

Online Supplement to Transportation Labor Cost Reduction via Vehicle Platooning: Alternative Models and Solution Methods

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Appendix A: Proofs of Structural and Computational-Complexity Properties

Herein we provide detailed proofs for the properties presented within the main manuscript.

A.1. Proof of Property 1

Without loss of generality, take a 3DM instance with corresponding sets A , B , and C each having equal cardinality β .

Build a polynomial reduction from 3DM to Problem UF as follows. (1) For each $a \in A$, $b \in B$, and $c \in C$, create nodes s_a , s_b , s_c , and d_a , d_b , d_c in graph $G(N, A)$, respectively. (2) For each $k \in A \cup B \cup C$, create an arc $(i, j) := (s_k, d_k)$ with corresponding weight $w_{ij} = 1$. (3) For each $r = (a, b, c) \in R$, create two more nodes in $G(N, A)$, $1r$ and $2r$, as well as the following arcs:

$$(s_a, 1r), (s_b, 1r), (s_c, 1r), (1r, 2r), (2r, d_a), (2r, d_b), \text{ and } (2r, d_c),$$

wherein $(1r, 2r)$ has weight one and all others have weight zero; see Figure 1.

We now show that the 3DM instance outputs an affirmative response if and only if the corresponding instance of UF allows for an objective function value of β .

(\leftarrow) Provided a perfect matching (i.e. a witness) for the 3DM, $\Phi \subseteq R$, let the routing of each task in $G(N, A)$ correspond to the associated arcs connecting elements of each tuple $\phi \in \Phi$. This induces a cost β across $G(N, A)$ for Problem UF.

(\rightarrow) Assume we have a witness of Problem UF with objective function value β . There exists no path between any s_k and d_k with weight less than one, and each path of weight one is accessible by either one or three tasks (i.e., a solo or platooning path). Therefore, for the objective function to equal β , the 3β tasks must platoon in groups of three to their respective destinations such that each platoon incurs a cost of one. Since these paths were generated from elements of R , each task can follow only one such path, and we have selected β of these paths, the corresponding elements of R provide a perfect matching. \square

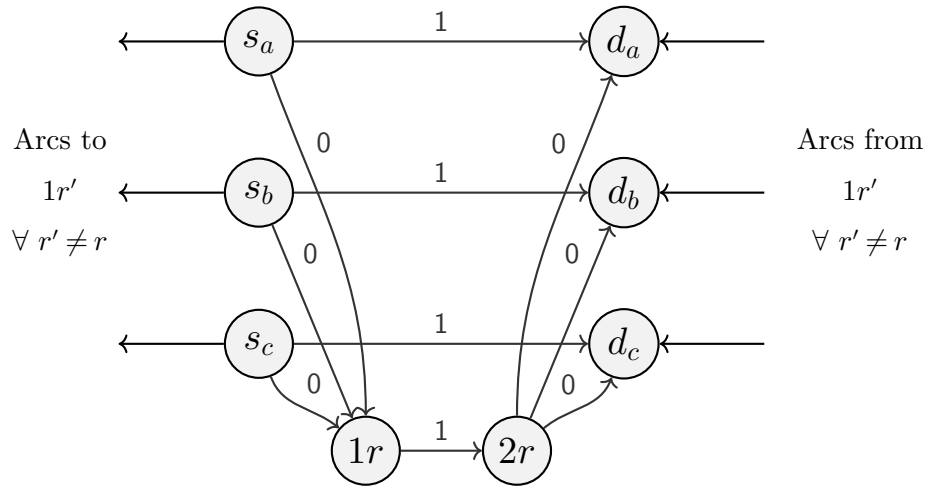


Figure 1 Partial Graph Constructed in Proof of Property 1 for some $r \in R$

A.2. Proof of Property 2

Build a polynomial reduction from 3DM as follows. For each $a \in A$, $b \in B$, and $c \in C$, create nodes s_a , s_b , s_c , and d_a , d_b , d_c in graph $G(N, A)$. For each $r = (a, b, c) \in R$, create six more nodes in $G(N, A)$, $1r, \dots, 6r$. Create arcs

$$(1r, 2r), (3r, 4r), (5r, 6r).$$

with weight one, and arcs

$$(s_a, 1r), (s_b, 1r), (s_c, 5r), (4r, d_c), (4r, d_b), (6r, d_a), (2r, 3r), (2r, 5r), (6r, 3r),$$

with weight zero; see Figure 2. Note that for each partial graph associated with a given r , there is a single feasible routing of tasks a , b , and c to their respective destinations that accrues a cost of three. Likewise, by construction, platoons can only be formed across disparate sets (e.g., tasks $a_1, a_2 \in A$ have no common arcs).

We now show that the 3DM instance outputs an affirmative response if and only if the corresponding instance of UF allows for an objective function value of 3β .

(\leftarrow) Given a perfect 3DM matching, $\Phi \subseteq R$, route the tasks in accordance with the arcs constructed from each tuple $\phi \in \Phi$. This yields a platooning solution with objective function value 3β .

(\rightarrow) Assume we have a platooning solution with objective function value 3β . In any feasible solution to Problem UI with $l = 2$, an individual task (e.g., $a \in A$) incurs a cost of two if it does not platoon, a pair of any two tasks (e.g., $(a, b) \in A \times B$) incurs a cost of three if they platoon across some arc in $G(N, A)$, and a triple of tasks (e.g., $(a, b, c) \in A \times B \times C$) also incurs as cost of three if each of the tasks platoons with one of the others across some arc in $G(N, A)$. Therefore, the routing cost, \mathcal{C} , can be calculated as

$$\mathcal{C} = 2\nu_1 + 3\nu_2 + 3\nu_3$$

where ν_1 , ν_2 , and ν_3 are the number of single tasks, pairs, and triples in the feasible solution. Since we have objective function value 3β , this implies that $\nu_3 = \beta$. Since all tasks are routed and no two triples repeat elements, we have a perfect 3DM matching. \square

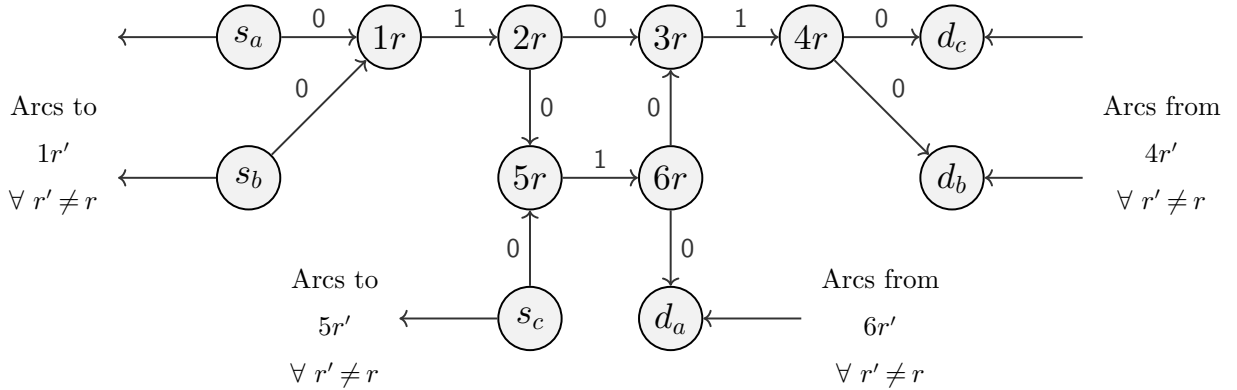


Figure 2 Partial Graph Constructed in Proof of Property 2 for some $r \in R$

A.3. Proof of Property 3

The 3DM transformation utilized in the proof of Property 1 constructs a special case of Problem UI with platoons of length $l \geq 3$. \square

A.4. Proof of Property 4

Build a polynomial reduction from 3DM as in the proof of Property 2. However, assign the following arcs weight one:

$$(s_c, 5r), (1r, 2r), (2r, 3r), (3r, 4r), (5r, 6r), (5r, 6r), (6r, d_a)$$

and the remaining arcs weight zero. Given an associated 3DM or Problem UI witness, the remainder of the proof follows that of Property 2, except the Problem UI decision problem corresponds with an objective function value of 6β . \square

A.5. Proof of Property 5

Build a polynomial time reduction from UF to PF by assigning an additional edge cost $\hat{w}_{ij} = 0$, $\forall (i, j) \in A$.

(\rightarrow) Given a witness of Problem UF having cost μ , route Problem PF's tasks from s_k to d_k $\forall k \in K$ according to the Problem UF solution. Select an arbitrary route from d_k to s_k $\forall k \in K$. Since the return cost $\hat{w}_{ij} = 0$ $\forall (i, j) \in A$, this constructed solution corresponds with a Problem PF tour of cost μ .

(\leftarrow) Given a witness of Problem PF having cost μ , note that all positive-value costs are associated with the s_k to d_k tour leg $\forall k \in K$. Therefore, a Problem UF solution routing every task k in accordance with these legs incurs a cost of μ .

In conjunction with Properties 1 and 2, this implies that Problem PF is \mathcal{NP} -hard for any platoon length. \square

A.6. Proof of Property 6

A polynomial reduction from UI to PI can be similarly constructed by assigning an additional edge cost $\hat{w}_{ij} = 0$ $\forall (i, j) \in A$. Given an associated Problem UI or PI witness, the proof proceeds as in Property 4. Therefore, by Properties 3 and 4, Problem PI is \mathcal{NP} -hard for platoons of any length. \square

A.7. Proof of Property 7

Provided an optimal solution of Problem UF, we have

$$\begin{aligned}
f^* &= \min_{t,x,y} \sum_{(i,j) \in A} \left(\sum_{k \in K} w_{ij} x_{ijk} - \sum_{k \in K} \sum_{k' \in K | k' > k} \delta w_{ij} y_{ijkk'} \right) \\
&= \min_{t,x,y} \sum_{(i,j) \in A} \left(\sum_{k \in \bar{K}} w_{ij} x_{ijk} - \sum_{k \in \bar{K}} \sum_{k' \in \bar{K} | k' > k} \delta w_{ij} y_{ijkk'} \right) \\
&\quad + \sum_{(i,j) \in A} \left(\sum_{k \in K \setminus \bar{K}} w_{ij} x_{ijk} - \sum_{k \in K \setminus \bar{K}} \sum_{k' \in K \setminus \bar{K} | k' > k} \delta w_{ij} y_{ijkk'} \right) \\
&= f_{\bar{K}}^* + f_{K \setminus \bar{K}}^* .
\end{aligned}$$

The first and the third equality hold by definition, whereas the second equality is due to the fact that \bar{K} is separate, i.e., $y_{ijk\bar{k}}^* = \hat{y}_{ijk\bar{k}}^* = 0$ holds. Therefore, the equality of the property holds.

Assume, for sake of contradiction, that $f_{\bar{K}}^* > \sum_{k \in \bar{K}} (p_{s_k d_k} + \hat{p}_{d_k s_k})$, and consider an alternative solution to the problem such task $k \in K \setminus \bar{K}$ follows the same path as in (x^*, y^*, t^*) , but each task $k \in \bar{K}$ follows its respective shortest path. Compared to the optimal solution, the objective function's partial sum for tasks $K \setminus \bar{K}$ does not change; however, its partial sum for tasks \bar{K} is reduced. This contradicts the optimality of (x^*, y^*, t^*) .

The same calculations can readily be extended to Problems UI, PF, and UI to confirm the validity of the bound. \square

A.8. Proof of Property 8

Assume, for sake of contradiction, that the set of arcs $A_{\bar{K}}$ is not connected. Given that the paths of all tasks from their origin to their destination are connected (e.g., see Constraint (??) in Problem UF), the nodes represented in $A_{\bar{K}}$ can be expressed as disjoint sets $\{i, \dots, i'\}$ and $\{j', \dots, j\}$ ordered by their precedence along

the platoon's path such that at least two tasks in \bar{K} take different routes between i' and j' . Let $k' \in \hat{K}$ be the task that follows a cost minimum path from i' to j' . Since $A_{\bar{K}}$ is not connected, there exists another task in the platoon following an alternate path from i' to j' having length greater than or equal to that used by k' . However, the cost of following this alternate path is greater than if it were to platoon with k' for $\delta > 0$ contradicting the optimality of the solution. \square

A.9. Proof of Property 9

For Problem UF, the objective function value induced by a separate subset of tasks $\bar{K} \subset K$ consists of each task's shortest-path costs from its origin to the platoon-formation node, the shortest-path cost along the platooning route, and the each task's shortest-path costs from the platoon-disbandment node to its destination. That is, the objective function equals

$$p_{ij}\delta(|\bar{K}|-1) + \sum_{k \in \bar{K}} (p_{s_k i} + p_{j d_k})$$

Since the pair (i, j) is chosen to ensure optimality, we have

$$\begin{aligned} f^* &= \sum_{i'=1}^{n'} \min_{i, j \in N} \left\{ p_{ij}\delta(|\bar{K}_{i'}|-1) + \sum_{k \in \bar{K}_{i'}} (p_{s_k i} + p_{j d_k}) \right\} \\ &= \sum_{i'=1}^{n'} \left(\sum_{k \in \bar{K}_{i'}} p_{s_k d_k} \right) - \eta(\bar{K}_{i'}) \\ &\geq \sum_{k \in K} (p_{s_k d_k} + \hat{p}_{s_k d_k}) - \sum_{\bar{K} \subseteq K, |\bar{K}| \leq l} (\min\{\eta(\bar{K}), 0\} + \min\{\hat{\eta}(\bar{K}), 0\}) \\ &= \xi \end{aligned}$$

implying the validity of the bound for Problem UF. Moreover, given that Problem PF's optimal solution lower bounds that of Problem UI, it is clear that ξ also lower bounds Problem UI's optimal solution. The same calculations can be readily extended to Problems PF and PI to confirm the validity of the bound as well. \square

A.10. Proof of Property 10

Each task can either travel alone (i.e., traversing its shortest path) or conjoin with one other task to platoon. As each task can be part of at most one platoon, we must find a matching of maximal (saving) weight in a complete graph with the tasks representing the nodes.

To calculate the savings efficiently, shortest paths between all pairs of nodes should be calculated in a preprocessing step; this can be accomplished in $O(\mathcal{N}^3)$ using the Floyd-Warshall algorithm. Computing the savings for one pair of tasks costs $O(\mathcal{N}^2)$ because we must search for the minimum over node pairs. As we have $O(\mathcal{K}^2)$ pairs of tasks, the computation of all savings costs $O(\mathcal{N}^2 \mathcal{K}^2)$. Finally, finding a maximal weight matching in a graph with \mathcal{K} nodes can be done in $O(\mathcal{K}^2)$ by applying adaptations of Edmonds' algorithm. \square