138.8*
C1_4_6	7140.2	7140.1	7140.1	2m11s	0	1	2m11s	7140.1*
C1_4_7	7136.3	7135.9	7136.2	2m27s	0	1	2m27s	7136.2*
C1_4_8	7107.1	7030.0	7071.2	13m02s	277	61	2h07m	<u>7083.0</u>*
C1_4_9	6927.9	6888.3	6927.8	22m43s	355	1	22m43s	6927.8*
C1_4_10	6825.5	6798.8	6819.2	44m59s	347	13	1h02m	6825.4*

[c] this instance has been solved before by Kallehauge, Larsen, and Madsen (2006)

Table 8: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class C1

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB
C2_4_1	4100.4	4088.4	4100.3	15m08s	35	1	15m08s	4100.3*
C2_4_2	3914.2	3897.5	3914.1	9h39m	283	1	9h39m	3914.1*
C2_4_3	3755.3	3722.7	3740.8	60h00m	566	1	>60h	3755.3
C2_4_4	3524.1	3483.3	3498.4	3h00m	288	11	>60h	3524.1
C2_4_5	3923.3	3919.7	3923.2	23m01s	58	1	23m01s	3923.2*
C2_4_6	3860.2	3843.8	3860.1	39m33s	99	1	39m33s	3860.1*
C2_4_7	3871.0	3861.6	3870.9	29m47s	82	1	29m47s	3870.9*
C2_4_8	3773.8	3743.4	3770.6	1h11m	397	9	4h29m	3773.7*
C2_4_9	3843.8	3782.0	3815.9	4h09m	542	15	>60h	3843.8
C2_4_10	3665.2	3616.9	3656.2	2h33m	682	73	>60h	3665.1

Table 9: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class C2

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB

R1_4_1	10313.9	10243.5	10284.1	4m00s	181	25	16m48s	10305.8*
R1_4_2	8876.0	8776.2	8834.3	17m52s	534	1683	>60h	8876.0
R1_4_3	7794.8	7673.7	7732.2	54m14s	658	327	>60h	7794.8
R1_4_4	7281.8	7182.4	7239.2	2h06m	1095	123	>60h	7281.8
R1_4_5	9188.7	9082.3	9145.8	6m35s	355	2393	>60h	9188.7
R1_4_6	8366.6	8257.8	8314.1	28m04s	596	1139	>60h	8366.6
R1_4_7	7609.1	7489.2	7558.6	1h25m	1020	185	>60h	7609.1
R1_4_8	7253.5	7148.5	7200.0	1h57m	967	147	>60h	7253.5
R1_4_9	8674.8	8545.7	8618.4	13m45s	611	1265	>60h	8674.8
R1_4_10	8088.9	7944.9	8024.2	31m57s	782	611	>60h	8088.9

Table 10: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class R1

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB
R2_4_1	7522.1	7485.3	7520.7	42m42s	96	1	42m42s	7520.7*
R2_4_2	6482.9	6466.3	6482.8	1h33m	127	1	1h33m	6482.8*
R2_4_3	5373.0	5349.3	5372.9	6h18m	333	1	6h18m	5372.9*
R2_4_4	4213.8	4144.1	4169.8	12h06m	1131	15	>60h	4213.8
R2_4_5	6572.1	6498.0	6559.6	10h06m	450	3	>60h	6572.1
R2_4_6	5814.6	5763.1	5811.8	24h11m	1423	3	>60h	5814.6
R2_4_7	4893.6	4843.2	4878.4	15h39m	1492	11	>60h	4893.6
R2_4_8	4001.1	3897.7	3913.9	20h11m	444	9	>60h	4001.1
R2_4_9	6070.3	6001.5	6059.4	3h44m	704	25	58h55m	6067.8*
R2_4_10	5665.1	5561.6	5619.5	4h09m	758	21	>60h	5665.1

Table 11: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class R2

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB
RC1_4_1	8523.0	8368.9	8470.0	16m38s	762	667	>60h	8523.0
RC1_4_2	7887.2	7770.3	7843.0	49m29s	1123	197	>60h	7887.2
RC1_4_3	7525.2	7423.3	7444.7	44m01s	252	269	>60h	7525.2
RC1_4_4	7295.3	7207.8	7242.9	1h43m	805	37	>60h	7295.3
RC1_4_5	8152.4	8033.8	8101.5	44m15s	1115	37	>60h	8152.4
RC1_4_6	8148.6	8005.9	8089.7	47m39s	1192	127	>60h	8148.6
RC1_4_7	7937.0	7830.6	7878.9	48m09s	841	235	>60h	7937.0
RC1_4_8	7765.8	7639.3	7679.0	42m19s	574	201	>60h	7765.8
RC1_4_9	7724.4	7604.8	7639.1	48m15s	455	209	>60h	7724.4
RC1_4_10	7583.2	7479.2	7521.6	1h42m	931	33	>60h	7583.2

Table 12: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class RC1

Instance	Initial	Robust	Root			Nodes	Total	Final
	UB	bound	bound	time	#R1C	number	time	UB

RC2_4_1	6147.4	6120.1	6146.6	3h18m	238	3	3h37m	6147.3*
RC2_4_2	5407.6	5389.5	5407.5	4h06m	458	1	4h06m	5407.5*
RC2_4_3	4573.1	4549.6	4573.0	35h25m	1294	1	35h25m	4573.0*
RC2_4_4	3598.0	3535.9	3535.9	8h25m	0	5	>60h	3598.0
RC2_4_5	5396.7	5363.0	5388.6	4h47m	466	7	>60h	5396.7
RC2_4_6	5332.1	5270.9	5308.5	9h04m	738	3	>60h	5332.1
RC2_4_7	4987.9	4944.9	4968.6	6h08m	453	19	>60h	4987.9
RC2_4_8	4703.2	4602.0	4618.7	4h53m	216	31	>60h	4703.2
RC2_4_9	4519.1	4448.9	4457.0	5h35m	278	21	>60h	4519.1
RC2_4_10	4252.4	4157.0	4157.0	2h11m	0	83	>60h	4252.4

Table 13: Detailed results for 400-customer Gehring and Homberger VRPTW instances of class RC2

Inst.	n	m	Initial UB	Robust bound	Root			Nod. num.	Total time	Final UB	Time CM	Impr. factor
					bound	time	#R1C					
p08	249	2	4372.90	4294.01	4347.67	49m01s	567	29	5h43m	4372.78*,[a]	–	–
p09	249	3	3858.80	3798.47	3849.96	1h34m	650	19	2h25m	3858.66*	–	–
p10	249	4	3631.30	3582.48	3626.26	37m26s	522	3	41m46s	3631.11*,[a]	–	–
p11	249	5	3546.20	3488.64	3540.43	44m15s	597	3	49m46s	3546.06*,[a]	–	–
p13	80	2	1319.10	1318.95	1318.95	1s	0	1	1s	1318.95	31s	17.5
p14	80	2	1360.30	1360.12	1360.12	1s	0	1	1s	1360.12	38s	20.5
p16	160	4	2572.40	2572.23	2572.23	12s	0	1	12s	2572.23	8m27s	39.7
p17	160	4	2709.20	2709.09	2709.09	55s	0	1	55s	2709.09	8m07s	8.8
p19	240	6	3827.20	3822.92	3827.06	1m25s	0	1	1m25s	3827.06	37m15s	26.3
p20	240	6	4058.20	4058.07	4058.07	2m30s	0	1	2m30s	4058.07	38m47s	15.5
p22	360	9	5702.30	5693.87	5702.16	2m28s	0	1	2m28s	5702.16*	–	–
p23	360	9	6078.90	6078.75	6078.75	5m52s	0	1	5m52s	6078.75*	–	–

[a] a solution with smaller value is claimed in Escobar et al. (2014), however the authors were not able to communicate it to us

Table 14: Detailed results for Cordeau et al. MDVRP instances of class "p"

Inst.	n	m	Initial UB	Robust bound	Root			Nod. num.	Total time	Final UB	Time CM	Impr. factor
					bound	time	#R1C					
pr01	48	4	861.40	861.32	861.32	3s	0	1	3s	861.32	0s	0.03
pr02	96	4	1307.50	1287.52	1307.34	4m18s	110	1	4m18s	1307.34*	>75h	>1042.8
pr03	144	4	1803.90	1773.42	1803.80	10m22s	185	1	10m22s	1803.80	31h11m	180.5
pr04	192	4	2058.50	2029.87	2058.31	22m52s	327	1	22m52s	2058.31*	>75h	>196.7
pr05	240	4	2331.30	2293.32	2318.80	1h14m	628	27	6h19m	2331.20*	–	–
pr06	288	4	2676.40	2635.08	2662.92	1h18m	522	21	6h39m	2676.30*	–	–
pr07	72	6	1089.70	1085.59	1089.56	17s	0	1	17s	1089.56	18m28s	63.8
pr08	144	6	1665.00	1640.18	1664.85	8m42s	105	1	8m42s	1664.85	11h53m	82.0
pr09	216	6	2133.30	2109.20	2133.20	23m06s	242	1	23m06s	2133.20*	>75h	>194.8
pr10	288	6	2868.40	2818.03	2850.68	1h40m	583	77	24h06m	2867.26*	–	–

Table 15: Detailed results for Cordeau et al. MDVRP instances of class "pr"

Instance	n	m	Initial UB	Robust bound	Root bound	Root time	Root #R1C	Nodes number	Total time	Final UB
pr01	48	4	1380.90	1377.09	1380.77	19s	0	1	19s	1380.77*
pr02	96	4	2304.00	2283.37	2303.89	2m51s	23	1	2m51s	2303.89*
pr03	144	4	2575.50	2538.93	2565.67	19m01s	248	7	24m12s	2574.56*
pr04	192	4	3450.00	3396.73	3432.33	42m22s	453	9	1h47m	3449.84*
pr05	240	4	4377.50	4288.21	4332.64	46m35s	442	217	>60h	4375.67
pr06	288	4	4422.20	4326.08	4329.25	21m45s	21	353	>60h	4421.95
pr07	72	6	1890.00	1868.89	1889.82	6m30s	0	1	6m30s	1889.82*
pr08	144	6	2971.20	2933.81	2962.61	25m44s	222	7	32m37s	2969.92*
pr09	216	6	3536.30	3511.38	3527.76	1h00m	239	5	1h03m	3536.20*
pr10	288	6	4639.80	4537.82	4559.48	48m34s	246	297	>60h	4639.80

Table 16: Detailed results for Cordeau and Laporte SDVRP instances

Instance	n	Initial UB	Robust bound	Root bound	Root time	Root #R1C	Nodes number	Total time	Final UB
CMT6	50	555.60	540.52	555.43	7s	49	1	7s	555.43*
CMT7	75	909.80	898.95	909.68	4s	35	1	4s	909.68*
CMT8	100	866.10	842.15	857.86	6m48s	319	17	54m47s	865.94*
CMT9	150	1162.70	1133.47	1153.37	7m58s	429	447	13h48m	1162.55*,[b]
CMT10	199	1396.00	1368.79	1387.35	16m22s	579	443	17h29m	1395.85*
CMT13	120	1541.30	1458.00	1475.23	10m39s	281	149	>60h	1541.30
CMT14	100	866.50	851.24	866.37	45m22s	318	1	45m22s	866.37*

[b] solution with value 1158.41, claimed in Dell'Amico et al. (2016), is infeasible (different rounding convention was used)

Table 17: Detailed results for Christofides et al. DCVRP instances

Instance	n	m	Initial UB	Robust bound	Root bound	Root time	Root #R1C	Nodes number	Total time	Final UB
DLP_75	20	3	453	448.43	452.85	0s	6	1	0s	452.85*
DLP_92	35	3	565	553.05	559.82	4m58s	110	5	9m39s	564.39*
DLP_93	39	3	1037	1011.57	1023.69	12s	46	11	1m39s	1036.99*
DLP_94	46	5	1379	1369.07	1378.25	17s	42	1	17s	1378.25*
DLP_55	56	3	10245	10134.23	10244.34	38s	29	1	38s	10244.34*
DLP_52	59	3	4028	3912.72	3931.40	2m59s	53	11	12m57s	4027.27*
DLP_10	69	4	2108	2048.67	2060.75	3m00s	77	13	18m38s	2107.55*
DLP_39	77	5	2922	2874.88	2902.55	6m03s	302	25	28m47s	2918.87*
DLP_70	78	4	6685	6561.07	6659.88	5m02s	252	3	13m53s	6684.56*
DLP_82	79	3	4767	4630.03	4648.96	1m10s	107	123	1h12m	4718.27*
DLP_08	84	3	4592	4569.97	4591.75	2s	0	1	2s	4591.75*
DLP_36	85	6	5685	5610.15	5684.62	1m29s	115	1	1m29s	5684.62*
DLP_43	86	7	8738	8686.41	8737.02	43s	31	1	43s	8737.02*
DLP_01	92	4	9211	8974.44	9081.63	10m33s	158	3	24m37s	9210.14*
DLP_11	95	4	3368	3288.08	3296.82	3m19s	31	9	20m54s	3367.41*

Table 18: Detailed results for Duhamel et al. HFVRP instances with up to 100 customers

Instance	n	m	Initial UB	Robust bound	Root bound	Root time	Root #R1C	Nodes number	Total time	Final UB
DLP_90	102	4	2347	2249.36	2257.75	5m05s	136	655	40h29m	2278.05*
DLP_17	105	3	5363	5278.74	5326.71	2m50s	131	7	3m30s	5362.83*
DLP_84	105	4	7228	7117.02	7142.28	2m02s	136	19	21m01s	7227.88*
DLP_81	106	4	10687	10483.95	10521.23	3m06s	182	313	5h36m	10583.50*
DLP_2B	107	6	8463	8300.92	8332.22	4m17s	37	1639	>60h	8463.00
DLP_07	108	4	8090	8009.35	8049.38	3m51s	104	19	12m07s	8071.97*
DLP_87	108	4	3754	3658.33	3692.55	6m32s	236	7	29m51s	3753.87*
DLP_47	111	5	16157	15971.69	16123.03	3m54s	250	5	5m49s	16156.12*
DLP_48	111	5	21310	21041.94	21228.34	19m51s	321	7	28m25s	21257.38*
DLP_61	111	3	7293	6997.61	7027.77	3m15s	33	1435	>60h	7293.00
DLP_12	112	4	3544	3410.14	3410.14	1m53s	0	25	1h39m	3543.99*
DLP_30	112	3	6314	6185.97	6207.60	4m11s	186	1489	31h29m	6278.62*
DLP_2A	113	6	7794	7708.25	7793.16	1m42s	77	1	1m42s	7793.16*
DLP_53	115	3	6435	6281.51	6376.47	12m26s	209	31	2h14m	6434.83*
DLP_05	116	5	10877	10764.90	10869.04	29s	30	1	29s	10869.04*
DLP_13	119	5	6697	6618.19	6649.75	5m20s	160	1261	>60h	6696.43
DLP_06	121	8	11689	11618.62	11682.98	2m37s	53	1	2m37s	11682.98*
DLP_03	124	4	10710	10635.69	10669.77	55s	106	5	1m13s	10709.66*
DLP_83	124	4	10020	9871.11	9899.78	2m33s	127	59	51m57s	10001.80*
DLP_68	125	4	8971	8799.96	8852.89	5m39s	192	89	3h00m	8889.03*
DLP_74	125	5	11587	11543.40	11563.64	41s	78	7	4m18s	11586.58*
DLP_21	126	3	5140	5079.13	5110.52	8m11s	289	11	25m05s	5139.84*
DLP_26	126	5	6434	6352.66	6382.52	6m00s	230	453	10h10m	6406.16*
DLP_88	127	5	12389	12328.90	12385.74	29s	25	1	29s	12385.74*
DLP_16	129	6	4157	4000.74	4006.42	4m33s	16	1573	>60h	4157.00
DLP_51	129	3	7722	7628.81	7666.22	10m11s	123	3	29m17s	7721.47*
DLP_31	131	8	4092	4045.03	4083.80	15m20s	284	5	18m24s	4091.52*
DLP_40	132	5	11119	10974.62	11015.52	2m32s	160	317	2h59m	11056.13*
DLP_89	134	5	7087	6982.53	7023.43	4m11s	239	1027	>60h	7087.00
DLP_41	135	7	7598	7444.36	7444.36	4m48s	0	3305	>60h	7598.00
DLP_34	136	6	5748	5700.07	5727.33	7m35s	306	27	1h10m	5739.02*
DLP_60	137	4	17037	16768.33	16934.97	5m40s	285	37	44m38s	17012.42*
DLP_73	137	5	10196	10138.46	10174.15	57s	88	3	2m07s	10195.33*
DLP_28	141	5	5531	5478.74	5499.59	4m24s	208	4705	>60h	5525.90
DLP_25	143	6	7207	7030.07	7042.25	5m46s	100	959	>60h	7207.00
DLP_85	146	4	8774	8690.09	8726.51	7m27s	295	111	2h40m	8763.90*
DLP_79	147	4	7260	7132.58	7165.36	8m22s	187	21	1h13m	7257.97*
DLP_66	150	4	12784	12484.59	12587.01	8m31s	104	999	>60h	12776.24

Table 19: Detailed results for Duhamel et al. HFVRP instances with 101–150 customers

Instance	n	m	Initial UB	Robust bound	Root bound	Root time	Root #R1C	Nodes number	Total time	Final UB
DLP_69	152	4	9148	9003.97	9086.90	7m52s	335	147	3h36m	9127.16*
DLP_76	152	8	11995	11900.23	11933.72	5m28s	212	1195	>60h	11994.22
DLP_56	153	4	31031	30571.34	30764.30	8m26s	250	1591	58h50m	30905.95*
DLP_86	153	5	9028	8935.26	8957.90	8m23s	274	249	8h45m	9020.63*
DLP_37	161	5	6851	6751.15	6787.92	7m57s	354	2017	>60h	6838.72
DLP_64	161	3	17136	16552.72	16552.72	6m38s	25	995	>60h	17135.16
DLP_24	163	4	9102	8990.07	8990.07	3m26s	0	1907	>60h	9102.00
DLP_57	163	4	44782	44302.32	44409.19	8m53s	194	1565	>60h	44782.00

DLP_29	164	4	9143	9044.54	9078.84	9m59s	182	43	2h18m	9132.03*
DLP_09	167	5	7604	7543.04	7563.84	12m52s	274	135	2h16m	7599.72*
DLP_35	168	6	9556	9492.98	9521.16	2m55s	114	3	3m17s	9522.45*
DLP_45	170	3	10477	10294.91	10343.43	8m34s	124	929	>60h	10477.00
DLP_80	171	3	6817	6712.75	6716.88	8m39s	29	219	13h07m	6816.89*
DLP_44	172	3	12192	11895.60	11895.60	12m31s	19	1187	>60h	12192.00
DLP_54	172	4	10352	10089.37	10089.37	7m45s	21	1323	>60h	10352.00
DLP_67	172	5	10885	10604.14	10662.94	9m53s	233	325	23h44m	10772.81*
DLP_63	174	5	19952	19670.12	19799.92	8m54s	108	97	5h24m	19890.65*
DLP_14	176	4	5645	5511.85	5511.85	4m37s	0	1023	>60h	5639.98
DLP_42	178	7	10818	10615.08	10615.08	7m00s	0	889	>60h	10805.94
DLP_02	181	4	11676	11603.82	11632.01	4m27s	104	25	9m34s	11649.81*
DLP_04	183	4	10749	10582.45	10673.39	9m41s	277	985	13h53m	10714.84*
DLP_95	183	2	6176	6100.98	6119.06	6m53s	290	1709	>60h	6170.20
DLP_71	186	3	9835	9711.77	9771.86	10m30s	292	87	4h08m	9798.06*
DLP_72	186	4	5904	5780.39	5831.66	9m35s	322	365	31h31m	5870.43*
DLP_50	187	6	12371	12035.29	12035.29	9m56s	0	1521	>60h	12371.00
DLP_15	188	7	8221	8083.44	8083.44	3m27s	0	873	>60h	8221.00
DLP_33	189	7	9411	9248.07	9283.05	9m51s	135	1129	>60h	9333.39
DLP_77	190	3	6917	6821.36	6826.04	7m49s	10	1215	>60h	6917.00
DLP_78	190	4	7036	6899.96	6899.96	6m40s	0	1099	>60h	7036.00
DLP_59	193	6	14283	13786.96	13786.96	23m42s	0	737	>60h	14283.00
DLP_91	196	4	6375	6228.90	6244.50	10m26s	157	1167	>60h	6348.36

Table 20: Detailed results for Duhamel et al. HFVRP instances with 151–200 customers

Instance	n	m	Initial	Robust	Root			Nodes	Total	Final
			UB	bound	bound	time	#R1C	number	time	UB
DLP_23	203	4	7742	7590.28	7603.76	8m46s	70	1043	>60h	7742.00
DLP_38	205	5	11195	11048.66	11092.13	14m28s	375	869	>60h	11195.00
DLP_58	220	6	23371	22800.97	22800.97	36m42s	10	969	>60h	23371.00
DLP_27	220	5	8423	8315.95	8343.41	10m15s	174	687	>60h	8423.00
DLP_65	223	3	13044	12648.64	12648.64	41m17s	24	657	>60h	13044.00
DLP_19	224	5	11687	11503.00	11526.84	8m25s	98	1053	>60h	11687.00
DLP_62	225	5	23011	22594.36	22645.25	13m36s	47	1047	>60h	22987.90
DLP_22	239	2	13069	12923.02	12956.02	13m12s	357	1193	>60h	13050.00
DLP_32	244	8	9383	9205.76	9235.38	14m29s	108	695	>60h	9383.00
DLP_49	246	8	16182	15681.80	15681.80	15m27s	0	735	>60h	16182.00
DLP_46	250	5	24567	23856.71	23856.71	14m14s	21	913	>60h	24567.00
DLP_18	256	5	9653	9491.55	9547.80	22m29s	519	1159	>60h	9649.05

Table 21: Detailed results for Duhamel et al. HFVRP instances with more than 200 customers

References

Contardo C, Martinelli R, 2014 *A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints. Discrete Optimization* 12:129 – 146.

Dell'Amico M, Díaz JCD, Hasle G, Iori M, 2016 *An adaptive iterated local search for the mixed capacitated general routing problem. Transportation Science* 50(4):1223–1238.

Escobar JW, Linfati R, Toth P, Baldoquin MG, 2014 *A hybrid granular tabu search algorithm for the multi-depot vehicle routing problem. Journal of Heuristics* 20(5):483–509.

Kallehauge B, Larsen J, Madsen OB, 2006 *Lagrangian duality applied to the vehicle routing problem with time windows. Computers and Operations Research* 33(5):1464 – 1487.

Pecin D, Contardo C, Desaulniers G, Uchoa E, 2017 *New enhancements for the exact solution of the vehicle routing problem with time windows. INFORMS Journal on Computing* 29(3):489–502.