

Electronic Companion

This electronic companion provides supplementary materials to the manuscript “*Liner Shipping Service Planning under Sulfur Emission Regulations*.”

EC.1. Proof of Proposition 1

PROOF. Instead of using sailing speeds as the decision variables, we define the sailing time of a type- k ship outside the ECA on path p of leg i of route r as t_{krip}^N , and thus the sailing time within the ECA of the leg is $t_{ri} - t_{krip}^N$. The fuel cost function, denoted by $h_{krip}(t_{krip}^N)$, for the ship to sail on path p of leg i of route r with sailing time t_{krip}^N outside the ECA, can be written as

$$h_{krip}(t_{krip}^N) = \alpha_k^E L_{rip}^E a_{kri} \cdot \left(\frac{L_{rip}^E}{t_{ri} - t_{krip}^N} \right)^{b_{kri}} + \alpha_k^N L_{rip}^N a_{kri} \cdot \left(\frac{L_{rip}^N}{t_{krip}^N} \right)^{b_{kri}},$$

$$\frac{L_{rip}^N}{V^{\max}} \leq t_{krip}^N \leq t_{ri} - \frac{L_{rip}^E}{V^{\max}}. \quad (\text{EC.1})$$

It is easy to prove that $h_{krip}(t_{krip}^N)$ is a convex function of t_{krip}^N . Note that the relationship between the optimal value of t_{krip}^N , denoted by t_{krip}^{N*} , that minimizes $h_{krip}(t_{krip}^N)$ and the optimal solution to model (12), denoted by v_{krip}^{E*} and v_{krip}^{N*} , is $v_{krip}^{E*} = L_{rip}^E / (t_{ri} - t_{krip}^{N*})$ and $v_{krip}^{N*} = L_{rip}^N / t_{krip}^{N*}$. The value of t_{krip}^N at which the first-order derivative of $h_{krip}(t_{krip}^N)$ is equal to 0, denoted by \bar{t}_{krip}^N , can be calculated as

$$\bar{t}_{krip}^N = \frac{L_{rip}^N}{\gamma_{kri} L_{rip}^E + L_{rip}^N} t_{ri}. \quad (\text{EC.2})$$

When $t_{ri} \in (\hat{t}_{krip}, \infty)$, we have $\frac{L_{rip}^N}{V^{\max}} < \bar{t}_{krip}^N < t_{ri} - \frac{L_{rip}^E}{V^{\max}}$, which implies $t_{krip}^{N*} = \bar{t}_{krip}^N$. We can therefore calculate

$$v_{krip}^{E*} = \frac{L_{rip}^E + \frac{1}{\gamma_{kri}} L_{rip}^N}{t_{ri}} < V^{\max} \quad (\text{EC.3})$$

$$v_{krip}^{N*} = \frac{\gamma_{kri} L_{rip}^E + L_{rip}^N}{t_{ri}} < V^{\max} \quad (\text{EC.4})$$

$$\frac{v_{krip}^{E*}}{v_{krip}^{N*}} = \frac{1}{\gamma_{kri}}. \quad (\text{EC.5})$$

When $t_{ri} \in [t_{rip}^{\min}, \hat{t}_{krip}]$, we have $\bar{t}_{krip}^N \leq \frac{L_{rip}^N}{V^{\max}}$, which implies $t_{krip}^{N*} = L_{rip}^N / V^{\max}$. We hence have

$$v_{krip}^{E*} = \frac{L_{rip}^E}{t_{ri} - \frac{L_{rip}^N}{V^{\max}}} \leq V^{\max} \quad (\text{EC.6})$$

$$v_{krip}^{N*} = V^{\max}. \quad (\text{EC.7})$$

EC.2. Proof of Proposition 2

PROOF. Substituting the optimal speeds within and outside the ECA on path p of leg i of route r (i.e., v_{krip}^{E*} and v_{krip}^{N*}) calculated in Proposition 1 into the objective function of model (12), we can obtain the two-piece continuous function $g_{krip}(t_{ri})$. \square

EC.3. Proof of Proposition 3

PROOF. The first-order derivative of $g_{krip}(t_{ri})$ is

$$\frac{dg_{krip}(t_{ri})}{dt_{ri}} = \begin{cases} -\alpha_k^E a_{kri} b_{kri} \cdot (L_{rip}^E)^{1+b_{kri}} \cdot (t_{ri} - \frac{L_{rip}^N}{V_{\max}})^{-(1+b_{kri})}, & t_{rip}^{\min} \leq t_{ri} \leq \hat{t}_{krip}, \\ -\alpha_k^N a_{kri} b_{kri} \cdot (\gamma_{kri} L_{rip}^E + L_{rip}^N)^{1+b_{kri}} \cdot (t_{ri})^{-(1+b_{kri})}, & t_{ri} \geq \hat{t}_{krip}. \end{cases} \quad (\text{EC.8})$$

In Eq. (EC.8), the derivative for $t_{ri} = t_{rip}^{\min}$ refers to the right derivative. When $t_{ri} = \hat{t}_{krip}$, the left derivative of $g_{krip}(t_{ri})$ can be acquired by the first piece of the function (EC.8), the right derivative can be obtained by the second piece, and the left and the right derivatives are equal, which indicates that $g_{krip}(t_{ri})$ is differentiable at $t_{ri} = \hat{t}_{krip}$.

The second-order derivative of $g_{krip}(t_{ri})$ is calculated as

$$\frac{d^2 g_{krip}(t_{ri})}{d(t_{ri})^2} = \begin{cases} \alpha_k^E a_{kri} b_{kri} \cdot (1 + b_{kri}) \cdot (L_{rip}^E)^{1+b_{kri}} \cdot (t_{ri} - \frac{L_{rip}^N}{V_{\max}})^{-(2+b_{kri})}, & t_{rip}^{\min} \leq t_{ri} \leq \hat{t}_{krip}, \\ \alpha_k^N a_{kri} b_{kri} \cdot (1 + b_{kri}) \cdot (\gamma_{kri} L_{rip}^E + L_{rip}^N)^{1+b_{kri}} \cdot (t_{ri})^{-(2+b_{kri})}, & t_{ri} \geq \hat{t}_{krip}. \end{cases} \quad (\text{EC.9})$$

Similarly, only the right second-order derivative exists when $t_{ri} = t_{rip}^{\min}$, and we can obtain the left second-order derivative by the first piece of the function (EC.9) and the right second-order derivative by the second piece when $t_{ri} = \hat{t}_{krip}$. In contrast to $dg_{krip}(t_{ri})/dt_{ri}$, $d^2 g_{krip}(t_{ri})/d(t_{ri})^2$ is discontinuous at $t_{ri} = \hat{t}_{krip}$ as the left and right second-order derivatives are not equal.

According to Eq. (EC.9), the second-order derivative of $g_{krip}(t_{ri})$ is greater than 0 when $t_{rip}^{\min} \leq t_{ri} < \hat{t}_{krip}$ and when $t_{ri} > \hat{t}_{krip}$. Therefore, $g_{krip}(t_{ri})$ is convex over $[t_{rip}^{\min}, \hat{t}_{krip})$ and over $(\hat{t}_{krip}, +\infty)$. In addition, given its differentiability at $t_{ri} = \hat{t}_{krip}$, $g_{krip}(t_{ri})$ is a convex function over the whole domain $[t_{rip}^{\min}, +\infty)$. \square

EC.4. Proof of Proposition 5

PROOF. Suppose that there is only one route $r = 1$ in the network. $\Gamma_r = 0$. The route has two legs. Leg 1 has two navigable paths. Path 1 is fully within the ECA, $L_{r,1,1}^E = 5000$, $L_{r,1,1}^N = 0$. Path 2 is fully outside the ECA, $L_{r,1,2}^E = 0$, $L_{r,1,2}^N = 12000$. Leg 2 has the same two navigable paths as leg 1. Consider two types of ships: type-1 is traditional ships and type-2 is scrubber-equipped ships. $\alpha_1^N = \alpha_2^E = \alpha_2^N = 0$, $\alpha_1^E > 0$. The fuel consumption rate functions are the same for the two types of

ships and the two legs, that is, $a_{kri} = a$ and $b_{kri} = b$ for $k = 1, 2$, $r = 1$, and $i = 1, 2$. $V^{\max} = 20$. We can calculate that

$$C_r(m_{1,r}, m_{2,r}) = \begin{cases} \frac{m_{1,r}}{m_{1,r} + m_{2,r}} \left\{ \alpha_1^E \cdot 2L_{r,1,1}^E \cdot a \cdot \left[\frac{2L_{r,1,1}^E}{W(m_{1,r} + m_{2,r})} \right]^b \right\}, & 3 \leq m_{1,r} + m_{2,r} \leq 7 \\ 0, & m_{1,r} + m_{2,r} \geq 8. \end{cases}$$

Define $\Xi := \alpha_1^E \cdot 2L_{r,1,1}^E \cdot a \cdot \left(\frac{2L_{r,1,1}^E}{W} \right)^b$. We have

$$C_r(5, 0) = 0.040\Xi$$

$$C_r(6, 0) = 0.028\Xi$$

$$C_r(7, 0) = 0.020\Xi$$

$$C_r(8, 0) = 0,$$

$2C_r(6, 0) < C_r(5, 0) + C_r(7, 0)$, and $2C_r(7, 0) > C_r(6, 0) + C_r(8, 0)$, which proves that $C_r(m_{1,r}, m_{2,r})$ is not convex or concave in $m_{1,r}$. We further have

$$C_r(5, 0) = 0.040\Xi$$

$$C_r(5, 1) = 0.023\Xi$$

$$C_r(5, 2) = 0.015\Xi$$

$$C_r(5, 3) = 0,$$

$2C_r(5, 1) < C_r(5, 0) + C_r(5, 2)$, and $2C_r(5, 2) > C_r(5, 1) + C_r(5, 3)$, which proves that $C_r(m_{1,r}, m_{2,r})$ is not convex or concave in $m_{2,r}$. \square

EC.5. Proof of Lemma 1

PROOF. Denote by \mathbf{t}_r^* the optimal schedule for route r given ship deployment vector $\mathbf{m}_r - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}$. We have

$$C_r(\mathbf{m}_r - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) = \sum_{i=1}^{I_r} \left[\frac{m_{k_1,r} - 1}{\sum_{k' \in K} m_{k'r}} c_{k_1ri}(t_{ri}^*) + \frac{m_{k_2,r} + 1}{\sum_{k' \in K} m_{k'r}} c_{k_2ri}(t_{ri}^*) + \sum_{k \in K \setminus \{k_1, k_2\}} \frac{m_{kr}}{\sum_{k' \in K} m_{k'r}} c_{kri}(t_{ri}^*) \right]. \quad (\text{EC.10})$$

As \mathbf{t}_r^* is a feasible but not necessarily optimal schedule for route r when the ship deployment vector is \mathbf{m}_r or $\mathbf{m}_r - 2\mathbf{e}_{k_1} + 2\mathbf{e}_{k_2}$, we have

$$C_r(\mathbf{m}_r) \leq \sum_{i=1}^{I_r} \left[\frac{m_{k_1,r}}{\sum_{k' \in K} m_{k'r}} c_{k_1ri}(t_{ri}^*) + \frac{m_{k_2,r}}{\sum_{k' \in K} m_{k'r}} c_{k_2ri}(t_{ri}^*) + \sum_{k \in K \setminus \{k_1, k_2\}} \frac{m_{kr}}{\sum_{k' \in K} m_{k'r}} c_{kri}(t_{ri}^*) \right] \quad (\text{EC.11})$$

and

$$C_r(\mathbf{m}_r - 2\mathbf{e}_{k_1} + 2\mathbf{e}_{k_2}) \leq \sum_{i=1}^{I_r} \left[\frac{m_{k_1,r} - 2}{\sum_{k' \in K} m_{k'r}} c_{k_1 r i}(t_{ri}^*) + \frac{m_{k_2,r} + 2}{\sum_{k' \in K} m_{k'r}} c_{k_2 r i}(t_{ri}^*) + \sum_{k \in K \setminus \{k_1, k_2\}} \frac{m_{kr}}{\sum_{k' \in K} m_{k'r}} c_{k r i}(t_{ri}^*) \right]. \quad (\text{EC.12})$$

Adding the left-hand sides and right-hand sides of Eqs. (EC.10)–(EC.12) we obtain $C_r(\mathbf{m}_r) - C_r(\mathbf{m}_r - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) \leq C_r(\mathbf{m}_r - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) - C_r(\mathbf{m}_r - 2\mathbf{e}_{k_1} + 2\mathbf{e}_{k_2})$. \square

EC.6. Proof of Theorem 1

PROOF. Suppose we have an optimal solution \mathbf{m}^* in which there are two routes r_1 and r_2 and two types of ships k_1 and k_2 such that $m_{k_1, r_1}^* \geq 1$, $m_{k_2, r_1}^* \geq 1$, $m_{k_1, r_2}^* \geq 1$, and $m_{k_2, r_2}^* \geq 1$.

Without loss of generality, suppose

$$C_{r_1}(\mathbf{m}_{r_1}^*) - C_{r_1}(\mathbf{m}_{r_1}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) \geq C_{r_2}(\mathbf{m}_{r_2}^*) - C_{r_2}(\mathbf{m}_{r_2}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}). \quad (\text{EC.13})$$

We consider three cases.

Case (i): $m_{k_1, r_1}^* < m_{k_2, r_2}^*$. Consider the new solution denoted by $\hat{\mathbf{m}}$ in which $\hat{\mathbf{m}}_{r_1} = \mathbf{m}_{r_1}^* - m_{k_1, r_1}^* \mathbf{e}_{k_1} + m_{k_1, r_1}^* \mathbf{e}_{k_2}$, $\hat{\mathbf{m}}_{r_2} = \mathbf{m}_{r_2}^* + m_{k_1, r_1}^* \mathbf{e}_{k_1} - m_{k_1, r_1}^* \mathbf{e}_{k_2}$, and $\hat{\mathbf{m}}_r = \mathbf{m}_r^*$, $r \in \{1, \dots, R\} \setminus \{r_1, r_2\}$. According to Lemma 1,

$$\begin{aligned} & C_{r_1}(\mathbf{m}_{r_1}^*) - C_{r_1}(\mathbf{m}_{r_1}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) \\ & \leq C_{r_1}(\mathbf{m}_{r_1}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) - C_{r_1}(\mathbf{m}_{r_1}^* - 2\mathbf{e}_{k_1} + 2\mathbf{e}_{k_2}) \\ & \leq C_{r_1}(\mathbf{m}_{r_1}^* - 2\mathbf{e}_{k_1} + 2\mathbf{e}_{k_2}) - C_{r_1}(\mathbf{m}_{r_1}^* - 3\mathbf{e}_{k_1} + 3\mathbf{e}_{k_2}) \\ & \leq \dots \\ & \leq C_{r_1}(\mathbf{m}_{r_1}^* - (m_{k_1, r_1}^* - 1)\mathbf{e}_{k_1} + (m_{k_1, r_1}^* - 1)\mathbf{e}_{k_2}) - C_{r_1}(\mathbf{m}_{r_1}^* - m_{k_1, r_1}^* \mathbf{e}_{k_1} + m_{k_1, r_1}^* \mathbf{e}_{k_2}). \end{aligned} \quad (\text{EC.14})$$

Therefore,

$$C_{r_1}(\mathbf{m}_{r_1}^*) - C_{r_1}(\hat{\mathbf{m}}_{r_1}) \geq m_{k_1, r_1}^* [C_{r_1}(\mathbf{m}_{r_1}^*) - C_{r_1}(\mathbf{m}_{r_1}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2})]. \quad (\text{EC.15})$$

Similarly,

$$\begin{aligned} & C_{r_2}(\mathbf{m}_{r_2}^*) - C_{r_2}(\mathbf{m}_{r_2}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2}) \\ & \geq C_{r_2}(\mathbf{m}_{r_2}^* + \mathbf{e}_{k_1} - \mathbf{e}_{k_2}) - C_{r_2}(\mathbf{m}_{r_2}^*) \\ & \geq C_{r_2}(\mathbf{m}_{r_2}^* + 2\mathbf{e}_{k_1} - 2\mathbf{e}_{k_2}) - C_{r_2}(\mathbf{m}_{r_2}^* + \mathbf{e}_{k_1} - \mathbf{e}_{k_2}) \\ & \geq \dots \\ & \geq C_{r_2}(\mathbf{m}_{r_2}^* + m_{k_1, r_1}^* \mathbf{e}_{k_1} - m_{k_1, r_1}^* \mathbf{e}_{k_2}) - C_{r_2}(\mathbf{m}_{r_2}^* + (m_{k_1, r_1}^* - 1)\mathbf{e}_{k_1} - (m_{k_1, r_1}^* - 1)\mathbf{e}_{k_2}) \end{aligned} \quad (\text{EC.16})$$

and therefore

$$C_{r_2}(\hat{\mathbf{m}}_{r_2}) - C_{r_2}(\mathbf{m}_{r_2}^*) \leq m_{k_1, r_1}^* [C_{r_2}(\mathbf{m}_{r_2}^*) - C_{r_2}(\mathbf{m}_{r_2}^* - \mathbf{e}_{k_1} + \mathbf{e}_{k_2})]. \quad (\text{EC.17})$$

Eqs. (EC.13), (EC.15), and (EC.17) imply

$$C_{r_1}(\hat{\mathbf{m}}_{r_1}) + C_{r_2}(\hat{\mathbf{m}}_{r_2}) \leq C_{r_1}(\mathbf{m}_{r_1}^*) + C_{r_2}(\mathbf{m}_{r_2}^*). \quad (\text{EC.18})$$

Hence, this new solution $\hat{\mathbf{m}}$, in which route r_1 does not have ships of type k_1 , is at least as good as \mathbf{m}^* .

Case (ii): $m_{k_1, r_1}^* = m_{k_2, r_2}^*$. Consider the new solution denoted by $\hat{\mathbf{m}}$ in which $\hat{\mathbf{m}}_{r_1} = \mathbf{m}_{r_1}^* - m_{k_1, r_1}^* \mathbf{e}_{k_1} + m_{k_1, r_1}^* \mathbf{e}_{k_2}$, $\hat{\mathbf{m}}_{r_2} = \mathbf{m}_{r_2}^* + m_{k_1, r_1}^* \mathbf{e}_{k_1} - m_{k_1, r_1}^* \mathbf{e}_{k_2}$, and $\hat{\mathbf{m}}_r = \mathbf{m}_r^*$, $r \in \{1, \dots, R\} \setminus \{r_1, r_2\}$. This new solution, in which route r_1 does not have ships of type k_1 and route r_2 does not have ships of type k_2 , is at least as good as \mathbf{m}^* .

Case (iii): $m_{k_1, r_1}^* > m_{k_2, r_2}^*$. Consider the new solution denoted by $\hat{\mathbf{m}}$ in which $\hat{\mathbf{m}}_{r_1} = \mathbf{m}_{r_1}^* - m_{k_2, r_2}^* \mathbf{e}_{k_1} + m_{k_2, r_2}^* \mathbf{e}_{k_2}$, $\hat{\mathbf{m}}_{r_2} = \mathbf{m}_{r_2}^* + m_{k_2, r_2}^* \mathbf{e}_{k_1} - m_{k_2, r_2}^* \mathbf{e}_{k_2}$, and $\hat{\mathbf{m}}_r = \mathbf{m}_r^*$, $r \in \{1, \dots, R\} \setminus \{r_1, r_2\}$. This new solution, in which route r_2 does not have ships of type k_2 , is at least as good as \mathbf{m}^* .

By repeating the above procedure, we can obtain an optimal solution in which any two routes have at most one common type of ship deployed. \square

EC.7. Proof of Proposition 7

PROOF. Denote by (y^*, \mathbf{m}^*) the optimal solution to [P3'], $y^* \geq 1$. Suppose that in the optimal solution, there exists a route \hat{r} such that $m_{k', \hat{r}}^* \geq 1$ and $m_{k'', \hat{r}}^* \geq 1$. Without loss of generality, suppose that one of the $m_{k'', \hat{r}}^*$ ships of type k'' deployed on route \hat{r} is a retrofitted one. Compared with not retrofitting that ship, the optimality of (y^*, \mathbf{m}^*) implies

$$u + (c_{k'} \cdot m_{k', \hat{r}}^* + c_{k''} \cdot m_{k'', \hat{r}}^*) + C_{\hat{r}}(\mathbf{m}_{\hat{r}}^*) \leq [c_{k'} \cdot (m_{k', \hat{r}}^* + 1) + c_{k''} \cdot (m_{k'', \hat{r}}^* - 1)] + C_{\hat{r}}(\mathbf{m}_{\hat{r}}^* + \mathbf{e}_{k'} - \mathbf{e}_{k''}). \quad (\text{EC.19})$$

That is,

$$C_{\hat{r}}(\mathbf{m}_{\hat{r}}^* + \mathbf{e}_{k'} - \mathbf{e}_{k''}) - C_{\hat{r}}(\mathbf{m}_{\hat{r}}^*) \geq u + c_{k''} - c_{k'}. \quad (\text{EC.20})$$

Lemma 1 implies

$$C_{\hat{r}}(\mathbf{m}_{\hat{r}}^*) - C_{\hat{r}}(\mathbf{m}_{\hat{r}}^* - m_{k', \hat{r}}^* \mathbf{e}_{k'} + m_{k', \hat{r}}^* \mathbf{e}_{k''}) \geq m_{k', \hat{r}}^* [C_{\hat{r}}(\mathbf{m}_{\hat{r}}^* + \mathbf{e}_{k'} - \mathbf{e}_{k''}) - C_{\hat{r}}(\mathbf{m}_{\hat{r}}^*)]. \quad (\text{EC.21})$$

Eqs. (EC.20)–(EC.21) imply

$$m_{k', \hat{r}}^* \cdot u + [c_{k'} \cdot 0 + c_{k''} \cdot (m_{k', \hat{r}}^* + m_{k'', \hat{r}}^*)] + C_{\hat{r}}(\mathbf{m}_{\hat{r}}^* - m_{k', \hat{r}}^* \mathbf{e}_{k'} + m_{k', \hat{r}}^* \mathbf{e}_{k''}) \leq (c_{k'} \cdot m_{k', \hat{r}}^* + c_{k''} \cdot m_{k'', \hat{r}}^*) + C_{\hat{r}}(\mathbf{m}_{\hat{r}}^*). \quad (\text{EC.22})$$

Therefore, further retrofitting all the $m_{k'\hat{r}}^*$ ships of type 1 on route \hat{r} will not increase the total cost. Retrofitting all ships of type k' on all the routes $r = 1, \dots, R$ that satisfy $m_{k''r}^* \geq 1$, we obtain a new solution that is at least as good as (y^*, \mathbf{m}^*) and in which no route has both type- k' and type- k'' of ships deployed. \square