

# Appendix to: Efficient Formulations and Decomposition Approaches for Power Peak Reduction in Railway Traffic via Timetabling

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## A Further Computational Statistics

In this appendix, we give some further computational statistics on the solution process of our models and algorithms as well as results for varying the upper bound on instantaneous peak consumption in Problem (TTMAPI).

Tables 1 and 2 detail the computational behaviour of formulations NA, TU and DF for Problem (TTMAPBR), distinguishable by Column *Form.*. Column *BB* shows the number of branch-of-bounds nodes solved while Column *Root-IP gap* shows the gap between the value of the root LP relaxation and the best integer solution found within a time limit of 10 minutes in per cent. Column *Next sol.* shows the time which passed until Gurobi finds a solution improving upon the initial timetable, which was passed to Gurobi as the first incumbent. Column *Next sav.* then shows the savings achieved with this improved solution, and, finally, Column *Best sav.* shows the best savings achieved by the best solution found within 10 minutes.

We see that the gap between the LP relaxation and the best solution found within 10 minutes by the two totally unimodular formulations TU and DF is always smaller than the produced by NA. On several of the regional and national instances, (*Regio Nordost*, *Regio Südost*, *Regio BW*, *S-Bahn Hamburg* and *Deutschland*), a significant gap remains when using formulation NA, whereas TU and DF close the gap (almost) completely. Furthermore, it becomes apparent the next better found solution for all formulations already brings large savings in most cases and is almost always already close to the savings in the optimal solution. However, we see that the totally unimodular formulations generally produce these improved solutions much faster in comparison to NA, with DF being far superior. The same holds for the number of solved branch-and-bound nodes. Note that the sparsity of DF gives it an additional edge in both respects, as Gurobi's heuristics take less time to execute and the LP relaxations can be solved much faster. In summary, these statistics explain very well why DF performed best in our computations.

Instance	Form.	#BB	Root-IP gap [%]	Next sol. [s]	Next sav. [%]	Best sav. [%]
Zeil	NA	1,297	0.33	0	7.57	14.89
Zeil	TU	19	0.32	0	6.80	14.89
Zeil	DF	14	0.32	0	11.04	14.89
Bayreuth Hbf	NA	1	0.65	0	18.24	22.18
Bayreuth Hbf	TU	31	0.65	0	18.99	22.18
Bayreuth Hbf	DF	1	0.65	0	19.71	22.18
Passau	NA	40,638	1.57	0	6.54	14.48
Passau	TU	65,317	0.30	0	10.95	14.48
Passau	DF	4,806	0.30	0	11.82	14.48
Jena Paradies	NA	12,068	0.56	2	6.90	12.46
Jena Paradies	TU	5,294	0.13	0	10.98	12.46
Jena Paradies	DF	2,887	0.13	0	10.94	12.46
Lichtenfels	NA	14,750	0.90	1	9.60	15.25
Lichtenfels	TU	10,117	0.29	1	13.76	15.25
Lichtenfels	DF	2,016	0.29	0	14.34	15.25
Erlangen	NA	1,384	2.45	10	7.13	14.83
Erlangen	TU	21,091	0.18	6	14.29	15.28
Erlangen	DF	3,523	0.18	2	14.90	15.28
Bamberg	NA	972	0.89	6	4.95	12.97
Bamberg	TU	5,203	0.05	4	11.46	13.07
Bamberg	DF	1,496	0.05	1	12.69	13.08
Aschaffenburg	NA	6,233	0.02	5	11.51	12.95
Aschaffenburg	TU	1	0.01	2	12.54	12.95
Aschaffenburg	DF	1	0.01	1	12.40	12.95
Kiel Hbf	NA	3,440	0.07	2	4.20	11.20
Kiel Hbf	TU	1	0.01	2	10.54	11.19
Kiel Hbf	DF	1	0.01	0	10.84	11.20
Leipzig Hbf (tief)	NA	1	2.25	3	5.13	6.40
Leipzig Hbf (tief)	TU	1	0.69	3	5.13	6.40
Leipzig Hbf (tief)	DF	1	0.58	0	5.13	6.40
Würzburg Hbf	NA	1,840	0.60	16	0.97	7.85
Würzburg Hbf	TU	12,089	0.03	8	8.08	8.31
Würzburg Hbf	DF	34,801	0.02	3	8.00	8.32
Dresden	NA	1	0.82	123	6.35	8.81
Dresden	TU	21,596	0.06	30	8.48	9.29
Dresden	DF	5,330	0.05	3	8.48	9.30
Ulm Hbf	NA	26	0.02	26	5.95	11.22
Ulm Hbf	TU	1	0.01	7	10.77	11.23
Ulm Hbf	DF	1	0.01	3	11.03	11.23
Stuttgart Hbf (tief)	NA	1	1.10	538	0.70	0.93
Stuttgart Hbf (tief)	TU	75	0.13	121	0.66	0.93
Stuttgart Hbf (tief)	DF	23	0.13	9	0.80	0.93
Berlin Hbf (S-Bahn)	NA	0	0.00	19	2.97	2.97
Berlin Hbf (S-Bahn)	TU	0	0.00	23	2.97	2.97
Berlin Hbf (S-Bahn)	DF	0	0.00	13	2.97	2.97
Hamburg-Altona(S)	NA	1	1.73	533	0.44	1.29
Hamburg-Altona(S)	TU	1	0.06	129	1.25	1.29
Hamburg-Altona(S)	DF	1	0.06	9	1.25	1.29
Frankfurt(Main)Hbf	NA	81	1.14	136	4.10	9.34
Frankfurt(Main)Hbf	TU	1,213	0.01	23	10.09	10.28
Frankfurt(Main)Hbf	DF	32	0.01	5	10.09	10.28
Nürnberg	NA	0	-	-	-	-
Nürnberg	TU	1	0.01	43	7.10	7.10
Nürnberg	DF	1	0.01	5	6.92	7.10

Table 1: Statistics on the solution process for the local instances for the different formulations of Problem (TTMAPBR). A dash ('-') indicates hitting the time limit of 10 minutes and no feasible solution and/or bound found.

Instance	Form.	#BB	Root-IP gap [%]	Next sol. [s]	Next sav. [%]	Best sav. [%]
S-Bahn Hamburg	NA	0	4.47	-	-	-
S-Bahn Hamburg	TU	1	0.03	273	2.28	2.36
S-Bahn Hamburg	DF	1	0.04	18	2.27	2.36
Regio Nord	NA	1	0.12	68	10.94	12.79
Regio Nord	TU	1	0.01	20	12.78	12.79
Regio Nord	DF	1	0.01	7	12.43	12.79
Regio Nordost	NA	47	15.59	-	-	-
Regio Nordost	TU	1	0.01	28	15.50	15.50
Regio Nordost	DF	1	0.01	9	15.39	15.50
Regio Hessen	NA	1	0.01	26	5.64	5.64
Regio Hessen	TU	1	0.01	31	5.12	5.65
Regio Hessen	DF	1	0.01	10	5.61	5.64
Regio Südwest	NA	1	0.06	85	12.98	13.00
Regio Südwest	TU	1	0.01	65	12.95	13.00
Regio Südwest	DF	1	0.01	10	12.88	13.00
Regio Südost	NA	5,930	8.96	-	-	-
Regio Südost	TU	1	0.00	72	8.81	8.96
Regio Südost	DF	1	0.01	17	8.86	8.95
Regio BW	NA	0	14.30	-	-	-
Regio BW	TU	1	0.01	286	13.09	13.35
Regio BW	DF	1	0.00	22	13.34	13.36
S-Bahn Berlin	NA	1	0.01	84	1.30	1.73
S-Bahn Berlin	TU	1	0.01	195	1.55	1.73
S-Bahn Berlin	DF	1	0.01	83	1.67	1.73
Regio NRW	NA	0	-	-	-	-
Regio NRW	TU	0	-	-	-	-
Regio NRW	DF	1	0.01	28	5.08	5.13
Regio Bayern	NA	1	0.21	513	10.65	10.73
Regio Bayern	TU	1	0.01	94	10.72	10.72
Regio Bayern	DF	1	0.01	41	10.60	10.72
Fernverkehr	NA	1	0.01	59	2.93	5.38
Fernverkehr	TU	1	0.01	17	5.38	5.38
Fernverkehr	DF	1	0.01	11	5.24	5.38
Regionalverkehr	NA	0	-	-	-	-
Regionalverkehr	TU	0	-	-	-	-
Regionalverkehr	DF	0	-	-	-	-
Deutschland	NA	0	5.06	-	-	-
Deutschland	TU	0	-	-	-	-
Deutschland	DF	1	0.01	481	5.02	5.05

Table 2: Statistics on the solution process for the regional and national instances for the different formulations of Problem (TTMAPBR). A dash ('-') indicates hitting the time limit of 10 minutes and no feasible solution and/or bound found.

Tables 3 and 4 give the same statistics as before for Problem (TTMAPI), comparing methods MIP and Benders over a time limit of 10 hours. The results can be seen as exemplary for (TTMAP) as well. We see that Benders can always produce the first improved solution faster than MIP, often by one or two orders of magnitude. For the local instances, Benders finds superior first improving solutions in addition. On the other hand, the first improving solution found by MIP for the regional instances is usually much better than the one found by Benders. For instances *Regionalverkehr* and *Deutschland*, we see that only Benders is able to produce an improving solution at all. In general, Benders can afford to solve many more branch-and-bound nodes within the time limit due to the reduced problem size. For the local instances, this leads to a better overall performance. For the regional instances, MIP takes a long time to solve the root LP relaxation, but then the information gained there is superior to that obtained by Benders, and better first improving solutions are found. However, for instances *Regionalverkehr*

and *Deutschland*, MIP cannot solve the root relaxation within 10 hours, while Benders performs much better. Altogether, the reduction of problem size by Benders enables us to find very good LP bounds and solutions early on, while after 10 hours the MIP is able to catch up on all but the largest two instances.

Instance	Method	#BB	Root-IP gap [%]	Next sol. [s]	Next sav. [%]	Best sav. [%]
Zeil	MIP	149	37.42	257	7.93	29.56
Zeil	Benders	1,076,763	29.98	2	13.96	40.85
Bayreuth Hbf	MIP	18,549	12.42	49	15.92	28.27
Bayreuth Hbf	Benders	3,616,285	16.15	2	7.23	29.69
Passau	MIP	9,372	8.02	674	8.13	27.74
Passau	Benders	966,406	9.71	18	10.02	28.29
Jena Paradies	MIP	41,932	0.41	307	12.17	15.23
Jena Paradies	Benders	1,461,635	0.43	1	13.52	15.32
Lichtenfels	MIP	3,970	0.29	1,347	21.72	26.31
Lichtenfels	Benders	37,287	0.49	1	18.91	26.31
Erlangen	MIP	4,746	0.37	1,552	21.90	25.98
Erlangen	Benders	1,337,382	0.50	6	22.32	25.98
Bamberg	MIP	42,640	0.15	2,857	18.26	20.84
Bamberg	Benders	1,356,763	0.22	4	17.23	20.85
Aschaffenburg	MIP	22,916	0.15	5,710	13.01	15.85
Aschaffenburg	Benders	2,115,654	0.17	11	13.91	15.90
Kiel Hbf	MIP	23,397	0.63	2,772	14.12	24.07
Kiel Hbf	Benders	3,112,494	0.57	8	21.01	24.24
Leipzig Hbf (tief)	MIP	6,422	1.08	169	10.77	20.12
Leipzig Hbf (tief)	Benders	34,916,658	3.04	5	5.71	20.12
Würzburg Hbf	MIP	13,659	0.22	8,174	15.83	18.33
Würzburg Hbf	Benders	1,283,635	0.24	15	17.09	18.37
Dresden	MIP	5,780	0.59	11,717	18.79	22.64
Dresden	Benders	556,324	0.84	33	12.19	22.80
Ulm Hbf	MIP	7,687	0.15	15,153	13.30	17.45
Ulm Hbf	Benders	1,264,683	0.13	11	14.13	17.52
Stuttgart Hbf (tief)	MIP	363	2.00	596	7.48	9.88
Stuttgart Hbf (tief)	Benders	37	3.05	18	6.54	9.88
Berlin Hbf (S-Bahn)	MIP	15,272	0.42	103	7.74	9.30
Berlin Hbf (S-Bahn)	Benders	22,488,370	4.50	20	2.10	9.30
Hamburg-Altona(S)	MIP	3,811	5.61	631	4.39	10.99
Hamburg-Altona(S)	Benders	1,461,544	9.80	19	3.32	10.99
Frankfurt(Main)Hbf	MIP	4,034	0.01	2,600	15.13	15.46
Frankfurt(Main)Hbf	Benders	6,230	0.01	30	15.43	15.46
Nürnberg	MIP	23,062	0.04	3,696	13.18	14.21
Nürnberg	Benders	394,527	0.03	31	13.49	14.22

Table 3: Statistics on the solution process for the regional and national instances for the different formulations of Problem (TTMAP) over a time limit of 10 hours

Instance	Method	#BB	Root-IP gap [%]	Next sol. [s]	Next sav. [%]	Best sav. [%]
S-Bahn Hamburg	MIP	1,428	3.63	1,321	10.30	13.70
S-Bahn Hamburg	Benders	71,830	7.51	63	11.01	13.70
Regio Nord	MIP	5,854	0.31	5,262	11.95	14.27
Regio Nord	Benders	260,334	2.98	283	0.06	14.04
Regio Nordost	MIP	5,863	0.36	7,093	28.17	28.47
Regio Nordost	Benders	91,600	2.87	357	0.83	28.06
Regio Hessen	MIP	23,000	0.20	1,789	13.86	16.52
Regio Hessen	Benders	358,009	1.94	85	5.11	16.44
Regio Südwest	MIP	5,451	0.19	6,944	14.38	16.57
Regio Südwest	Benders	22,069	3.13	663	2.78	16.10
Regio Südost	MIP	1,045	0.22	10,081	23.99	24.87
Regio Südost	Benders	18,994	1.71	362	5.22	24.59
Regio BW	MIP	5	0.61	10,854	19.71	21.91
Regio BW	Benders	9,693	1.40	191	5.64	21.91
S-Bahn Berlin	MIP	75,450	0.12	1,192	4.46	5.07
S-Bahn Berlin	Benders	1,077,974	8.31	192	1.13	5.04
Regio NRW	MIP	5,972	0.29	9,786	14.14	18.18
Regio NRW	Benders	340,403	1.06	178	1.98	18.08
Regio Bayern	MIP	3,540	0.08	13,456	20.88	21.66
Regio Bayern	Benders	44,497	1.68	147	4.63	21.66
Fernverkehr	MIP	5,914	0.02	11,572	9.61	10.23
Fernverkehr	Benders	31,656	0.01	118	10.22	10.24
Regionalverkehr	MIP	0	-	-	-	-
Regionalverkehr	Benders	5,803	3.28	6,504	3.66	15.37
Deutschland	MIP	0	-	-	-	-
Deutschland	Benders	1	0.09	3,082	4.22	9.32

Table 4: Statistics on the solution process for the regional and national instances for the different formulations of Problem (TTMAP). A dash (‘-’) indicates hitting the time limit of 10 hours and no feasible solution and/or bound found.

For instance *Deutschland*, we finally present an additional study with different values for the allowable upper bound on instantaneous peak power consumption, see Table 5. Here, we took the largest power consumption in any second of the planning horizon, which was 774.42 MW, and multiplied it by different factors (see Column *Factor*) to obtain the value for  $U$ . The resulting maximally allowed instantaneous power consumption is shown in Column *Max U*. In Column *Max U solution*, we compare it to the maximum instantaneous power consumption actually occurring in the resulting solution. Column *Savings* shows the achieved reduction in maximum average peak consumption compared to the initial timetable, and Column *Time* shows the solution time needed by Benders. It becomes clear that it is possible to reduce the maximum instantaneous power consumption significantly while only losing little in the savings with respect to maximum average power consumption. We also see that with smaller values for  $U$ , it becomes more difficult to solve the resulting instances. Our experiments show that it becomes increasingly difficult to find a first feasible solution, which is to be expected, as the search space shrinks considerably. We suspect that for even smaller factors than 0.87, the problem is infeasible.

Figure 1 shows that, indeed, both aims of reducing maximum peak consumption and instantaneous consumption can be realized in Germany-wide railway traffic. The depicted solution reduces the former by 9.16% and the latter by 13%.

Instance	Factor	Max $U$ [MW]	Max. $U$ solution [MW]	Savings [%]	Time [s]
Deutschland	0.87	673.75	673.71	9.16	19,339
Deutschland	0.88	681.49	681.46	9.18	15,593
Deutschland	0.90	696.98	696.97	9.22	12,627
Deutschland	1.00	774.42	774.39	9.32	5,132
Deutschland	1.10	851.87	851.75	9.36	4,216
Deutschland	1.20	929.31	928.95	9.38	3,679
Deutschland	no limit	-	1,001.91	9.38	3,483

Table 5: Statistics for the solutions of Problem (TTMAPI) for instance *Deutschland* for different values of  $U$  using Benders decomposition.

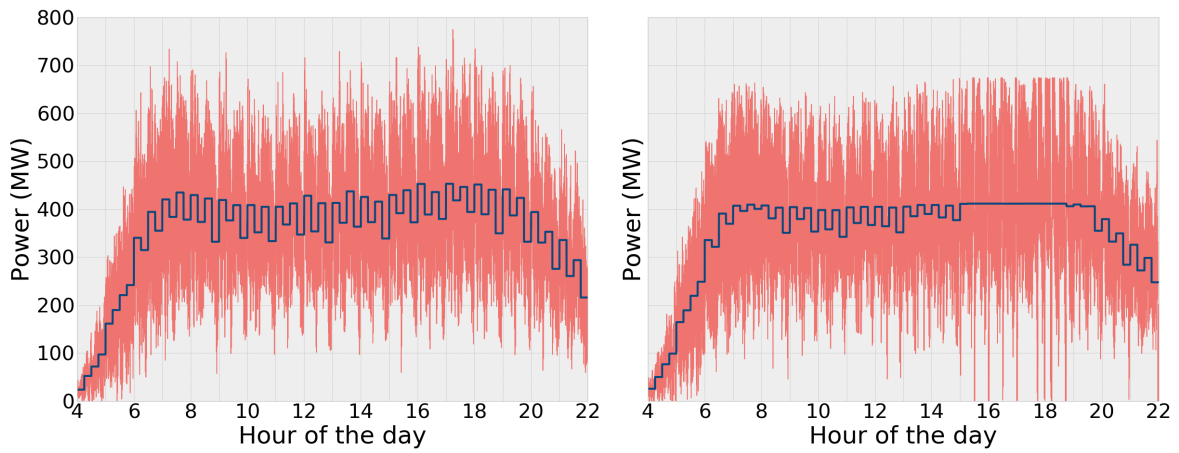


Figure 1: Power consumption profile of instance *Deutschland* before (left) and after (right) optimization according to (TTMAPI) with a peak reduction factor of 0.87 after 10 hours of computation via Benders decomposition