

1. Appendix

1.1. IIWM model

$$\min \sum_{d \in \mathcal{D}} (c_d^l \cdot \delta^{less} + c_d^m \cdot \delta^{more}) + \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} r_{t,d} \cdot \alpha_{d_t,d} + \sum_{t \in \mathcal{T}} x_t \cdot t_{d_t} \quad (1)$$

subject to:

$$\sum_{t \in \mathcal{T}_o^2} y_{t,o} = 1 \quad \forall o \in \mathcal{O} \quad (2)$$

$$\sum_{\substack{o \in \mathcal{O} \\ d_o = d \\ d_o \neq st_t^o}} y_{t,o} \cdot si_o \leq r_{t,d} \cdot cap^{truck} \quad \forall t \in \mathcal{T}, d \in \mathcal{D} \quad (3)$$

$$x_t \geq y_{t,o} \quad t \in \mathcal{T}, o \in \mathcal{O} \quad (4)$$

$$\sum_{o \in \mathcal{O}} y_{t,o} \cdot si_o \leq cap^{train} \cdot x_t \quad \forall t \in \mathcal{T} \quad (5)$$

$$c_d^l \geq L_d - \sum_{t \in \mathcal{T}_d} x_t \quad \forall d \in \mathcal{D} \quad (6)$$

$$c_d^m \geq \sum_{t \in \mathcal{T}_d} x_t - U_d \quad \forall d \in \mathcal{D} \quad (7)$$

$$\sum_{o \in \mathcal{O}_c} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} y_{t,o} I_{t,o,s} \leq (1 - sl_c) \cdot |\mathcal{O}_c| \cdot |\mathcal{S}| \quad \forall c \in \mathcal{C} \quad (8)$$

$$x_t, y_{t,o}, t_{o,s} \in \{0, 1\} \quad \forall t \in \mathcal{T}, o \in \mathcal{O}, s \in \mathcal{S} \quad (9)$$

$$c_d^l, c_d^m \in \mathbb{R}^+ \quad \forall t \in \mathcal{T}, o \in \mathcal{O} \quad (10)$$

$$r_{t,d} \in \mathbb{N} \quad \forall t \in \mathcal{T}, d \in \mathcal{D}, s \in \mathcal{S} \quad (11)$$

1.2. Proof of Proposition

We restate proposition ?? hereby for convenience.

PROPOSITION 1. *Consider the following two polytopes.*

$$\mathcal{P}_1 = \begin{cases} \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} \tau_{tos\phi} y_{tos\phi} \leq dt_o + M z_{os}, & \forall o \in \mathcal{O}, \forall s \in \mathcal{S} & (12a) \\ \sum_{o \in \mathcal{O}_c} \sum_{s \in \mathcal{S}} z_{os} \leq (1 - sl_c) |\mathcal{O}_c| |\mathcal{S}|, & \forall c \in \mathcal{C} & (12b) \\ \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} y_{tos\phi} = 1, & \forall o \in \mathcal{O}, \forall s \in \mathcal{S} & (12c) \\ 0 \leq z_{os} \leq 1, & \forall o \in \mathcal{O}, \forall s \in \mathcal{S} & (12d) \\ 0 \leq y_{tos\phi} \leq 1, & \forall t \in \mathcal{T}, \forall o \in \mathcal{O}, \forall s \in \mathcal{S}, \forall \phi \in \Phi_t & (12e) \end{cases}$$

$$\mathcal{P}_2 = \begin{cases} \sum_{o \in \mathcal{O}_c} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} I_{tos\phi} y_{tos\phi} \leq (1 - sl_c) |\mathcal{O}_c| |\mathcal{S}|, & \forall c \in \mathcal{C} & (13a) \\ \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} y_{tos\phi} = 1, & \forall o \in \mathcal{O}, \forall s \in \mathcal{S} & (13b) \\ 0 \leq y_{t,o,s,\phi} \leq 1, & \forall t \in \mathcal{T}, \forall o \in \mathcal{O}, \forall s \in \mathcal{S}, \forall \phi \in \Phi_t & (13c) \end{cases}$$

Where $I_{tos\phi} = \{1, \text{if } \tau_{tso\phi} > dt_o; 0, \text{ otherwise}\}$.

Then, $\mathcal{P}_2 \subset \text{proj}_y(\mathcal{P}_1)$, where $\text{proj}_y(\mathcal{P})$ denotes the projection of polytope \mathcal{P} on the y -space.

Proof. The proof consists of two parts. First, we show that $\mathcal{P}_2 \subseteq \text{proj}_y(\mathcal{P}_1)$, and then we show that there exists a point $y_{t,o,s,\phi} \in \text{proj}_y(\mathcal{P}_1)$ which does not belong to \mathcal{P}_2 .

For the first part, we consider an arbitrary $y_{t,o,s,\phi} \in \mathcal{P}_2$ and show that there exists a $z_{s,o}$ such that $(y_{t,o,s,\phi}, z_{o,s}) \in \mathcal{P}_1$. Indeed, setting $z_{o,s} = \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} I_{tos\phi} y_{tos\phi}$ makes (12b) identical to (13a). Also, $0 \leq \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} I_{tos\phi} y_{tos\phi} \leq 1$, because of Constraints (12c) and (12e).

Obviously, Constraints (12c) holds, because it is identical to 13b. Finally, substituting z_{os} in (12a) gives:

$$\begin{aligned} & \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} \tau_{tos\phi} y_{tos\phi} \leq dt_o + M z_{os} \\ \Rightarrow & \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} \tau_{tos\phi} y_{tos\phi} \leq dt_o + M \left(\sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} I_{tos\phi} y_{tos\phi} \right) \\ \Rightarrow & \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} (y_{t,o,s,\phi} (\tau_{t,o,s,\phi} - M I_{t,o,s,\phi})) \leq dt_o \\ \Rightarrow & \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} (y_{t,o,s,\phi} (M I_{t,o,s,\phi} - \tau_{t,o,s,\phi}) + dt_o) \geq 0 \end{aligned}$$

which is always satisfied for $M \geq \max\{\tau_{tos\phi}\}$. Note that M could be a little smaller, but that does not matter regarding the proof. This completes the first part of the proof.

For the second part, we consider an instance where:

- $sl_c > 0$ for some $c \in \mathcal{C}$
- $\tau_{t,o,s,\phi} = dt_o + \epsilon, \forall t, o, s, \phi$ and $\epsilon > 0$

For this instance, it holds by definition that $I_{tos\phi} = 1, \forall t, o, s, \phi$, because every order is delivered too late in every scenario. We employ the following parametric family of points:

$$(y_{tos\phi}, z_{os}) = \left(\frac{1}{|\mathcal{T}| |\Phi_t|}, \frac{1}{\lambda |\mathcal{O}_c| |\mathcal{S}|} \right)$$

Here, $\lambda > 0$. The y -projection of this family is clearly not in \mathcal{P}_2 because (13a) is infeasible:

$$\begin{aligned} & \sum_{o \in \mathcal{O}_c} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} 1 \cdot \frac{1}{|\mathcal{T}| |\Phi_t|} \leq (1 - sl_c) |\mathcal{O}_c| |\mathcal{S}| \\ \Rightarrow & |\mathcal{O}_c| |\mathcal{S}| \leq (1 - sl_c) |\mathcal{O}_c| |\mathcal{S}| \end{aligned} \quad (14)$$

Which is infeasible, because $0 \leq (1 - sl_c) < 1$.

Now we must show that this point is in \mathcal{P}_1 . First of all, substituting it in (12a) gives:

$$\begin{aligned} \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} (dt_o + \epsilon) \frac{1}{|\mathcal{T}||\mathcal{Q}_t|} &\leq dt_o + \frac{M}{\lambda|\mathcal{O}_s||\mathcal{S}|} \\ M &\geq \lambda|\mathcal{O}_s||\mathcal{S}|(5dt_o - \sum_{t \in \mathcal{T}} \sum_{\phi \in \Phi_t} \frac{dt_o + \epsilon}{|\mathcal{T}||\Phi_t|}) \\ M &\geq \lambda|\mathcal{O}_s||\mathcal{S}|\epsilon \end{aligned}$$

Thus shows that every $M \geq \epsilon\lambda|\mathcal{O}_c||\mathcal{S}| > 0$ guarantees feasibility for constraint (12a). Finally, substituting $z_{o,s}$ in (12b) gives us:

$$\begin{aligned} \sum_{o \in \mathcal{O}_c} \sum_{s \in \mathcal{S}} \frac{1}{\lambda|\mathcal{O}_c||\mathcal{S}|} &\leq (1 - sl_c)|\mathcal{O}_c||\mathcal{S}| \\ \frac{1}{\lambda} &\leq (1 - sl_c)|\mathcal{O}_c||\mathcal{S}| \\ \lambda &\geq \frac{1}{(1 - sl_c)|\mathcal{O}_c||\mathcal{S}|} \end{aligned}$$

Thus every $\lambda \geq 1/((1 - sl_c)|\mathcal{O}_c||\mathcal{S}|) > 0$ makes Constraints (12a) feasible. This shows that some members of this parametric family are feasible for \mathcal{P}_1 , which completes the proof. \square

1.3. Efficient number of scenarios

In theory, an infinite number of disruption scenarios could be used in the models in order to take all possible delays into account. However, taking a large number of disruption scenarios into account makes the model very challenging. Furthermore, the effects of adding an extra disruption scenario diminishes with the number of scenarios present. Therefore, in this section we investigate after how much scenarios the value of the objective function remains constant. To that end, 20 different instances with 3 destinations, 10 customers, one trip to each destination per day, and 4 orders on average per customer are used.

In total 150 disruption scenarios are available for each instance. Each of the 20 instances is solved while including 10, 20, 30, \dots , 100, and 150 disruption scenarios. The ADLM is not able to solve these instances up to 150 disruption scenarios. Therefore, the RHH will be used to tackle the problem instances and Figure 1 gives an overview of the results. The average objective value over all 20 instances is displayed in the figure for every number of disruption scenarios included.

As can be seen, the average objective value stabilizes after around 60 scenarios. The objective function is increasing before, but now seems to stabilize. In order to be on the safe side, we decided to use 70 disruption scenarios for the large instances.

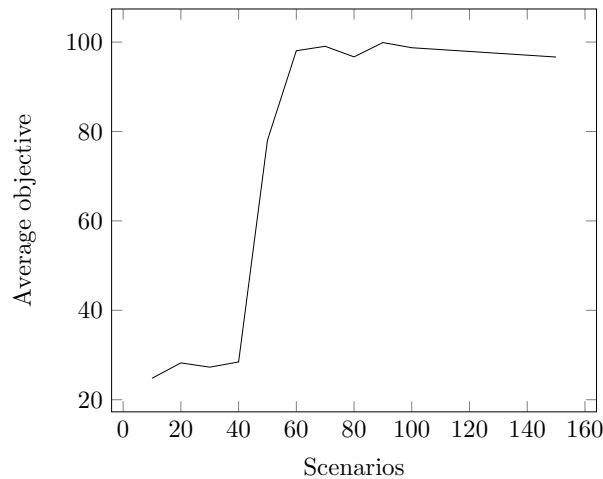


Figure 1 Average objective value over the 20 instances while including 10 up to 150 scenarios.

1.4. Impact penalty External Resources

The approach used in this paper so far has been motivated by practice and one of the aspects included is the possibility to use external resources able to transport customers orders that are late anyway. All computational experiments so far consider the use of these external resources to be just as expensive as the use of a single train. This is selected in this way because the model focuses on the tactical planning phase and in practice the orders that are scheduled to be delivered by external resources are usually squeezed in during the operational planning phase. So far, in this paper the main disadvantage of using external resources is that the orders delivered on them are always delivered late and influences the service level consequently. In this paragraph we will test whether the assumption of having a relatively small external resource penalty will influence the results significantly.

The costs of using external resources depends on the distance between origin and destination. The larger the distance is, the larger the costs of using these connections will be. On average the costs of using external resources is now five times higher than using a regular train. Consequently, the model tends to not use the external resources, unless the benefits in terms of additional trains and using trucks between destinations outweigh the extra costs.

Table 1 shows the same type of results as in Table ??, only now using a larger penalty for the external resources. As can be seen, the objective value has slightly increased while the use of external resources diminishes. Consequently, the number of trips and number of trucks used throughout the planning horizon have increased. However, there are never additional trains used and the adjustment in the tactical schedule is not significant.

There is only one noticeable difference between the two situations. In case of a large penalty on external resources there is a relatively small difference in the resulting tactical schedule between different service levels in case *no additional trains* are necessary. For instance, if the service level is 75%, the model will try to send all customer orders to its destination by train in case this does not lead to using any additional trains. Consequently, the resulting tactical schedule is similar to other service levels in case the same number of trains are used.

Service level	Objective	# Trains	# Trips	# Trucks	Orders Last
0.75	3711	3.50	15.10	12.10	11.20
0.80	4115	3.90	15.80	14.90	9.50
0.85	4621	4.40	17.30	16.60	6.60
0.90	5008	4.80	17.80	17.40	3.10
0.95	5807	5.60	18.70	18.10	0.90

Table 1 Average results regarding service level.

Our methods are able to include both small and large penalties on using external resources, without results completely changing. It is dependent on the setting used by a practitioner which of the two situations will actually be used.