

## Appendix. Algorithm for Section 5.3

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**Algorithm 1:** Short-distance tariff: finding a short-distance path.

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**Input** : PTN  $(V, E)$ , upper bounds  $S_{\max}, L_{\max}$ , two stations  $x, y \in V$   
**Output:** shortest  $x$ - $y$ -path  $W$  with  $s(W) \leq S_{\max}$  and  $l(W) \leq L_{\max}$  if one exists  
// Initialization

- 1  $S_{\max} := \min\{S_{\max}, |V| - 1\}, L_{\max} := \min\{L_{\max}, \max_{e \in E} l(e) \cdot |V|\}$
- 2 **for** all  $v \in V$  **do**
- 3      $d_0(v) := \infty$
- 4      $\pi_0(v) := \text{None}$
- 5  $d_0(x) := 0$

    // Compute distances and predecessors (Bellman-Ford)

- 6 **for**  $s = 1, \dots, S_{\max}$  **do**
- 7     **for** all  $v \in V$  **do**
- 8          $d_s(v) := d_{s-1}(v)$
- 9          $\pi_s(v) := \pi_{s-1}(v)$
- 10        **for** all edges  $(w, v) \in E$  **do**
- 11            **if**  $d_s(v) > d_{s-1}(w) + l(w, v)$  **then**
- 12                 $d_s(v) := d_{s-1}(w) + l(w, v)$
- 13                 $\pi_s(v) := w$

    // Check if a feasible  $x$ - $y$ -path exists and compute it if necessary

- 14 **if**  $d_{S_{\max}}(y) \leq L_{\max}$  **then**
- 15     // Determine the path  $W$  from  $x$  to  $y$  by backtracking the predecessors
- 16      $W' = [y]$  // list of all predecessors starting from  $y$
- 17      $\text{current} := y$
- 18      $s := S_{\max}$
- 19     **while**  $\text{current} \neq x$  **do**
- 20          $\text{current} := \pi_s(\text{current})$
- 21          $W'.\text{append}(\text{current})$  // add current to the end of  $W'$
- 22          $s := s - 1$
- 23     Set  $W := (W')^{-1}$
- 24     **return**  $W$
- 25 **else**
- 26     **return** None

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