Supplemental material

This e-companion is structured as follows: Section EC.1 contains the proofs of all statements of the main article. Section EC.2 contains further details concerning the separation of valid inequalities within the branch-and-cut framework. Sections EC.3 to EC.5 contain additional and detailed computational results for the MDSDVRP, the MDTSP, and the SDVRP, respectively. Finally, Section EC.6 contains result showing the impact of the heuristic used to initialize the B&C algorithm.

EC.1. Proofs

EC.1.1. Proofs of Section 1.2 (solution properties)

PROPERTY 4 *There exists an optimal solution to the MDSDVRP in which at least one route contains at most one split node.*

Proof. Assume that there exists a solution satisfying Property 1 in which every route contains at least two split nodes. Let R_1 be one such route containing split nodes s_1 and s_2. Split node s_2 must be contained in at least one additional route, say R_2. Due to Property 1, the latter route must contain a split node s_3 that cannot be contained in a previously considered route, i.e., $s_3 \notin \{s_1, s_2\}$. Continuing the argument one concludes that the last split node encountered in this procedure (say s_ℓ) is contained in only one route which contradicts the fact that s_ℓ is a split node. □

EC.1.2. Proofs of Section 4 (capacity constraints)

THEOREM 1 *Capacity constraints* (9) *are valid for the MDSDVRP.*

Proof. Let $S' \subseteq S$ be the subset of S such that $q_i < Q$ for each $i \in S'$ and whose nodes in S' are served via single-customer routes, i.e., $\bar{y}_i = 1$, $\forall i \in S'$. Then, inequality (9) can be rewritten as

$$\bar{x}(E(S)) \leq \bar{z}(S) - \frac{q(S \setminus S')}{Q} - |S'|.$$

Since $q_i < Q$ for each $i \in S'$, the demand of each node in S' can be satisfied by one single-customer route. Therefore $\bar{x}(\delta(i)) = 0$, $\forall i \in S'$, $\bar{x}(E(S')) = 0$, and $\bar{z}(S) = \bar{z}(S \setminus S') + |S'|$ which imply that the latter inequality reduces to

$$\bar{x}(E(S \setminus S')) \leq \bar{z}(S \setminus S') - \frac{q(S \setminus S')}{Q}$$

which is a weaker version of (8) for set $S \setminus S'$ and therefore valid for the MDSDVRP. □

THEOREM 2 *Consider formulation* (1) *when generic capacity constraints* (1f) *are replaced by rounded capacity constraints* (8) *and* y*CAPs* (9)*. There exist solutions to this formulation that are infeasible for the MDSDVRP even if the depot-consistency constraints are met.*

Proof. Consider the candidate solution given in Figure 2a. As discussed in Section 2, this candidate solution may only violate capacity constraints and is indeed infeasible when assuming $Q = 7$, $q_6 = 6$, and $q_i = 2$, $i \in C \setminus \{6\}$. Since no single-customer routes are used, yCAPs (9) are implied by rounded capacity constraints (8) and it therefore remains to show that none of the latter inequalities is violated. To this end, observe that $\lceil q(S)/Q \rceil \leq 2$ for each $S \subseteq C$ and $\bar{z}(S) = |S| + 1$ for each S such that $\bar{x}(E(S)) > 0$. Thus, $\bar{z}(S) - \lceil q(S)/Q \rceil \geq |S| - 1$ holds for each set $S \subseteq C$ and the rounded capacity constraint (8) is therefore dominated by the subtour elimination constraint $\bar{x}(E(S)) \leq |S| - 1$. We conclude that the candidate solution does not violate the rounded capacity constraints (8) nor the yCAPS (9) despite exceeding the capacity limits of at least one vehicle. Since the depot-consistency constraints are trivially satisfied (as there is only one depot), the theorem follows. □

EC.1.3. Proofs of Section 5.1 (path-elimination based constraints)

THEOREM 3 *Multi-leg PE constraints* (10) *are valid for the MDSDVRP.*

Proof. Consider an arbitrary solution $\mathcal{U}$ to the MDSDVRP and let

$$\sum_{j=1}^{k} x(\delta(i_j, I_j)) + y(S) + x(E(S)) \leq z(S) \tag{EC.1}$$

be an arbitrary multi-leg PE inequality for $S \subseteq C$, $i_j \in S$, and $I_j \subseteq D$, $i_j \neq i_p$, $1 \leq j \neq p \leq k$, as defined in (10). We will show that $\mathcal{U}$ satisfies this inequality by induction on the number of vehicle routes from $\mathcal{U}$ visiting at least one node from S.

We observe that all variables values implied by a set of empty routes are equal to zero. In this case, the left- and right-hand side of (EC.1) are equal to zero providing a proper start for our argument. Thus, assume that (EC.1) holds for the variable values associated with an arbitrary subset of the vehicle routes of $\mathcal{U}$. Let $R = (d, u_1, u_2, \ldots, u_\ell, d)$, $d \in D$, $u_i \in C$, $u_i \neq u_j$, $1 \leq i \neq j \leq \ell$, be an arbitrary, not yet added route from $\mathcal{U}$. We will show that (EC.1) holds after adding R.

We first consider the case when R is a single-customer route visiting only one customer, i.e., $\ell = 1$ and $R = (d, u_1, d)$. If $u_1 \in S$, both the left-hand side and the right-hand side of (EC.1) increase by one while neither of the two sides changes if $u_1 \notin S$.

Next, assume $\ell \geq 2$ and let (V', E') be the subgraph induced by intersection of edges from S and R, i.e., $V' = S \cap \{u_1, u_2, \ldots, u_\ell\}$ and $E' = E(S) \cap \{\{u_i, u_{i+1}\} \mid 1 \leq i < \ell\}$. As depot $d \in R$ is considered in at most one leg of (EC.1), adding route R can increase its left-hand side by at most $1 + |E'|$ while its right-hand side is increased by $|V'|$. The theorem follows since $1 + |E'| \leq |V'|$ holds as R is a route and (V', E') is therefore acyclic. □

Lemma EC.1 derives a condition under which (10) are redundant that is useful for separation routines and also used in the proof of Theorem 4.

LEMMA EC.1. *Multi-leg PE constraints* (10) *are redundant if the subgraph induced by* $\bar{x}(E(S))$ *is not connected.*

Proof Consider an arbitrary candidate solution and a set $S \subseteq C$ for which the subgraph induced by $\bar{x}(E(S))$ contains two (disconnected) node sets $S' \subset S$, $\emptyset \neq S' \neq S$, and $T = S \setminus S'$, i.e., for which $\bar{x}(S', T) = 0$. Let $\bigcup_{j=1}^{k} \delta(i_j, I_j)$ be the legs of an arbitrary multi-leg inequality and assume that $\mathcal{I} = \{i_1, i_2, \ldots, i_k\} \cap S'$ and $\mathcal{I}' = \{i_1, i_2, \ldots, i_k\} \setminus \mathcal{I} \subseteq T$. Note that the definition of set $\mathcal{I}'$ includes the empty set and that multi-leg inequalities are valid (but redundant) for the case of zero legs (i.e., when $k = 0$). The theorem follows since the multi-leg PE inequality

$$\sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) + \bar{y}(S) + \bar{x}(E(S)) \leq \bar{z}(S)$$

is obtained as the sum of the two multi-leg PE constraints

$$\sum_{j: i_j \in \mathcal{I}} \bar{x}(\delta(i_j, I_j)) + \bar{y}(S') + \bar{x}(E(S')) \leq \bar{z}(S')$$
$$\sum_{j: i_j \in \mathcal{I}'} \bar{x}(\delta(i_j, I_j)) + \bar{y}(T) + \bar{x}(E(T)) \leq \bar{z}(T)$$

for S' and T. This holds true since $S = S' \cup T$, $S' \cap T = \emptyset$, ensures that $\bar{y}(S) = \bar{y}(S') + \bar{y}(T)$, $\bar{z}(S) = \bar{z}(S') + \bar{z}(T)$ while $\bar{x}(S', T) = 0$ implies that $\bar{x}(E(S)) = \bar{x}(E(S')) + \bar{x}(E(T))$. □

THEOREM 4 *Consider formulation* (1) *when generic depot-consistency constraints* (1g) *are replaced by the subset of multi-leg PE constraints* (10) *for* $k = 2$*. There exist solutions to this formulation that violate at least one multi-leg PE constraint* (10) *for* $k \geq 3$*.*

Proof. Consider an instance with depots $D = \{1, 2, 3\}$ and customer nodes $C = \{4, 5, \ldots, 11\}$ and the candidate solution to that instance corresponding to the graph given in Figure EC.1. A solid edge between two nodes u and v indicates $\bar{x}_{uv} = 1$ and we assume the vehicle capacity is not restricting. Clearly, this graph does not represent a feasible solution to the MDSDVRP since the vehicle(s) traversing edges $\{1, 9\}$ and $\{2, 8\}$ cannot return to their initial depot without visiting other depots. We also observe that all nodes have even degree and that the subgraph induced by the customer nodes C is acyclic. Thus, only depot-consistency constraints may be violated.

Indeed, a multi-leg PE inequality (10) is violated for $k = 3$, $I_1 = \{1\}$, $I_2 = \{2\}$, $I_3 = \{3\}$, $i_1 = 9$, $i_2 = 8$, $i_3 = 10$, and $S = \{8, 9, 10\}$, since its left-hand side is equal to five while its right-hand side is equal to four. Observe that no customer node is adjacent to two depots in Figure EC.1. Thus, for $k = 1$ all multi-leg PE inequalities are satisfied and $\ell := \sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) \leq 2$ if $k \leq 2$. Using Lemma EC.1 and $\bar{y}_i = 0$, $\forall i \in C$, we conclude that $\ell + |S| - 1 > \bar{z}(S)$ and therefore $\bar{z}(S) \leq |S|$ must hold for set $S \subseteq C$ if a multi-leg PE inequality would be violated. Each connected subgraph

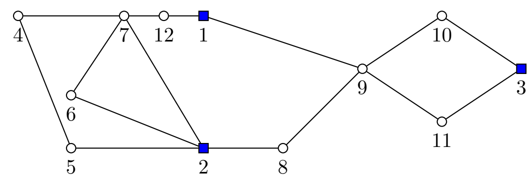

Figure EC.1 **A candidate solution which is infeasible for the MDSDVRP which violates multi-leg PE constraints** (10)**, but satisfies those considering at most two legs.**

consisting of customer nodes in Figure EC.1 that has edges to at least two depots does, however, contain a customer node that is visited twice (i.e., node 7 or 9). Thus, $\bar{z}(S) \geq |S|+1$ for each relevant customer subset S and the theorem follows. □

THEOREM 5 *Consider formulation* (1) *when generic depot-consistency constraints are replaced by multi-leg PE constraints* (10). *There exist solutions to this formulation that are infeasible for the MDSDVRP even if the capacity restrictions of all vehicles are met.*

Proof. Consider an instance with depots $D=\{1,2\}$ and customers $C=\{3,4,\ldots,10\}$ and the candidate solution to that instance corresponding to the graph given in Figure 3. A solid edge between nodes u and v indicates $\bar{x}_{uv}=1$ and we assume that the vehicle capacity is not restricting. We observe that all node degrees are even and the subgraph induced by the customers C is acyclic. All constraints of (1) except depot-consistency inequalities are thus satisfied. It can be easily seen that the graph from Figure 3 does not represent a feasible solution to the MDSDVRP since, e.g., the vehicle traversing edge $\{1,3\}$ cannot return to depot 1 without visiting depot 2. Also observe that each connected subgraph induced by a set $S \subseteq C$ that is connected to both depots consists of exactly one node with degree four while all other nodes in S have degree two, i.e., $\bar{z}(S)=|S|+1$. Since $|D|=2$ and no edge is traversed multiple times the left-hand side value of a PE constraint cannot be larger than $|S|+1$. Therefore, all multi-leg PE constraints are satisfied. □

EC.1.4. Proofs of Section 5.2 (path-forcing based constraints)

THEOREM 6 *Multi-leg PF constraints* (12) *are valid for the MDSDVRP.*

Proof. Consider an arbitrary solution of the MDSDVRP and let $S \subseteq C$, $i_j \in S$, $D' = \cup_{j=1}^{k} I_j$, $I_j \cap I_l = \emptyset$, $1 \leq j \neq l \leq k$, as defined in the multi-leg PF constraints (12). Assume that $\sum_{j=1}^{k} \bar{x}(i_j, I_j) = \ell$, $\ell \in \mathbb{N}_0$, and recall that (with the exception of single-customer routes) each route traverses each edge at most once. Since each depot is contained in at most one set I_j, $1 \leq j \leq k$, these ℓ edge traversals correspond to ℓ different routes. Each such route enters S by an edge $\{d, i_j\}$, $d \in I_j$, and leaves S via an edge $\{u,v\}$ with $u \in S$, and $v \in (C \cup \{d\}) \setminus S$. If $v=d$, we have $u \neq i_j$ since each edge is traversed only once by each route. Edge $\{u,v\}$ is not contained in one of the legs considered on

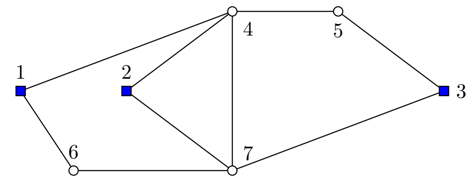

Figure EC.2 **A candidate solution which is infeasible for the MDSDVRP that violates multi-leg PF constraints** (12)**, but does not violate PF constraints** (11)**.**

the left-hand side of (12) since depot d is contained in set I_j and thus in leg $\delta(i_j, I_j)$. Considering all ℓ relevant tours, it follows that

$$\bar{x}(\delta(S, C \setminus S)) + \bar{x}(\delta(S, D')) - \sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) \geq \ell.$$

Thus, we have

$$\bar{x}(\delta(S, (C \cup D') \setminus S)) - \sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) \geq \sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j))$$

which is equivalent to (12) for the considered solution. □

Theorem 7 *Consider formulation* (1) *when generic depot-consistency constraints* (1g) *are replaced by PF constraints* (11)*. There exist solutions to this formulation that violate at least one multi-leg PF constraint* (12)*.*

Proof. Consider an instance with depots $D = \{1, 2, 3\}$ and customer nodes $C = \{4, 5, 6, 7\}$ and the candidate solution to that instance corresponding to the graph given in Figure EC.2. A solid edge between nodes u and v indicates $\bar{x}_{uv} = 1$ and we assume that the vehicle capacity is not restricting. As all node-degrees are even and the subgraph induced by the customer nodes C is acyclic, only depot-consistency inequalities may be violated. Observe that (single-leg) PF constraints (11) are not violated since each depot-edge $\{d, i\} \in E_D$, $d \in D$, part of this solution is contained in a cycle starting and ending at the same depot and not passing through any other depot. PF constraints (11) do, however, not ensure that these cycles are edge disjoint. Indeed, the candidate solution is infeasible for the MDSDVRP since the edge $\{4, 7\}$ is contained in all such cycles. It also violates a multi-leg PF inequality (12) for $S = \{4, 5\}$, $D' = \{1, 2, 3\}$ and $k = 2$ with $I_1 = \{1, 2\}$, $I_2 = \{3\}$, $i_1 = 4$, $i_2 = 7$. □

Theorem 8 *Multi-leg PF constraints* (12) *imply multi-leg PE constraints* (10)*.*

Proof. We first observe that each multi-leg PF inequality (12) can be rewritten as

$$2\sum_{j=1}^{k} x(\delta(i_j, I_j)) \leq x(\delta(S)) - x(\delta(S, D \setminus D'))$$

since $x(\delta(S,(C\cup D')\setminus S))=x(\delta(S))-x(\delta(S,D\setminus D'))$. Using $x(\delta(S))=2z(S)-2y(S)-2x(E(S))$ which is obtained by summing up the degree equations (1b) for all customers $i\in S$, it follows that

$$2\sum_{j=1}^{k}x(\delta(i_j,I_j))+x(\delta(S,D\setminus D'))\le 2z(S)-2x(E(S))-2y(S).$$

Rearranging the terms and dividing by two, we obtain

$$\sum_{j=1}^{k}x(\delta(i_j,I_j))+0.5x(\delta(S,D\setminus D'))+x(E(S))+y(S)\le z(S) \tag{EC.2}$$

which shows that multi-leg PF constraints (12) correspond to lifted versions of multi-leg PE constraints (10) with an additional term $0.5x(\delta(S,D\setminus D'))$ on the left-hand side. This result also implies that there is a one-to-one correspondence between multi-leg PE constraints (10) and the subset of multi-leg PF constraints with $D'=D$. □

Lemma EC.2 shows that we can focus on sets $S\subseteq C$ for which the subgraph induced by $\bar{x}(E(S))$ is connected when trying to identify violated constraints (12). This result will be used in the proofs of Lemma EC.3 and Theorem 9.

LEMMA EC.2. *Multi-leg PF constraints* (12) *are redundant if the subgraph induced by* $\bar{x}(E(S))$ *is not connected.*

Proof Consider an arbitrary candidate solution and a set $S\subseteq C$ for which the subgraph induced by $\bar{x}(E(S))$ contains two (disconnected) sets $S_1\subset S$, $\emptyset\ne S_1\ne S$, and $S_2=S\setminus S_1$. Let $\bigcup_{j=1}^{k}\delta(i_j,I_j)$ be the legs of an arbitrary multi-leg PF constraint for set S and assume that there exists ℓ, $1\le\ell<k$ such that $\{i_1,i_2,\ldots,i_\ell\}\subseteq S_1$ and $\{i_{\ell+1},\ldots,i_k\}\subseteq S_2$. Consider the two multi-leg PF inequalities

$$2\sum_{j=1}^{\ell}\bar{x}(\delta(i_j,I_j))\le\bar{x}(\delta(S_1,(C\cup D_1)\setminus S_1)) \tag{EC.3}$$

$$2\sum_{j=\ell+1}^{k}\bar{x}(\delta(i_j,I_j))\le\bar{x}(\delta(S_2,(C\cup D_2)\setminus S_2)) \tag{EC.4}$$

for S_1 and S_2 where $D_1=\cup_{j=1}^{\ell}I_j$ and $D_2=\cup_{j=\ell+1}^{k}I_j$ denote the subsets of depots assigned to nodes in S_1 and S_2, respectively. Summing up the latter two inequalities, we obtain

$$2\sum_{j=1}^{k}\bar{x}(\delta(i_j,I_j))\le\bar{x}(\delta(S_1,(C\cup D_1)\setminus S_1))+\bar{x}(\delta(S_2,(C\cup D_2)\setminus S_2)). \tag{EC.5}$$

We also observe that the relations

$$\bar{x}(\delta(S_1,(C\cup D_1)\setminus S_1))=\bar{x}(\delta(S_1,(C\cup D_1)\setminus(S_1\cup S_2)))=\bar{x}(\delta(S_1,(C\cup D_1)\setminus S))\le\bar{x}(\delta(S_1,(C\cup D)\setminus S))$$
$$\bar{x}(\delta(S_2,(C\cup D_2)\setminus S_2))=\bar{x}(\delta(S_2,(C\cup D_2)\setminus(S_1\cup S_2)))=\bar{x}(\delta(S_2,(C\cup D_2)\setminus S))\le\bar{x}(\delta(S_2,(C\cup D)\setminus S))$$

hold since $\bar{x}(\delta(S_1, S_2)) = 0$, $D_1 \subseteq D'$ and $D_2 \subseteq D'$. Using above inequalities in (EC.5), we obtain

$$2\sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) \leq \bar{x}(\delta(S_1, (C \cup D_1) \setminus S_1)) + \bar{x}(\delta(S_2, (C \cup D_2) \setminus S_2))$$
$$\leq \bar{x}(\delta(S_1, (C \cup D) \setminus S)) + \bar{x}(\delta(S_2, (C \cup D) \setminus S)) = \bar{x}(\delta(S, (C \cup D) \setminus S))$$

which shows that the multi-leg PF inequality for set S is implied by the two inequalities for sets S_1 and S_2. Finally, we note that if $\ell = k$ (which is the case excluded above), the multi-leg PF constraint (EC.3) for set S_1 implies the one for S since $\bar{x}(\delta(S_1, (C \cup D_1) \setminus S_1)) \leq \bar{x}(\delta(S_1, (C \cup D) \setminus S)) \leq \bar{x}(\delta(S, (C \cup D) \setminus S))$. □

The following lemma will be used in the proof of Theorem 9.

LEMMA EC.3. *Consider formulation* (1) *with generic depot-consistency constraints* (1g) *replaced by multi-leg PF constraints* (12). *Assume that a candidate solution contains two customer nodes i and j such that $\bar{x}_{ij} = 1$, $\bar{z}_i = 1$, $\bar{z}_j \geq 2$ and which are both connected to a depot $d \in D$ by an edge traversed exactly once (i.e., $\bar{x}_{di} = 1$ and $\bar{x}_{dj} = 1$). If the considered candidate solution violates a multi-leg PF constraint* (12) *with $i \in S$, then it also violates one for set $S' = S \setminus \{i\}$.*

Proof Consider nodes $i \in C$, $j \in C$, and $d \in D$, satisfying the conditions in the lemma. Since the inequality (12) for $S = \{i\}$ is not violated, $i \in S$ implies that $j \in S$ (due to Lemma EC.2). If edge $\{d, i\}$ is not part of the legs considered on the left-hand side of the multi-leg PF inequality, both the left-hand side and the right-hand side value remains identical when removing node i from S, i.e., considering $S' = S \setminus \{i\}$. Similarly, if edge $\{d, i\}$ is contained in one leg of the multi-leg PF inequality, we can instead consider a leg containing edge $\{d, j\}$ which leaves the left-hand side value unchanged and then remove node i from set S as in the first case. □

THEOREM 9 *Consider formulation* (1) *with generic depot-consistency constraints replaced by multi-leg PF constraints* (12). *There exist solutions to this formulation that are infeasible for the MDSDVRP and which do not violate the capacity restrictions of any vehicle.*

Proof. Consider the candidate solution given in Figure 2b. As discussed in Section 2, this candidate solution is not feasible for the MDSDVRP and may only violate depot-consistency constraints. It therefore remains to show that no multi-leg PF constraint (12) is violated. To simplify our arguments, we first observe that due to Lemma EC.3 we do not need to consider inequalities for which set S contains a customer node in $\{4, 9, 10, 11, 12, 14\}$. Thus, we assume without loss of generality that $S \subseteq S' = \{5, 6, 7, 8, 13\}$ and that subgraph induced by $\bar{x}(E(S))$ is connected, cf. Lemma EC.2.

We first observe that all multi-leg PF constraints (12) with $k = 1$ and $|D'| = 1$ are satisfied. They correspond to PF constraints (11) which are satisfied since there exists a route starting and ending at depot $d \in D$ that does not visit any other depots for each depot edge $e = \{d, i\} \in E_D$ with $\bar{x}_e = 1$.

Next, we discuss the case when set S contains only one customer node, i.e., $S = \{i\}$, $i \in S'$. Here, we focus on customer nodes $i \in \{6,7,8\}$ for which $\bar{x}(\delta_D(i)) = 2$ and assume that $|D'| = 2$ since customers with one adjacent depot are covered by the previous case and customers adjacent to more than two depots do not exist. Thus, the left-hand side of each relevant multi-leg PF constraint is equal to four. Since $\bar{x}(\delta(\{i\}, C \setminus \{i\})) \geq 2$ for each $i \in \{6,7,8\}$ and the two depot edges considered on the left-hand side are also contained in the cut $\bar{x}(\delta(\{i\}, (C \cup D') \setminus \{i\}))$, the right-hand side of each such inequality is at least four. Thus, all multi-leg PF constraints with $|S| = 1$ are satisfied.

We now turn our attention to the case $|D'| = 2$ and $S \subseteq S'$ such that $|S| \geq 2$ and the subgraph induced by $\bar{x}(E(S))$ is connected. We assume w.l.o.g. that set S contains nodes i_1 and i_2 (which may be identical) incident to two depot-edges $\{d_1, i_1\}$ and $\{d_2, i_2\}$ which are considered on the left-hand side of (12). We will show that no multi-leg PF constraint can be violated in this case since $\bar{x}(\delta(S, C \setminus S)) \geq 2$ and thus $\bar{x}(\delta(S, (C \cup D') \setminus S)) \geq 4$ holds (because of the two depot edges $\{d_1, i_1\}$ and $\{d_2, i_2\}$) for all relevant sets S. Observe, that $8 \notin S$, since $8 \in S$ would imply that $\{\{8,10\},\{8,11\}\} \subseteq \delta(S, C \setminus S)$. Next, assume that $5 \in S$ for which we discuss two subcases based on the above assumptions of connectivity and $|S| \geq 2$. Node 7 cannot be in S too since $\{5,7\} \subseteq S$ implies that $\{\{4,5\},\{7,12\}\} \subseteq \delta(S, C \setminus S)$. Node 6 cannot be in S either since $\{5,6\} \subseteq S$ and $8 \notin S$ (due to the previous result) implies that $\{\{4,5\},\{6,8\}\} \subseteq \delta(S, C \setminus S)$. We conclude that $S \cap \{5,8\} = \emptyset$ and thus $S \subseteq \{6,7,13\}$ which does not contain a connected set of at least two nodes and therefore contradicts the assumptions made in this case.

It remains to show that all multi-leg PF constraints considering all three depots (i.e., $D' = \{1,2,3\}$, $\sum_{j=1}^{k} \bar{x}(\delta(i_j, I_j)) = 3$) with $|S| \geq 2$ such that the subgraph induced by $\bar{x}(E(S))$ is connected are satisfied. Here, we observe that the right-hand side of each associated inequality (12) simplifies to $\bar{x}(\delta(S, V \setminus S))$ and that such an inequality is violated if $\bar{x}(\delta(S, V \setminus S)) \leq 5$. Furthermore, $\{7,13\} \cap S \neq \emptyset$ as depot 3 is connected only to nodes 7 and 13. Node 13 cannot be in set S of a violated multi-leg PF inequality since $13 \in S$ implies $\{8,13\} \subseteq S$ (since $|S| \geq 2$) and thus $\bar{x}(\delta(S, V \setminus S)) \geq \bar{x}(\delta(\{13,8\}, V \setminus S)) \geq 7 > 5$ for any $S \in S'$ such that $\{8,13\} \subseteq S$. Thus, assume that $7 \in S$ and observe that $\bar{x}(\delta(S, V \setminus S)) = 6$ if $S = \{5,7\}$. Further adding node 6 (i.e., $\{5,6,7\} \subseteq S$), we have $\bar{x}(\delta(S, V \setminus S)) \geq \bar{x}(\delta(\{5,6,7\}, V \setminus S)) \geq 7 > 5$ for any such $S \in S'$. The theorem follows since $S \cap \{7,13\} = \emptyset$ which implies that no violated multi-leg PF inequalities can exist in case $D' = 3$. $\square$

EC.2. Separation of valid inequalities

EC.2.1. Exact separation of yCAPs (9)

We first observe that for each set $S \subset C$ the cut form

$$x(\delta(S)) + 2y(S) \geq \frac{2}{Q}\left(q(S) + \sum_{i \in S: q_i < Q} (Q - q_i) \cdot y_i\right)$$

of (9) is obtained using the degree constraints (1b) of all customers $i \in S$. Multiplying this inequality with $Q/2$ and re-arranging the terms, we obtain

$$q(C \setminus S) + \sum_{i \in C \setminus S : q_i < Q} (Q - q_i) \cdot y_i + \frac{Q}{2} x(\delta(S)) + Q y(S) \geq q(C) + \sum_{i \in C : q_i < Q} (Q - q_i) \cdot y_i \qquad \text{(EC.6)}$$

which suggests the following, cut-based separation of yCAPs based on a directed support graph with node set $V \cup \{s,t\}$ where s is the source and t is the target node. For each depot $d \in D$, arc (s,d) is included with infinite capacity to ensure that all depots are outside set S. For each customer $i \in C$, arc (i,t) is added that has capacity $q_i + (Q - q_i) \cdot \bar{y}_i$ if $q_i < Q$ and capacity q_i otherwise. These arcs account for the first two terms on the left-hand side of (EC.6) for customers outside of S. Depot edges $e = \{d,i\} \in \bar{E}_D$, $d \in D$, $i \in C$, are represented by arcs (d,i) with capacity $\frac{Q}{2}\bar{x}_e + Q\bar{y}_i$ if d is the depot from which single-customer routes to i are performed and with capacity $\frac{Q}{2}\bar{x}_e$ otherwise. Finally, we include arcs (i,j) and (j,i) with capacity $\frac{Q}{2}\bar{x}_e$ for each customer edge $e = \{i,j\} \in \bar{E}_C$. Together with aforementioned depot arcs, they account for the last two terms on the left-hand side of (EC.6). A minimum cut from s to t in this graph corresponds to a violated yCAP if its capacity is less than the right-hand side of (EC.6).

EC.2.2. Exact separation of rounded capacity cuts (8)

Formulation (EC.7), inspired by Martinelli et al. (2013), is used for the exact separation of rounded capacity cuts (8). Variables $s_i \in \{0,1\}$, $i \in C$, indicate customers in S, variables $h_e \in \{0,1\}$, $e \in \bar{E}_C$, indicate edges within set S, variables $\kappa \in \{0,\ldots,K\}$, indicate the number of vehicles needed to serve S, and variable $\rho \in \{0,\ldots,Q-1\}$ indicates the total residual vehicle capacity.

$$\max \quad \sum_{e \in \bar{E}} \bar{x}_e h_e - \sum_{j \in C} \bar{z}_j s_j + \kappa \qquad \text{(EC.7a)}$$

$$\text{s.t} \quad h_e \leq s_i \qquad \forall i \in C, \forall e = \{i,j\} \in \bar{E}_C \qquad \text{(EC.7b)}$$

$$\sum_{i \in C} q_j s_j + \rho = Q\kappa \qquad \text{(EC.7c)}$$

$$s(C) \geq 2 \qquad \text{(EC.7d)}$$

All solutions with an objective value larger than zero correspond to a violated cut since the objective function (EC.7a) maximizes the violation of an inequality (8). The associated set S is defined by all nodes $i \in C$ such that $\bar{s}_i = 1$. Constraints (EC.7b) guarantee that edge $\{i,j\} \in \bar{E}_C$ can only be in $E(S)$ if both incident nodes are in S. Equation (EC.7c) links the demand of all nodes in S to the number of vehicles required to serve them and inequality (EC.7d) makes sure that set S contains at least two nodes since the case when $|S| = 1$ is covered by (1k).

Even though the MIP is quite sparse, for large instances the runtime can be long. All solutions found by the MIP with an objective value greater than zero are sorted according to their objective value (non-increasing) and considered in an iterative manner. The capacity cut corresponding to each considered solution is added if its node set S is disjoint to those of all such cuts already added for the current candidate solution. Subsequently, we iterate re-solving the MIP while forcing $s_i = 0$ for all customers i already contained in set S of an already added inequality until no more violated cuts are found. For each identified set S, we also check whether a corresponding yCAP constraint (9) is violated in which case we add this constraint, too.

EC.2.3. Separation of multi-leg depot-consistency constraints (10) and (12)

Formulation (EC.8) used for the exact separation of multi-leg PF constraints (12) is based on their subtour form (EC.2). As we detail below, it is also used to separate multi-leg PE constraints (10) after small modifications. Binary variables s_i, $i \in C$, indicate customers in set S, h_e, $e \in \bar{E}_C$, edges within set S, binary variables l_e, $e \in \bar{E}_D$, indicate edges included in a leg, binary variables f_e, $e \in \bar{E}_D$, indicate depot edges connecting a depot in $D \setminus D'$ and a customer in S, and binary variables t_d, $d \in D$, indicate depots in $D \setminus D'$.

$$\max \sum_{e \in \bar{E}_D} (\bar{x}_e l_e + 0.5 \bar{x}_e f_e) + \sum_{e \in \bar{E}_C} \bar{x}_e h_e + \sum_{j \in C} (\bar{y}_j - \bar{z}_j) s_j \tag{EC.8a}$$

$$h_e \le s_i \qquad \forall i \in C, \forall \{i,j\} \in \bar{E}_C \tag{EC.8b}$$

$$l_e \le s_j \qquad \forall e = \{d,j\} \in \bar{E}_D \tag{EC.8c}$$

$$t_d + l(\delta(d)) \le 1 \qquad \forall d \in D \tag{EC.8d}$$

$$f_e \le t_d \qquad \forall e = \{d,j\} \in \bar{E}_D, d \in D \tag{EC.8e}$$

$$f_e \le s_j \qquad \forall e = \{d,j\} \in \bar{E}_D, j \in C \tag{EC.8f}$$

$$s(C) \ge 2 \tag{EC.8g}$$

All solutions with an objective value larger than zero correspond to a violated cut since the objective function (EC.8a) maximizes the violation of an inequality (12). Constraints (EC.8b) guarantee that edge $\{i,j\} \in \bar{E}_C$ can only be in $E(S)$ if both incident nodes are in S. Similarly, inequalities (EC.8c) make sure that customer j is in S whenever $\{d,j\} \in \bar{E}_D$ is contained in a leg. Constraints (EC.8d) state that each depot is either contained in exactly one leg or in $D \setminus D'$ while (EC.8e) and (EC.8f) are forcing constraints ensuring that depot edges not part of a leg (but counting with a factor of 0.5 in the objective function) are incident to a depot in $D \setminus D'$ and a customer in S. Finally, inequality (EC.8g) make sure that set S contains at least two nodes since multi-leg PF constraints with $|S| = 1$ are equivalent to constraints (3).

As shown in the proof of Theorem 8, multi-leg PE constraints (10) are a special case of multi-leg PF constraints in which $D' = D$. Thus, the variant of formulation (EC.8) obtained by removing variables t_d, $d \in D$, and f_e, $e \in \bar{E}_D$, is used to separate inequalities (10).

Table EC.1 LP gaps [%] of MDSDVRP instances (set mdsd40) for capacity and depot-consistency constraints.

instance	$\|D\|$	D_R	R	RY	RYE2	RYE	RYF1	RYF
mdsd40-1	2	[.1,.9]	2.34	1.94	1.90	1.90	1.94	1.90
		[.3,.7]	3.74	3.01	3.00	3.00	3.01	3.00
		[.7,.9]	1.86	0.75	0.70	0.70	0.75	0.70
	4	[.1,.9]	3.92	3.02	2.68	2.66	2.95	2.65
		[.3,.7]	3.95	2.91	2.81	2.80	2.91	2.79
		[.7,.9]	2.13	0.69	0.59	0.59	0.66	0.58
	6	[.1,.9]	2.66	2.32	2.17	2.09	2.31	2.07
		[.3,.7]	4.90	3.94	3.81	3.81	3.93	3.80
		[.7,.9]	1.33	0.31	0.06	0.05	0.08	0.05
mdsd40-2	2	[.1,.9]	1.99	1.14	1.08	1.08	1.14	1.08
		[.3,.7]	4.28	3.55	3.46	3.46	3.55	3.46
		[.7,.9]	1.75	0.71	0.65	0.65	0.69	0.65
	4	[.1,.9]	2.69	2.08	2.01	2.01	2.08	2.01
		[.3,.7]	5.64	4.34	4.12	4.12	4.34	4.12
		[.7,.9]	1.41	0.29	0.24	0.23	0.25	0.23
	6	[.1,.9]	4.41	3.95	3.45	3.37	3.69	3.37
		[.3,.7]	6.27	5.03	4.58	4.47	4.92	4.46
		[.7,.9]	1.42	0.45	0.32	0.31	0.37	0.31
mdsd40-6	2	[.1,.9]	4.40	3.58	3.58	3.58	3.58	3.58
		[.3,.7]	3.97	2.97	2.97	2.97	2.97	2.97
		[.7,.9]	2.65	1.06	1.05	1.05	1.06	1.05
	4	[.1,.9]	4.34	3.58	3.56	3.54	3.58	3.54
		[.3,.7]	4.48	3.01	2.88	2.83	3.01	2.83
		[.7,.9]	1.71	0.92	0.42	0.42	0.60	0.42
	6	[.1,.9]	4.40	3.55	3.47	3.46	3.54	3.44
		[.3,.7]	3.52	2.52	1.87	1.78	2.32	1.74
		[.7,.9]	0.60	0.41	0.20	0.20	0.22	0.20

EC.3. Computational results for the MDSDVRP

This section contains computational results for the MDSDVRP that accompany the figures and discussion in the article.

Table EC.1 details LP-relaxation gaps [%] of formulation (1) when including different sets of capacity and depot-consistency inequalities. Notation `R` indicates that a variant includes rounded capacity-cuts (8), `Y` indicates inclusion of yCAPs (9), `E2` of two-leg PE constraints, `E` of multi-leg PE constraints (10), `F1` of (single-leg) PF constraints (11), and `F` of multi-leg PF constraints (12).

Table EC.2 compares different configurations of the B&C algorithm on MDSDVRP instance set `mdsd40`. Their abbreviations indicate whether they consider rounded capacity cuts (`R`), yCAPs (`Y`), (multi-leg) PE constraints (`E`), (multi-leg) PF constraints (`F`), or all depot-consistency constraints (`D`). The lifting detailed in Section 6.2 is applied for all variants including depot-consistency constraints. In addition to solution times (`time`) or optimality gaps (`gap`) [%] after two hours, the table also reports numbers of candidate solutions identified by the feasibility check that violate capacity (`# capacity infeasible`) or depot-consistency (`# depot infeasible`) constraints.

EC.4. Computational results for the MDTSP

This section contains detailed computational results for the MDTSP that accompany the figures and discussion in the article. Table EC.3 details LP-relaxation gaps [%] of formulation (1) when including different sets of depot-consistency inequalities. Notation `B` indicates that a variant

Table EC.2 Comparison of B&C configurations for the MDSDVRP on instance set `mdsd40` with $|C| = 40$.

			time (gap)				# capacity infeasible				# depot infeasible			
instance	$\lvert D\rvert$	D_{R}	RY	RYE	RYF	RYD	RY	RYE	RYF	RYD	RY	RYE	RYF	RYD
mdsd40-1	2	[.1,.9]	(0.1)	218	176	**158**	0	1	0	0	11017	0	0	0
		[.3,.7]	(0.4)	650	618	**534**	0	0	0	0	7057	42	7	0
		[.7,.9]	159	158	210	**117**	0	0	1	0	5	0	0	0
	4	[.1,.9]	(1.0)	2516	816	**794**	0	0	0	0	4266	0	0	0
		[.3,.7]	367	**171**	176	173	0	0	0	0	131	0	0	0
		[.7,.9]	138	52	**31**	57	0	2	0	0	327	0	0	0
	6	[.1,.9]	(0.5)	275	204	**193**	0	0	0	0	3778	0	0	0
		[.3,.7]	(0.8)	**449**	525	461	0	0	0	0	3769	0	0	0
		[.7,.9]	74	**10**	11	**10**	0	0	0	0	0	0	0	0
mdsd40-2	2	[.1,.9]	78	82	79	**48**	0	0	0	0	0	0	0	0
		[.3,.7]	(0.4)	1159	987	**735**	59	115	221	33	6481	134	0	0
		[.7,.9]	235	95	92	**90**	0	0	0	0	51	0	0	0
	4	[.1,.9]	72	66	100	**63**	0	0	0	0	4	0	0	0
		[.3,.7]	(0.7)	1787	**1720**	2095	0	0	0	0	4452	0	0	0
		[.7,.9]	**74**	219	173	183	0	0	0	0	0	0	0	0
	6	[.1,.9]	(1.6)	292	**261**	408	0	0	0	0	4864	0	0	0
		[.3,.7]	(2.5)	(0.8)	6360	**4933**	0	0	0	0	4914	0	0	0
		[.7,.9]	71	55	**36**	56	0	0	0	0	8	0	0	0
mdsd40-6	2	[.1,.9]	(1.6)	(1.5)	(1.4)	**(0.7)**	102	150	477	480	4559	3160	2180	3105
		[.3,.7]	(1.7)	**(0.6)**	**(0.6)**	**(0.6)**	21	138	189	265	5419	7723	6953	6554
		[.7,.9]	(0.0)	**209**	287	317	0	0	0	0	5002	4	0	0
	4	[.1,.9]	(1.8)	2386	2082	**1801**	0	302	304	317	4621	1127	0	0
		[.3,.7]	(1.3)	1027	**795**	1543	0	0	0	3	4893	11	6	4
		[.7,.9]	(0.6)	84	147	**81**	0	0	0	0	3273	0	0	0
	6	[.1,.9]	5983	414	**409**	697	723	690	696	790	6182	0	0	0
		[.3,.7]	671	114	129	**111**	0	1	1	0	1167	0	0	0
		[.7,.9]	203	40	57	**31**	0	0	0	0	83	0	0	0
mdsd40-9	2	[.1,.9]	(0.8)	(1.0)	(1.4)	**(0.7)**	20	39	442	251	6893	6754	4025	5599
		[.3,.7]	(0.5)	**(0.4)**	**(0.4)**	**(0.4)**	36	231	1152	1834	6780	6703	5478	5329
		[.7,.9]	(0.8)	**(0.4)**	(0.5)	**(0.4)**	14	127	16	78	1884	3781	2715	4385
	4	[.1,.9]	(1.3)	(0.8)	**(0.4)**	(0.5)	8	22	1670	1600	4014	5822	5762	4028
		[.3,.7]	(0.7)	(0.5)	**(0.2)**	**(0.2)**	15	256	3897	4786	7233	4633	198	71
		[.7,.9]	(1.0)	**2119**	2833	2967	0	0	0	0	3101	3925	3770	3759
	6	[.1,.9]	(0.6)	101	85	**63**	0	0	0	0	7877	0	0	0
		[.3,.7]	(1.3)	133	**98**	151	1	2	2	2	8676	0	0	0
		[.7,.9]	100	**51**	70	61	0	0	0	0	382	0	0	0

includes no such constraints, `E2` indicates inclusion of two-leg PE constraints, `E` of multi-leg PE constraints (10), `F1` of (single-leg) PF constraints (11), and `F` of multi-leg PF constraints (12).

EC.5. Computational results for the SDVRP

This section contains detailed computational results for the SDVRP that accompany the figures and discussion in the article. Tables EC.4 to EC.11 provide detailed results for SDVRP instances considered in the literature for the cases of a limited and unlimited fleet as well as with and without rounding up all costs to integer values. For the best performing B&C variant `RY`, the table reports obtained lower bounds (`LB`), upper bounds (`UB`), solutions times (`time`) or remaining gaps (`gap`) [%] after two hours, and numbers of cuts added for integer (`# lazy cons`) and fractional (`# fract cuts`) candidate solutions. The latter two categories are reported for symmetry breaking constraints (`SY`), subtour elimination constraints (`SE`), yCAPs (`Y`), rounded capacity constraints (`R`), and feasibility cuts (`F`). These tables also reports best known values (`LB`, `UB`, `time`) from the literature (`BK`) and objective values of the heuristic solutions (`heur`) used to initialize the B&C

Table EC.3 LP gaps [%] of MDTSP instances for different depot-consistency constraints.

type	instance	B	E2	E	F1	F
BGS	100-10-1	1.61	1.48	1.07	0.92	0.92
	100-10-2	0.85	0.80	0.36	0.35	0.35
	100-10-3	3.86	3.81	3.68	3.66	3.66
	100-20-1	1.62	1.37	0.98	0.89	0.89
	100-20-2	2.45	2.24	1.75	1.65	1.65
	100-20-3	4.05	3.60	3.10	2.91	2.91
	100-5-1	1.72	1.59	1.56	1.56	1.56
	100-5-2	0.50	0.41	0.41	0.41	0.41
	100-5-3	2.87	2.85	2.80	2.80	2.80
	200-10-1	0.86	0.86	0.78	0.78	0.78
	200-20-1	1.13	1.10	1.00	1.00	1.00
	200-40-1	2.15	1.90	1.55	1.46	1.46
	300-10-1	1.35	1.27	1.10	1.08	1.08
	300-20-1	1.56	1.39	1.24	1.19	1.19
	300-30-1	1.79	1.52	1.11	1.02	1.02
	300-40-1	2.00	1.71	1.12	0.99	0.99
	300-60-1	2.19	1.82	1.19	1.03	1.03
BM	100-10-1	1.41	1.16	0.84	0.84	0.84
	100-10-2	1.70	1.63	1.51	1.51	1.51
	100-10-3	2.29	1.77	1.41	1.10	1.10
	100-5-1	1.98	1.94	1.80	1.80	1.80
	100-5-2	1.22	1.13	0.64	0.58	0.58
	100-5-3	2.82	2.59	2.37	2.37	2.37
	200-10-1	1.42	1.36	1.25	1.14	1.14
	200-10-2	1.76	1.75	1.72	1.72	1.72
	200-10-3	1.66	1.66	1.66	1.66	1.66

algorithm. A dash instead of a best known lower or upper bounds indicates that no such bound has been reported in the literature for the corresponding instance.

EC.6. Computational results with / without heuristic

This section contains detailed computational results showing the impact of the initial heuristic described in Section 6.1. Tables EC.12 to EC.21 provide obtained lower bounds (`LB`), upper bounds (`UB`), and solution times (`time`) or remaining gaps (`gap`) [%]. For each considered benchmark problem these numbers are reported for the best known method from the literature (`BK`), the best performing B&C variant of the considered benchmark problem (`BC`), and a variant of this B&C without applying the heuristic detailed in Section 6.1 (`BC - no heur.`).

Table EC.4 Branch-and-cut results for the SDVRP with limited fleet and rounded costs, part 1/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
S101D1	100	716.00	716.00	3125	716.00	716.00	**2919**	0	5	0	135	412	0	114	142	7265	731.00
S101D2	100	**1337.10**	**1366.00**	-	1276.00	1374.00	(7.1)	7	21	0	44	0	44	148	597	7355	1374.00
S101D3	100	**1832.20**	**1864.00**	-	1773.00	1895.00	(6.4)	3	39	1	71	0	177	1175	1671	6560	1895.00
S101D5	100	**2737.10**	**2770.00**	-	2676.00	2823.00	(5.2)	0	104	0	42	0	0	3018	2062	5741	2823.00
S51D1	50	458.00	458.00	**0**	458.00	458.00	11	0	2	0	2	0	0	8	24	124	458.00
S51D2	50	703.00	703.00	800	703.00	703.00	**266**	0	10	0	84	130	0	118	217	1788	712.00
S51D3	50	935.17	942.00	-	**942.00**	942.00	**1019**	2	2	0	55	0	82	163	209	2876	954.00
S51D4	50	1551.00	1551.00	2102	1551.00	1551.00	**1174**	0	20	0	121	0	0	2309	406	2013	1575.00
S51D5	50	**1325.34**	1328.00	-	1318.00	1328.00	(0.8)	0	20	0	84	0	0	4466	290	3075	1329.00
S51D6	50	**2153.00**	2153.00	**4619**	2148.00	2153.00	(0.2)	1	44	0	167	0	357	8305	946	1860	2177.00
S76D1	75	592.00	592.00	**30**	592.00	592.00	73	0	3	0	26	0	0	20	58	1804	607.00
S76D2	75	**1061.10**	**1082.00**	-	1044.00	1089.00	(4.1)	0	59	0	172	107	0	843	466	9169	1103.00
S76D3	75	**1395.90**	**1420.00**	-	1375.00	1434.00	(4.1)	7	42	0	133	0	279	1797	1449	7712	1434.00
S76D4	75	**2046.10**	**2073.00**	-	2029.00	2097.00	(3.2)	4	34	0	108	0	588	3133	3277	4576	2097.00
SD1	8	228.00	228.00	**0**	228.00	228.00	1	0	0	0	2	0	0	0	2	2	240.00
SD10	64	**2688.00**	2688.00	**2577**	2677.00	2688.00	(0.4)	0	8	0	36	0	0	3736	679	2801	2772.00
SD11	80	13280.00	13280.00	5241	13280.00	13280.00	**107**	0	38	0	11	0	0	272	706	1056	13300.00
SD12	80	7102.36	7315.00	-	**7178.00**	**7216.00**	(0.5)	0	71	0	150	0	0	4431	1760	4891	7235.00
SD13	96	9933.21	-	-	**10023.00**	**10188.00**	(1.6)	0	93	0	53	0	0	3149	2239	5779	10226.00
SD14	120	-	-	-	**10493.00**	**10889.00**	(3.6)	0	69	0	11	0	0	2113	1584	3984	10889.00
SD15	144	-	-	-	**14738.00**	**15230.00**	(3.2)	0	100	0	3	0	0	1355	1951	4177	15230.00
SD16	144	-	-	-	**3384.00**	**3496.00**	(3.2)	0	72	0	137	0	0	1318	2526	6309	3496.00
SD17	160	-	-	-	**26198.00**	**26875.00**	(2.5)	0	72	0	0	0	0	722	1821	3847	26875.00
SD18	160	-	-	-	**13833.00**	**14534.00**	(4.8)	0	35	0	1	0	0	756	1700	3381	14534.00
SD19	192	-	-	-	**19465.00**	**20481.00**	(5.0)	0	18	0	0	0	0	1039	2686	4634	20481.00
SD2	16	708.00	708.00	**0**	708.00	708.00	1	0	0	0	0	0	0	2	25	25	708.00
SD20	240	-	-	-	**38725.80**	**41046.00**	(5.7)	0	0	0	0	0	0	1598	2365	4682	41046.00
SD21	288	-	-	-	**10887.60**	**11655.00**	(6.6)	0	0	0	0	0	0	1043	2259	3628	11655.00
SD3	16	432.00	432.00	**0**	432.00	432.00	1	0	0	0	2	0	0	0	11	11	452.00
SD4	24	630.00	630.00	**0**	630.00	630.00	1	0	0	0	15	0	0	1	38	38	642.00
SD5	32	1392.00	1392.00	11	1392.00	1392.00	**5**	0	0	0	0	0	0	47	156	257	1392.00
SD6	32	832.00	832.00	**1**	832.00	832.00	3	0	11	0	38	0	0	11	107	137	836.00
SD7	40	3640.00	3640.00	**2**	3640.00	3640.00	3	0	0	0	0	0	0	80	192	208	3640.00
SD8	48	5068.00	5068.00	19	5068.00	5068.00	**8**	0	23	0	20	0	0	98	224	251	5068.00
SD9	48	2046.00	2046.00	126	2046.00	2046.00	**80**	0	8	0	45	0	0	367	412	962	2109.00
eil22	21	375.00	375.00	**0**	375.00	375.00	2	0	0	0	0	0	0	2	35	83	375.00
eil23	22	569.00	569.00	**0**	569.00	569.00	1	0	0	0	0	0	0	3	14	21	569.00
eil30	29	510.00	510.00	14	510.00	510.00	**7**	0	5	0	3	15	0	8	39	48	514.00
eil33	32	835.00	835.00	**2**	835.00	835.00	5	0	0	0	0	0	0	17	115	316	835.00
eil51	50	521.00	521.00	36	521.00	521.00	**26**	0	0	0	5	0	0	8	58	791	526.00
eilA101	100	799.80	**814.00**	-	**807.00**	818.00	(1.3)	0	16	0	72	164	0	128	145	9922	819.00
eilA76	75	**807.60**	**818.00**	-	800.00	829.00	(3.5)	0	37	0	183	60	0	581	264	8602	840.00
eilB101	100	**1040.60**	**1061.00**	-	1011.00	1071.00	(5.6)	3	22	0	93	0	29	50	310	2703	1071.00
eilB76	75	**981.40**	**1002.00**	-	958.00	1012.00	(5.3)	0	48	0	79	0	0	871	353	8174	1012.00
eilC76	75	717.80	733.00	-	**724.00**	733.00	(1.2)	0	4	0	67	30	0	170	182	6755	737.00
eilD76	75	666.10	682.00	-	**679.00**	**679.00**	**1670**	0	2	0	35	0	0	64	131	5742	686.00

Table EC.5 Branch-and-cut results for the SDVRP with limited fleet and rounded costs, part 2/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\lvert C \rvert$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
p01_1030	50	753.00	753.00	6098	753.00	753.00	**570**	2	4	0	14	139	66	35	163	2355	757.00
p01_1050	50	**993.38**	998.00	-	993.00	998.00	(0.5)	1	22	0	172	0	133	1005	421	4362	1013.00
p01_1090	50	1480.00	1480.00	**5368**	1480.00	1480.00	6017	2	45	0	212	1	88	5172	440	2421	1494.00
p01_110	50	458.00	458.00	**1**	458.00	458.00	16	0	1	0	10	0	0	11	23	175	459.00
p01_3070	50	**1473.00**	**1473.00**	**5355**	1460.00	1479.00	(1.3)	2	21	0	88	0	235	5219	810	3543	1487.00
p01_7090	50	**2137.98**	**2142.00**	-	2133.00	2143.00	(0.5)	2	12	0	113	0	335	7466	1285	2183	2166.00
p02_1030	75	1059.86	1157.00	-	**1060.00**	**1117.00**	(5.1)	0	42	0	79	0	0	940	329	6909	1117.00
p02_1050	75	1449.11	-	-	**1451.00**	**1498.00**	(3.1)	0	31	0	49	0	0	3233	660	6253	1498.00
p02_1090	75	2241.79	-	-	**2244.00**	**2305.00**	(2.6)	0	117	0	130	0	23	4915	1430	5051	2305.00
p02_110	75	612.00	612.00	4284	612.00	612.00	**2706**	0	3	0	27	0	9	60	73	9423	619.00
p02_3070	75	**2166.59**	2431.00	-	2154.00	**2224.00**	(3.1)	2	63	0	76	0	238	4974	1833	4969	2232.00
p02_7090	75	3155.61	-	-	**3157.00**	**3252.00**	(2.9)	7	9	0	7	0	214	284	471	1112	3252.00
p03_1030	100	**1379.00**	-	-	1352.00	**1450.00**	(6.8)	11	34	0	65	0	54	139	545	6562	1450.00
p03_1050	100	**1914.80**	-	-	1890.00	**2013.00**	(6.1)	2	51	0	54	0	135	902	1458	4151	2013.00
p03_1090	100	**2994.36**	-	-	2964.00	**3098.00**	(4.3)	24	37	1	50	0	393	3021	2604	3936	3098.00
p03_110	100	740.76	760.00	-	**747.00**	**749.00**	(0.3)	0	4	0	52	49	0	110	98	7328	760.00
p03_3070	100	**2894.75**	-	-	2855.00	**3006.00**	(5.0)	18	66	0	45	0	240	3454	2621	5149	3006.00
p03_7090	100	**4269.98**	-	-	4249.00	**4385.00**	(3.1)	26	74	1	31	0	548	3094	2517	3447	4385.00
p04_1030	150	-	-	-	**1843.00**	**2041.00**	(9.7)	0	106	0	25	0	0	369	1026	2413	2041.00
p04_1050	150	-	-	-	**2646.00**	**2856.00**	(7.4)	0	70	0	6	0	0	511	1269	2769	2856.00
p04_1090	150	-	-	-	**4289.00**	**4573.00**	(6.2)	3	89	0	5	0	106	440	1300	2464	4574.00
p04_110	150	-	-	-	**890.00**	**926.00**	(3.9)	0	29	0	67	0	0	91	198	3752	926.00
p04_3070	150	-	-	-	**4098.00**	**4433.00**	(7.6)	1	58	0	5	0	81	679	1451	2752	4433.00
p04_7090	150	-	-	-	**6125.00**	**6447.00**	(5.0)	23	157	0	12	0	365	536	1072	1963	6447.00
p05_1030	199	-	-	-	**2226.00**	**2475.00**	(10.1)	0	26	0	0	0	0	813	2443	4766	2475.00
p05_1050	199	-	-	-	**3179.00**	**3545.00**	(10.3)	0	30	0	0	0	144	841	1574	2673	3545.00
p05_1090	199	-	-	-	**5182.00**	**5602.00**	(7.5)	0	17	0	0	0	47	1043	1856	3131	5604.00
p05_110	199	-	-	-	**1015.00**	**1089.00**	(6.8)	0	35	0	25	0	0	77	272	2861	1089.00
p05_3070	199	-	-	-	**5069.00**	**5520.00**	(8.2)	0	50	0	0	0	0	721	1749	3093	5523.00
p05_7090	199	-	-	-	**7832.00**	**8213.00**	(4.6)	9	28	0	0	0	584	1272	1323	2494	8213.00
p10_1030	199	-	-	-	**2226.00**	**2475.00**	(10.1)	0	27	0	0	0	0	813	2443	4766	2475.00
p10_1050	199	-	-	-	**3179.00**	**3545.00**	(10.3)	0	33	0	0	0	144	841	1574	2673	3545.00
p10_1090	199	-	-	-	**5183.00**	**5602.00**	(7.5)	0	31	0	0	0	47	1046	1858	3134	5604.00
p10_110	199	-	-	-	**1015.00**	**1089.00**	(6.8)	0	35	0	25	0	0	77	272	2861	1089.00
p10_3070	199	-	-	-	**5069.00**	**5520.00**	(8.2)	0	39	0	0	0	0	718	1747	3090	5523.00
p10_7090	199	-	-	-	**7832.00**	**8213.00**	(4.6)	10	28	0	0	0	584	1272	1323	2494	8213.00
p11_1030	120	-	-	-	**2788.00**	**3009.00**	(7.3)	0	102	0	30	0	0	383	1433	3166	3009.00
p11_1050	120	-	-	-	**4074.00**	**4290.00**	(5.0)	6	117	0	13	0	96	1753	1972	3722	4290.00
p11_1090	120	-	-	-	**6665.00**	**6930.00**	(3.8)	6	77	2	11	0	274	2916	2912	3989	6931.00
p11_110	120	-	-	-	**1014.00**	**1032.00**	(1.7)	0	74	1	365	159	30	730	795	4433	1032.00
p11_3070	120	-	-	-	**6415.00**	**6726.00**	(4.6)	0	116	0	5	0	221	3590	2920	3232	6726.00
p11_7090	120	-	-	-	**9924.00**	**10205.00**	(2.8)	46	82	0	6	0	679	1797	2143	2534	10205.00

Table EC.6 Branch-and-cut results for the SDVRP with limited fleet and non-rounded costs, part 1/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
S101D1	100	726.59	726.59	**2636**	726.59	726.59	2796	0	17	0	196	108	0	104	111	7038	766.40
S101D2	100	**1358.90**	**1378.43**	-	1298.71	1387.93	(6.4)	3	10	0	107	0	27	50	287	6558	1387.93
S101D3	100	**1853.10**	**1874.81**	-	1786.92	1924.85	(7.2)	9	18	0	96	0	55	108	699	6381	1924.85
S101D5	100	**2767.60**	**2791.22**	-	2712.98	2866.23	(5.3)	0	56	0	116	0	0	360	1384	6623	2866.23
S51D1	50	459.50	459.50	**0**	459.50	459.50	27	0	0	0	4	0	0	7	20	170	461.24
S51D2	50	708.42	708.42	368	708.42	708.42	**146**	0	1	0	24	20	0	22	87	1965	709.63
S51D3	50	940.82	947.97	-	**947.97**	947.97	**1398**	0	1	0	142	0	64	18	128	3435	957.14
S51D4	50	1553.47	1560.88	-	**1560.88**	1560.88	**695**	0	5	0	72	0	0	26	367	2057	1570.79
S51D5	50	1326.61	1333.67	-	**1330.24**	1333.67	(0.3)	0	1	0	169	0	0	35	250	4416	1360.67
S51D6	50	2165.64	2169.10	-	**2169.10**	2169.10	**2233**	1	3	1	93	0	267	223	1060	2473	2191.27
S76D1	75	598.94	598.94	164	598.94	598.94	**57**	0	2	0	15	0	0	12	39	1638	612.22
S76D2	75	**1071.30**	**1087.40**	-	1050.85	1102.03	(4.6)	0	10	0	179	42	0	163	186	10546	1113.00
S76D3	75	**1407.54**	**1427.86**	-	1387.57	1438.24	(3.5)	0	5	0	401	0	250	93	534	10392	1438.24
S76D4	75	**2059.80**	**2079.76**	-	2049.88	2099.94	(2.4)	3	5	0	89	0	375	100	2378	5192	2102.12
SD1	8	228.28	228.28	**0**	228.28	228.28	1	0	0	0	2	0	0	0	2	2	240.65
SD10	64	**2684.88**	**2684.88**	**70**	2670.17	2684.89	(0.5)	0	31	0	92	0	0	1738	757	3151	2759.67
SD11	80	13280.00	13280.00	5331	13280.00	13280.00	**134**	0	99	0	70	0	0	434	694	1154	13320.00
SD12	80	**7175.80**	**7213.61**	-	7153.52	7230.52	(1.1)	0	74	0	111	0	0	3862	1673	5226	7238.65
SD13	96	**10053.60**	**10110.58**	-	10015.90	10110.60	(0.9)	0	55	0	46	0	0	3428	2066	5566	10187.20
SD14	120	**10588.20**	**10725.38**	-	10499.60	10837.70	(3.1)	0	85	0	21	0	0	2054	1691	4090	10837.70
SD15	144	**14908.50**	**15129.68**	-	14755.80	15379.30	(4.1)	0	86	0	8	0	0	1391	2054	4291	15379.30
SD16	144	**3381.30**	**3381.30**	**6**	3381.08	3456.45	(2.2)	0	52	0	170	0	0	326	1701	5908	3511.02
SD17	160	**26317.20**	**26533.39**	-	26236.40	26997.60	(2.8)	0	38	0	1	0	0	1103	2095	4563	26997.60
SD18	160	**14029.20**	**14283.51**	-	13813.20	14619.30	(5.5)	0	58	0	6	0	0	724	1523	3164	14619.30
SD19	192	**19707.20**	**20191.20**	-	19424.90	20653.70	(5.9)	0	32	0	0	0	0	1089	2118	3874	20653.70
SD2	16	708.28	708.28	**0**	708.28	708.28	1	0	0	0	0	0	0	4	33	33	708.28
SD20	240	**39252.80**	**39840.00**	-	38724.40	40320.80	(4.0)	0	0	0	0	0	0	1271	2452	4634	40320.80
SD21	288	**11271.10**	**11271.10**	**205**	10974.80	11579.10	(5.2)	0	27	0	0	0	0	432	1290	2463	11579.10
SD3	16	430.58	430.58	**0**	430.58	430.58	2	0	0	0	0	0	0	0	8	8	430.58
SD4	24	631.05	631.05	**0**	631.05	631.05	3	0	0	0	0	0	0	2	41	41	631.05
SD5	32	1390.57	1390.57	13	1390.57	1390.57	**4**	0	0	0	0	0	0	39	168	285	1390.57
SD6	32	831.24	831.24	**0**	831.24	831.24	4	0	3	0	22	0	0	4	119	237	834.08
SD7	40	3640.00	3640.00	17	3640.00	3640.00	**3**	0	14	0	6	0	0	78	190	221	3640.00
SD8	48	5068.28	5068.28	67	5068.28	5068.28	**12**	0	5	0	9	0	0	103	247	264	5068.28
SD9	48	2044.20	2044.20	**11**	2044.20	2044.20	101	0	6	0	44	0	0	415	364	962	2081.45
eil22	21	375.28	375.28	**0**	375.28	375.28	2	0	0	0	0	0	0	4	34	76	375.28
eil23	22	568.56	568.56	**0**	568.56	568.56	1	0	0	0	0	0	0	2	14	17	568.56
eil30	29	512.72	512.72	**8**	512.72	512.72	9	0	3	0	0	81	0	9	48	66	512.72
eil33	32	837.06	837.06	**2**	837.06	837.06	8	0	1	0	5	20	0	16	105	263	837.06
eil51	50	524.61	524.61	**17**	524.61	524.61	22	0	0	0	8	3	0	16	52	953	527.67
eilA101	100	810.06	**826.14**	-	**816.94**	831.25	(1.7)	0	12	0	104	175	0	98	134	12841	835.79
eilA76	75	**809.67**	**823.89**	-	806.91	827.23	(2.5)	0	15	0	115	121	0	104	161	8258	849.01
eilB101	100	**1055.40**	**1076.26**	-	1023.93	1088.32	(5.9)	3	7	0	124	0	101	39	183	5396	1088.32
eilB76	75	**985.42**	**1009.04**	-	966.46	1019.12	(5.2)	0	4	0	92	0	0	102	241	10511	1019.12
eilC76	75	724.62	**738.67**	-	**729.07**	740.31	(1.5)	0	1	0	32	36	0	67	113	8087	740.31
eilD76	75	677.32	687.60	-	**682.67**	**686.70**	(0.6)	0	3	0	42	22	0	75	50	9188	696.34

Table EC.7 Branch-and-cut results for the SDVRP with limited fleet and non-rounded costs, part 2/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
p01_1030	50	756.71	756.71	915	756.71	756.71	**343**	0	6	0	35	40	64	21	103	2142	763.04
p01_1050	50	996.93	1005.75	-	**999.21**	1005.75	(0.7)	0	1	0	45	0	60	40	237	5623	1017.76
p01_1090	50	1485.00	1487.18	-	**1487.18**	1487.18	**1326**	0	0	0	94	0	47	23	363	2597	1497.04
p01_110	50	459.50	459.50	**0**	459.50	459.50	11	0	0	0	1	0	0	5	26	231	464.64
p01_3070	50	**1474.10**	1481.71	-	1472.36	1481.71	(0.6)	1	0	1	114	0	157	81	423	5876	1497.66
p01_7090	50	2149.05	2156.14	-	**2152.60**	2156.14	(0.2)	5	3	0	112	0	331	122	1208	2765	2183.33
p02_1030	75	**1093.56**	**1122.91**	-	1064.84	1125.36	(5.4)	0	22	0	180	149	0	131	228	10404	1125.36
p02_1050	75	**1483.17**	1509.79	-	1464.75	**1508.10**	(2.9)	0	2	0	82	0	0	83	365	9418	1509.09
p02_1090	75	**2270.44**	2372.22	-	2263.33	**2327.94**	(2.8)	0	70	0	536	35	21	325	1100	7384	2334.79
p02_110	75	616.58	617.85	-	**617.85**	617.85	**4680**	0	3	0	33	7	9	61	50	10285	635.21
p02_3070	75	**2192.25**	**2235.61**	-	2170.06	2239.89	(3.1)	1	31	2	165	0	246	317	1210	7288	2268.25
p02_7090	75	**3192.10**	**3253.71**	-	3179.68	3262.83	(2.5)	5	54	17	213	0	408	1154	2491	5118	3263.33
p03_1030	100	**1435.23**	1491.82	-	1368.80	**1480.01**	(7.5)	3	2	3	73	0	35	62	297	7127	1480.01
p03_1050	100	**1971.43**	**2018.09**	-	1911.33	2038.20	(6.2)	1	11	1	66	0	50	122	901	7541	2038.20
p03_1090	100	**3043.27**	3136.29	-	3008.80	**3119.57**	(3.6)	11	12	6	92	0	220	156	1236	5457	3119.57
p03_110	100	753.12	762.40	-	**756.25**	**760.00**	(0.5)	0	3	0	54	13	0	98	174	10187	765.25
p03_3070	100	**2945.76**	3044.92	-	2898.70	**3030.84**	(4.4)	15	23	2	104	0	168	476	1539	6089	3030.89
p03_7090	100	**4316.42**	4452.55	-	4302.24	**4417.50**	(2.6)	15	30	17	94	0	531	1167	2306	4129	4417.53
p04_1030	150	**1986.79**	2109.45	-	1886.15	**2052.27**	(8.1)	0	4	0	18	0	0	126	1032	8719	2052.27
p04_1050	150	**2811.64**	2956.18	-	2708.02	**2909.28**	(6.9)	0	7	0	12	0	0	234	1596	9837	2909.28
p04_1090	150	**4474.18**	4708.11	-	4389.91	**4622.66**	(5.0)	2	12	2	10	0	105	298	1724	6615	4622.75
p04_110	150	896.03	1065.94	-	**899.53**	**926.94**	(3.0)	0	4	0	37	0	0	72	90	5449	926.94
p04_3070	150	**4269.77**	4637.69	-	4174.84	**4464.12**	(6.5)	1	36	4	41	0	59	439	1609	4245	4465.86
p04_7090	150	**6287.09**	6529.63	-	6232.93	**6481.48**	(3.8)	19	64	1	53	0	352	538	1657	3253	6481.72
p05_1030	199	**2423.64**	2632.22	-	2259.42	**2524.65**	(10.5)	0	0	0	0	0	0	417	2322	7889	2524.65
p05_1050	199	**3420.17**	3548.13	-	3205.67	**3542.18**	(9.5)	1	32	0	3	0	115	689	1287	2272	3542.18
p05_1090	199	**5425.69**	5894.69	-	5240.85	**5656.48**	(7.3)	0	22	0	0	0	42	780	1875	3113	5656.93
p05_110	199	**1042.37**	1188.45	-	1029.40	**1112.60**	(7.5)	0	12	0	26	0	0	69	233	2715	1112.60
p05_3070	199	**5306.11**	5669.69	-	5138.50	**5557.55**	(7.5)	0	28	0	0	0	0	922	1900	3321	5558.31
p05_7090	199	**8062.24**	8400.74	-	7936.82	**8323.17**	(4.6)	10	25	0	5	0	493	840	1354	2295	8324.13
p10_1030	199	-	-	-	**2259.42**	**2524.65**	(10.5)	0	0	0	0	0	0	417	2322	7889	2524.65
p10_1050	199	-	-	-	**3205.67**	**3542.18**	(9.5)	1	32	0	3	0	115	689	1287	2272	3542.18
p10_1090	199	-	-	-	**5240.85**	**5656.48**	(7.3)	0	32	0	0	0	42	780	1875	3113	5656.93
p10_110	199	-	-	-	**1029.62**	**1112.60**	(7.5)	0	15	0	48	0	0	70	233	2785	1112.60
p10_3070	199	-	-	-	**5138.50**	**5557.55**	(7.5)	0	28	0	0	0	0	922	1900	3321	5558.31
p10_7090	199	-	-	-	**7937.69**	**8323.17**	(4.6)	9	28	0	5	0	493	841	1355	2297	8324.13
p11_1030	120	**2879.63**	**2988.61**	-	2845.77	3012.26	(5.5)	0	16	0	107	0	0	241	1289	6891	3012.26
p11_1050	120	4162.99	**4308.17**	-	**4167.34**	4350.64	(4.2)	0	19	0	86	0	88	252	1916	6532	4350.64
p11_1090	120	**6808.07**	**7020.87**	-	6732.76	7034.31	(4.3)	5	59	3	31	0	192	1344	2354	4681	7034.31
p11_110	120	1023.37	1063.73	-	**1027.54**	**1043.57**	(1.5)	1	58	0	349	105	43	612	740	5196	1049.30
p11_3070	120	**6584.11**	6860.65	-	6537.36	**6748.64**	(3.1)	3	32	10	55	0	128	1089	2183	3951	6748.64
p11_7090	120	**10111.11**	10456.19	-	10065.00	**10322.50**	(2.5)	10	56	8	37	0	428	2547	3216	3628	10322.90

Table EC.8 Branch-and-cut results for the SDVRP with unlimited fleet and rounded costs, part 1/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\vert C\vert$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
S101D1	100	714.87	716.00	-	**716.00**	716.00	**4374**	0	14	0	219	240	5	76	176	6523	746.00
S101D2	100	**1301.93**	**1366.00**	-	1277.00	1374.00	(7.1)	3	16	0	58	0	68	143	677	6362	1374.00
S101D3	100	**1803.51**	**1864.00**	-	1772.00	1890.00	(6.2)	7	47	0	47	0	146	769	1230	5654	1890.00
S101D5	100	**2709.48**	**2770.00**	-	2668.00	2804.00	(4.9)	11	43	0	36	0	343	2804	2590	4994	2804.00
S51D1	50	458.00	458.00	20	458.00	458.00	**14**	0	1	0	9	0	6	9	59	218	470.00
S51D2	50	703.00	703.00	674	703.00	703.00	**198**	0	12	0	50	33	25	45	184	1907	716.00
S51D3	50	933.07	943.00	-	**942.00**	**942.00**	**944**	1	5	0	87	0	99	152	295	2647	954.00
S51D4	50	1547.44	1551.00	-	**1551.00**	1551.00	**2266**	0	22	0	57	0	263	2481	758	2240	1569.00
S51D5	50	**1326.73**	1328.00	-	1316.00	1328.00	(0.9)	0	22	0	97	0	123	4387	350	2948	1329.00
S51D6	50	**2153.00**	**2153.00**	**4110**	2147.00	2166.00	(0.9)	1	46	0	289	0	377	5813	1500	2682	2175.00
S76D1	75	592.00	592.00	134	592.00	592.00	**101**	0	1	0	29	0	3	26	72	2493	607.00
S76D2	75	**1040.67**	**1082.00**	-	1039.00	1092.00	(4.9)	0	21	0	33	0	126	565	486	9695	1092.00
S76D3	75	**1379.57**	**1420.00**	-	1375.00	1434.00	(4.1)	8	44	0	176	0	212	1790	1233	7855	1434.00
S76D4	75	**2034.70**	**2073.00**	-	2030.00	2096.00	(3.1)	7	27	0	73	1	497	3176	3210	4489	2097.00
SD1	8	228.00	228.00	0	228.00	228.00	0	0	0	0	0	0	4	0	21	22	240.00
SD10	64	**2688.00**	**2688.00**	**2317**	2670.00	2698.00	(1.0)	6	19	0	139	0	160	2987	1593	3613	2744.00
SD11	80	13280.00	13280.00	999	13280.00	13280.00	**140**	0	49	0	33	0	202	248	755	1135	13340.00
SD12	80	7133.81	7279.00	-	**7168.00**	**7221.00**	(0.7)	15	46	0	72	0	505	3624	2614	3941	7274.00
SD13	96	9992.94	**10112.00**	-	**10033.00**	10232.00	(1.9)	3	70	0	40	0	781	3202	2627	4167	10232.00
SD14	120	-	-	-	**10485.00**	**10819.00**	(3.1)	24	32	1	17	0	343	2022	1554	3736	10819.00
SD15	144	-	-	-	**14776.00**	**15159.00**	(2.5)	1	61	0	12	0	517	1505	1546	3642	15159.00
SD16	144	-	-	-	**3348.00**	**3491.00**	(4.1)	6	55	0	374	0	217	1038	2446	5347	3491.00
SD17	160	-	-	-	**26269.00**	**26883.00**	(2.3)	7	58	0	1	0	680	970	2052	4122	26883.00
SD18	160	-	-	-	**13826.00**	**14507.00**	(4.7)	8	24	0	6	0	452	785	1528	3212	14507.00
SD19	192	-	-	-	**19418.00**	**20408.00**	(4.9)	1	24	1	1	0	539	963	1835	3516	20408.00
SD2	16	708.00	708.00	**0**	708.00	708.00	1	0	0	0	0	0	31	7	35	46	708.00
SD20	240	-	-	-	**38697.70**	**40657.00**	(4.8)	0	0	0	0	0	979	1453	2153	3949	40657.00
SD21	288	-	-	-	**10956.00**	**11669.00**	(6.1)	2	26	0	0	0	1411	524	2408	2213	11669.00
SD3	16	432.00	432.00	**0**	432.00	432.00	2	0	0	0	0	0	14	0	38	47	432.00
SD4	24	630.00	630.00	**1**	630.00	630.00	3	0	0	0	1	0	32	0	106	114	634.00
SD5	32	1392.00	1392.00	15	1392.00	1392.00	**4**	0	0	0	0	0	50	20	139	259	1392.00
SD6	32	832.00	832.00	**3**	832.00	832.00	4	0	0	0	0	0	39	2	81	198	835.00
SD7	40	3640.00	3640.00	26	3640.00	3640.00	**4**	0	0	0	0	0	77	65	142	210	3640.00
SD8	48	5068.00	5068.00	33	5068.00	5068.00	**11**	0	0	0	0	0	89	100	234	297	5068.00
SD9	48	2046.00	2046.00	174	2046.00	2046.00	**126**	0	0	0	14	0	101	129	254	697	2079.00
eil22	21	375.00	375.00	**1**	375.00	375.00	2	0	0	0	0	0	3	3	35	114	375.00
eil23	22	569.00	569.00	2	569.00	569.00	**1**	0	0	0	0	0	7	2	14	16	569.00
eil30	29	503.00	503.00	**1**	503.00	503.00	3	0	0	0	0	0	7	6	35	38	503.00
eil33	32	835.00	835.00	12	835.00	835.00	**5**	0	0	0	0	0	8	16	118	273	835.00
eil51	50	521.00	521.00	61	521.00	521.00	**20**	0	0	0	3	0	5	9	97	586	526.00
eilA101	100	792.40	**814.00**	-	**806.00**	817.00	(1.3)	0	3	0	51	0	182	113	209	9253	819.00
eilA76	75	792.71	**818.00**	-	**799.00**	823.00	(2.9)	0	19	0	58	0	65	233	264	7321	823.00
eilB101	100	**1017.77**	**1061.00**	-	1012.00	1071.00	(5.5)	2	20	0	112	0	36	66	328	3921	1071.00
eilB76	75	**957.60**	**1002.00**	-	956.00	1015.00	(5.8)	0	42	0	103	0	174	497	322	8796	1015.00
eilC76	75	714.24	**733.00**	-	**721.00**	734.00	(1.8)	0	9	0	60	170	33	150	283	7310	737.00
eilD76	75	667.93	682.00	-	**679.00**	**679.00**	**5746**	0	11	0	106	30	19	154	151	8706	695.00

Table EC.9 Branch-and-cut results for the SDVRP with unlimited fleet and rounded costs, part 2/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
p01_1030	50	752.00	753.00	-	**753.00**	753.00	**415**	1	12	0	23	156	52	43	120	1981	757.00
p01_1050	50	991.36	998.00	-	**998.00**	998.00	**6710**	0	4	0	19	0	99	597	338	4837	1013.00
p01_1090	50	1480.00	1480.00	3725	1480.00	1480.00	**3655**	1	20	0	127	0	176	2335	493	2374	1493.00
p01_110	50	458.00	458.00	**10**	458.00	458.00	17	0	1	0	3	0	4	10	39	194	458.00
p01_3070	50	**1473.00**	**1473.00**	**6953**	1458.00	1479.00	(1.4)	5	6	0	68	0	258	4305	694	3289	1482.00
p01_7090	50	**2134.87**	2142.00	-	2133.00	2142.00	(0.4)	3	9	0	79	0	415	7553	1360	2350	2166.00
p02_1030	75	**1062.36**	1172.00	-	1061.00	**1108.00**	(4.2)	0	21	0	49	0	63	817	493	8560	1108.00
p02_1050	75	**1456.12**	1557.00	-	1451.00	**1499.00**	(3.2)	0	29	0	28	0	108	2020	828	6248	1499.00
p02_1090	75	**2258.51**	**2304.00**	-	2246.00	2305.00	(2.6)	3	44	0	40	0	509	4320	2833	4796	2305.00
p02_110	75	609.19	612.00	-	**612.00**	612.00	**3254**	0	2	0	18	5	25	51	177	8608	619.00
p02_3070	75	**2177.37**	2253.00	-	2153.00	**2234.00**	(3.6)	3	30	0	57	0	349	4080	2022	4694	2243.00
p02_7090	75	**3171.95**	3233.00	-	3163.00	**3230.00**	(2.1)	8	94	0	125	0	525	4357	2495	3618	3244.00
p03_1030	100	**1379.18**	1542.00	-	1357.00	**1456.00**	(6.8)	6	41	0	23	0	98	131	670	6428	1456.00
p03_1050	100	**1925.21**	2053.00	-	1889.00	**2014.00**	(6.2)	1	36	0	20	0	149	1060	1336	4616	2014.00
p03_1090	100	**3005.48**	3152.00	-	2965.00	**3098.00**	(4.3)	18	45	0	75	0	435	3183	2695	3927	3098.00
p03_110	100	737.64	762.00	-	**744.00**	**750.00**	(0.8)	0	27	0	167	326	22	170	195	8206	789.00
p03_3070	100	**2909.22**	3049.00	-	2860.00	**3001.00**	(4.7)	23	61	0	30	0	273	3173	2413	4579	3001.00
p03_7090	100	**4251.65**	4404.00	-	4245.00	**4368.00**	(2.8)	16	65	3	39	0	563	4080	3289	4127	4368.00
p04_1030	150	-	-	-	**1846.00**	**2034.00**	(9.2)	1	23	0	5	0	86	421	1791	4770	2034.00
p04_1050	150	-	-	-	**2639.00**	**2860.00**	(7.7)	1	31	0	4	0	104	580	1735	3714	2860.00
p04_1090	150	-	-	-	**4299.00**	**4573.00**	(6.0)	1	47	0	0	0	192	448	1386	2617	4573.00
p04_110	150	-	-	-	**889.00**	**947.00**	(6.1)	0	23	0	108	0	13	39	203	2344	947.00
p04_3070	150	-	-	-	**4062.00**	**4416.00**	(8.0)	1	61	0	2	0	158	464	1289	2519	4416.00
p04_7090	150	-	-	-	**6113.00**	**6420.00**	(4.8)	26	68	0	15	0	425	1361	1711	3166	6420.00
p05_1030	199	-	-	-	**2194.51**	**2480.00**	(11.5)	0	0	0	0	0	75	415	2535	5168	2480.00
p05_1050	199	-	-	-	**3178.00**	**3510.00**	(9.5)	13	21	0	0	0	138	980	1419	2388	3510.00
p05_1090	199	-	-	-	**5167.00**	**5579.00**	(7.4)	0	9	0	0	0	213	1084	1911	3150	5580.00
p05_110	199	-	-	-	**1015.00**	**1126.00**	(9.9)	1	26	0	23	0	32	63	336	3186	1126.00
p05_3070	199	-	-	-	**5041.00**	**5484.00**	(8.1)	0	33	0	0	0	210	924	1711	2948	5486.00
p05_7090	199	-	-	-	**7807.00**	**8206.00**	(4.9)	15	25	0	0	0	589	741	1233	2036	8206.00
p10_1030	199	-	-	-	**2194.51**	**2480.00**	(11.5)	0	0	0	0	0	75	415	2535	5168	2480.00
p10_1050	199	-	-	-	**3178.00**	**3510.00**	(9.5)	11	21	0	0	0	138	980	1419	2388	3510.00
p10_1090	199	-	-	-	**5167.00**	**5579.00**	(7.4)	3	19	0	0	0	213	1087	1913	3152	5580.00
p10_110	199	-	-	-	**1015.00**	**1126.00**	(9.9)	0	23	0	17	0	32	63	336	3186	1126.00
p10_3070	199	-	-	-	**5041.00**	**5484.00**	(8.1)	0	27	0	0	0	210	924	1711	2948	5486.00
p10_7090	199	-	-	-	**7807.00**	**8206.00**	(4.9)	14	26	0	0	0	589	741	1233	2036	8206.00
p11_1030	120	-	-	-	**2791.00**	**2976.00**	(6.2)	0	82	0	13	0	62	351	1726	3793	2976.00
p11_1050	120	-	-	-	**4089.00**	**4296.00**	(4.8)	1	200	0	7	0	124	1349	1542	3690	4296.00
p11_1090	120	-	-	-	**6673.00**	**6982.00**	(4.4)	5	111	0	8	0	341	3427	3127	3567	6982.00
p11_110	120	-	-	-	**1013.00**	**1032.00**	(1.8)	0	103	0	407	70	54	1136	906	4681	1032.00
p11_3070	120	-	-	-	**6436.00**	**6663.00**	(3.4)	19	88	1	5	0	188	1712	2106	2883	6663.00
p11_7090	120	-	-	-	**9922.00**	**10193.00**	(2.7)	17	96	1	7	0	726	2876	2731	3028	10193.00

Table EC.10 Branch-and-cut results for the SDVRP with unlimited fleet and non-rounded costs, part 1/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
S101D1	100	716.92	726.59	-	**726.59**	726.59	**3318**	0	22	0	191	110	7	138	122	5419	768.72
S101D2	100	**1356.78**	**1378.43**	-	1296.13	1394.67	(7.1)	0	6	0	56	0	31	72	406	6711	1394.67
S101D3	100	**1845.07**	**1874.81**	-	1784.59	1921.54	(7.1)	0	4	1	75	0	89	82	718	7063	1921.59
S101D5	100	**2758.21**	**2791.22**	-	2687.48	2816.49	(4.6)	8	8	1	66	0	87	149	1007	5854	2816.86
S51D1	50	459.50	459.50	**13**	459.50	459.50	17	0	0	0	13	0	5	9	40	195	472.61
S51D2	50	708.42	708.42	2376	708.42	708.42	**248**	0	3	0	25	8	17	24	129	2263	711.42
S51D3	50	947.97	947.97	20738	947.97	947.97	**1486**	0	0	0	105	0	72	25	147	3195	957.14
S51D4	50	1560.88	1560.88	**3827**	1560.88	1560.88	3860	0	3	0	67	9	153	31	505	3232	1569.81
S51D5	50	**1333.67**	1333.67	**6310**	1329.62	1333.67	(0.3)	2	1	0	200	0	58	48	355	4544	1348.19
S51D6	50	2169.10	2169.10	16755	2169.10	2169.10	**6414**	7	7	4	87	0	304	322	941	2273	2191.75
S76D1	75	598.94	598.94	563	598.94	598.94	**88**	0	2	0	11	0	3	18	79	1744	613.99
S76D2	75	**1066.88**	**1087.40**	-	1044.45	1098.92	(5.0)	0	3	0	86	0	38	89	266	10587	1098.92
S76D3	75	**1406.85**	**1427.86**	-	1385.89	1438.24	(3.6)	2	2	0	132	0	257	97	445	10349	1438.24
S76D4	75	**2053.66**	**2079.76**	-	2046.84	2088.79	(2.0)	1	1	0	206	0	478	132	1868	5863	2101.76
SD1	8	228.28	228.28	0	228.28	228.28	0	0	0	0	0	0	4	0	19	18	240.00
SD10	64	**2684.86**	2684.88	**1903**	2667.57	2684.88	(0.6)	4	11	0	36	0	202	2034	1195	3503	2746.57
SD11	80	13280.00	13280.00	1340	13280.00	13280.00	**122**	0	23	0	19	0	232	200	655	1069	13340.00
SD12	80	7135.27	7270.87	-	**7159.98**	**7242.17**	(1.1)	15	25	1	75	0	433	3984	2298	5028	7242.17
SD13	96	9992.74	**10110.58**	-	**10035.20**	10110.60	(0.7)	4	54	0	47	0	582	2840	2362	4695	10230.20
SD14	120	**10502.76**	**10754.70**	-	10487.90	10849.10	(3.3)	34	32	1	19	0	369	1931	1833	4005	10849.10
SD15	144	**14787.05**	**15151.10**	-	14755.40	15342.60	(3.8)	14	31	1	7	0	501	1302	2002	3907	15342.60
SD16	144	**3379.33**	**3379.33**	**9772**	3349.78	3467.84	(3.4)	10	29	1	28	0	143	252	1134	3707	3467.84
SD17	160	26166.80	**26547.44**	-	**26226.00**	26798.50	(2.1)	8	36	0	0	0	669	1156	2031	4093	26798.50
SD18	160	**13892.74**	**14334.03**	-	13808.00	14438.30	(4.4)	21	53	0	7	0	492	750	1467	2965	14438.30
SD19	192	**19584.84**	**20191.20**	-	19457.50	20562.10	(5.4)	2	20	0	0	0	556	1503	1932	3755	20562.10
SD2	16	708.28	708.28	**0**	708.28	708.28	1	0	0	0	0	0	28	8	27	37	708.28
SD20	240	**38901.37**	**39840.00**	-	38734.90	40375.30	(4.1)	0	0	0	0	0	913	1497	2189	4182	40375.30
SD21	288	**11254.83**	**11271.10**	-	10971.40	11592.60	(5.4)	0	0	0	0	0	919	721	2066	3099	11592.60
SD3	16	430.58	430.58	**0**	430.58	430.58	2	0	0	0	0	0	15	0	36	54	430.58
SD4	24	631.05	631.05	**1**	631.05	631.05	4	0	0	0	0	0	30	0	73	85	633.44
SD5	32	1390.57	1390.57	51	1390.57	1390.57	**5**	0	0	0	0	0	46	16	194	298	1390.57
SD6	32	831.24	831.24	4	831.24	831.24	4	0	0	0	0	0	36	2	144	291	831.24
SD7	40	3640.00	3640.00	55	3640.00	3640.00	**4**	0	0	0	0	0	90	58	174	226	3640.00
SD8	48	5068.28	5068.28	72	5068.28	5068.28	**7**	0	24	0	9	0	97	103	255	335	5068.28
SD9	48	2044.20	2044.20	267	2044.20	2044.20	**114**	0	7	0	29	0	108	184	278	902	2078.86
eil22	21	375.28	375.28	6	375.28	375.28	**2**	0	0	0	0	0	4	3	43	109	375.28
eil23	22	568.56	568.56	1	568.56	568.56	1	0	0	0	0	0	9	3	14	18	568.56
eil30	29	505.01	505.01	**2**	505.01	505.01	3	0	0	0	0	0	3	10	42	55	505.01
eil33	32	837.06	837.06	46	837.06	837.06	**7**	0	1	0	6	20	10	15	107	320	837.06
eil51	50	524.61	524.61	95	524.61	524.61	**16**	0	0	0	4	0	2	7	68	729	527.67
eilA101	100	804.40	**826.14**	-	**815.00**	835.79	(2.5)	0	9	0	109	102	123	115	221	11076	835.79
eilA76	75	**809.58**	**823.89**	-	805.25	828.75	(2.8)	0	7	0	107	31	32	96	166	9369	834.12
eilB101	100	**1055.59**	**1076.26**	-	1023.72	1088.19	(5.9)	1	9	0	137	0	117	48	169	5076	1088.19
eilB76	75	**984.13**	**1009.04**	-	962.02	1015.83	(5.3)	0	6	0	125	1	70	131	229	9675	1015.83
eilC76	75	722.76	738.67	-	**728.73**	738.67	(1.3)	0	5	0	77	57	19	79	130	9574	747.83
eilD76	75	674.17	**687.60**	-	**681.31**	690.17	(1.3)	0	1	0	39	7	13	80	123	10800	692.66

Table EC.11 Branch-and-cut results for the SDVRP with unlimited fleet and non-rounded costs, part 2/2

		BK			BC			# lazy cons					# fract cuts				
instance	$\lvert C\rvert$	LB	UB	time	LB	UB	time (gap)	SY	SE	Y	R	F	SY	SE	Y	R	heur
p01_1030	50	756.70	756.71	20494	**756.71**	756.71	**281**	0	2	0	38	3	55	14	111	2126	763.04
p01_1050	50	**1005.75**	1005.75	**23257**	998.04	1005.75	(0.8)	7	2	0	106	0	59	65	173	5986	1016.84
p01_1090	50	1487.18	1487.18	7155	1487.18	1487.18	**1388**	0	0	0	54	0	105	25	330	3000	1494.88
p01_110	50	459.50	459.50	**8**	459.50	459.50	12	0	0	0	8	0	5	10	51	230	464.64
p01_3070	50	**1481.71**	1481.71	**12418**	1473.52	1481.71	(0.6)	0	1	0	128	0	168	61	432	6421	1497.66
p01_7090	50	2155.80	2155.80	42587	2155.80	2155.80	**7098**	1	0	0	32	0	262	94	783	2446	2179.23
p02_1030	75	**1095.65**	1121.82	-	1067.52	**1119.73**	(4.7)	0	22	0	164	136	52	163	265	10679	1124.12
p02_1050	75	**1482.50**	1514.39	-	1461.17	**1509.02**	(3.2)	0	4	0	81	0	88	98	510	9764	1509.02
p02_1090	75	**2272.05**	**2318.28**	-	2259.26	2329.53	(3.0)	2	13	0	126	0	416	238	1766	6292	2330.61
p02_110	75	612.45	617.85	-	**617.85**	617.85	**5415**	0	5	0	32	9	26	91	153	10496	635.21
p02_3070	75	**2190.16**	**2237.19**	-	2172.76	2253.51	(3.6)	4	11	0	86	0	224	278	1097	9406	2256.34
p02_7090	75	**3192.55**	**3232.15**	-	3181.27	3237.92	(1.7)	4	26	11	116	0	401	807	1748	4711	3253.97
p03_1030	100	**1437.78**	**1477.35**	-	1369.50	1480.01	(7.5)	3	7	0	90	0	40	62	484	7291	1480.01
p03_1050	100	**1971.34**	2040.92	-	1898.68	**2038.20**	(6.8)	2	19	0	89	0	79	105	1074	7349	2038.20
p03_1090	100	**3042.93**	3127.06	-	3001.04	**3115.42**	(3.7)	23	19	3	88	0	322	276	1740	5668	3115.54
p03_110	100	749.42	769.42	-	**759.38**	**760.00**	(0.1)	0	3	0	28	0	3	82	126	7885	761.77
p03_3070	100	**2945.42**	3030.66	-	2896.12	**3026.71**	(4.3)	15	18	2	106	0	275	314	1657	6422	3026.71
p03_7090	100	**4334.44**	**4417.57**	-	4305.20	4436.99	(3.0)	23	35	16	93	0	422	828	2186	4192	4437.00
p04_1030	150	**1986.34**	2066.46	-	1876.92	**2056.87**	(8.7)	0	1	0	9	0	61	122	1021	8821	2056.87
p04_1050	150	**2811.98**	2917.80	-	2699.47	**2901.97**	(7.0)	0	0	0	0	0	84	274	1770	10886	2901.97
p04_1090	150	**4474.92**	4665.87	-	4388.76	**4624.60**	(5.1)	6	17	0	5	0	181	321	1558	5402	4624.60
p04_110	150	**895.46**	**947.14**	-	895.02	966.90	(7.4)	0	14	0	121	0	15	33	137	2419	966.90
p04_3070	150	**4267.33**	**4438.76**	-	4153.56	4455.14	(6.8)	7	14	4	8	0	166	397	1487	4494	4455.14
p04_7090	150	**6284.76**	6482.19	-	6216.65	**6457.37**	(3.7)	23	46	2	16	0	365	485	1746	3598	6458.23
p05_1030	199	**2423.99**	2583.29	-	2254.17	**2528.69**	(10.9)	0	0	0	0	0	72	359	2236	7792	2528.69
p05_1050	199	**3420.23**	3568.25	-	3244.54	**3542.18**	(8.4)	0	0	0	0	0	121	660	2911	9022	3542.18
p05_1090	199	**5422.95**	5673.18	-	5257.30	**5627.28**	(6.6)	0	27	0	1	0	182	1033	1832	3096	5627.35
p05_110	199	**1042.37**	1148.27	-	1028.84	**1117.58**	(7.9)	0	16	0	28	0	22	73	222	2828	1117.58
p05_3070	199	**5304.09**	5559.29	-	5147.61	**5505.91**	(6.5)	0	14	0	6	0	181	1025	1867	3339	5506.85
p05_7090	199	**8062.14**	8359.65	-	7937.06	**8301.59**	(4.4)	2	22	0	1	0	538	1117	1500	2584	8301.59
p10_1030	199	-	-	-	**2254.17**	**2528.69**	(10.9)	0	0	0	0	0	72	359	2236	7792	2528.69
p10_1050	199	-	-	-	**3244.54**	**3542.18**	(8.4)	0	0	0	0	0	121	660	2911	9022	3542.18
p10_1090	199	-	-	-	**5257.30**	**5627.28**	(6.6)	0	22	0	1	0	182	1033	1832	3096	5627.35
p10_110	199	-	-	-	**1028.68**	**1117.58**	(8.0)	0	15	0	24	0	21	71	222	2735	1117.58
p10_3070	199	-	-	-	**5147.73**	**5505.94**	(6.5)	0	12	0	8	0	181	1025	1867	3339	5506.90
p10_7090	199	-	-	-	**7937.06**	**8301.59**	(4.4)	5	25	0	0	0	538	1117	1500	2584	8301.59
p11_1030	120	**2867.79**	**2983.82**	-	2844.93	3017.20	(5.7)	0	11	0	90	0	41	185	1665	6082	3017.20
p11_1050	120	4156.68	**4259.94**	-	**4168.03**	4350.64	(4.2)	0	9	0	111	0	72	270	1764	6234	4350.64
p11_1090	120	**6780.19**	**6995.85**	-	6738.52	7173.25	(6.1)	12	38	4	41	0	236	1059	1815	3518	7173.58
p11_110	120	1023.39	1055.28	-	**1024.49**	**1049.30**	(2.4)	0	44	0	572	18	50	565	744	3892	1049.30
p11_3070	120	**6593.28**	6822.31	-	6542.85	**6711.40**	(2.5)	12	34	5	38	0	138	839	1712	3685	6713.78
p11_7090	120	**10113.55**	10376.94	-	10111.30	**10342.90**	(2.2)	9	53	9	49	0	576	1446	2871	3418	10343.40

Table EC.12 Comparison with/without heuristic for the MDSDVRP on instances from the literature.

				BK	BC			BC - no heuristic		
D_R	instance	$\lvert C\rvert$	$\lvert D\rvert$	UB	LB	UB	time (gap)	LB	UB	time (gap)
[.1, .9]	mdsd1	50	4	1018.22	988.03	988.03	6534	988.03	988.03	4423
	mdsd2	75	5	1289.06	1249.19	1271.33	(1.7)	1242.06	1335.06	(7.0)
	mdsd3	100	2	2624.41	2444.27	2575.15	(5.1)	2435.64	2709.52	(10.1)
	mdsd4	100	2	2393.23	2222.92	2357.63	(5.7)	2220.18	2558.99	(13.2)
	mdsd5	100	3	1963.13	1838.94	1924.35	(4.4)	1838.92	-	(nan)
	mdsd6	100	4	1963.68	1844.85	1903.48	(3.1)	1840.95	2029.79	(9.3)
	mdsd7	249	2	16096.91	14628.10	15938.10	(8.2)	14650.20	-	(nan)
	mdsd8	249	3	13258.26	12068.70	13149.50	(8.2)	12117.40	-	(nan)
	mdsd9	249	4	11959.27	10758.60	11730.20	(8.3)	10768.10	-	(nan)
	mdsd10	249	5	11377.30	10424.20	11241.00	(7.3)	10517.90	-	(nan)
[.3, .7]	mdsd1	50	4	990.85	956.27	956.27	2871	956.27	956.27	2919
	mdsd2	75	5	1223.57	1162.18	1210.36	(4.0)	1158.93	1249.53	(7.3)
	mdsd3	100	2	2558.33	2388.33	2554.07	(6.5)	2388.76	2678.84	(10.8)
	mdsd4	100	2	2336.65	2178.54	2336.47	(6.8)	2178.22	-	(nan)
	mdsd5	100	3	1871.47	1749.03	1852.82	(5.6)	1739.57	1949.04	(10.7)
	mdsd6	100	4	1887.48	1750.92	1849.02	(5.3)	1752.30	2032.84	(13.8)
	mdsd7	249	2	16136.07	14624.30	16165.20	(9.5)	14776.00	-	(nan)
	mdsd8	249	3	13444.18	12172.70	13322.00	(8.6)	12217.40	-	(nan)
	mdsd9	249	4	12176.61	10956.70	12054.00	(9.1)	10956.70	-	(nan)
	mdsd10	249	5	11831.52	10519.40	11570.40	(9.1)	10519.40	-	(nan)
[.7, .9]	mdsd1	50	4	1344.99	1325.64	1325.64	537	1325.64	1325.64	290
	mdsd2	75	5	1705.98	1705.84	1705.84	1437	1705.84	1705.84	1326
	mdsd3	100	2	3878.34	3750.82	3890.69	(3.6)	3757.28	3993.27	(5.9)
	mdsd4	100	2	3525.24	3362.58	3501.13	(4.0)	3363.18	3635.66	(7.5)
	mdsd5	100	3	2772.58	2659.72	2731.22	(2.6)	2659.16	2805.38	(5.2)
	mdsd6	100	4	2696.47	2612.24	2654.70	(1.6)	2607.55	2729.45	(4.5)
	mdsd7	249	2	25502.49	23620.30	24915.40	(5.2)	23652.30	-	(nan)
	mdsd8	249	3	20915.02	19410.60	20566.40	(5.6)	19514.40	-	(nan)
	mdsd9	249	4	18844.77	17531.30	18516.40	(5.3)	17528.90	-	(nan)
	mdsd10	249	5	17777.76	16620.20	17445.50	(4.7)	16632.60	-	(nan)
SQ	sq1	32	2	1063.08	1052.58	1052.58	28	1052.58	1052.58	59
	sq2	48	3	1601.02	1578.87	1578.87	403	1578.87	1578.87	245
	sq5	64	2	3434.71	3394.32	3428.98	(1.0)	3385.19	3459.07	(2.1)
	sq3	64	4	2142.11	2098.47	2126.84	(1.3)	2098.21	2134.30	(1.7)
	sq4	80	5	2684.02	2622.64	2652.15	(1.1)	2621.25	2700.10	(2.9)
	sq9	96	2	7109.71	6906.32	7050.18	(2.0)	6908.81	7180.97	(3.8)
	sq6	96	3	5142.06	5045.43	5148.10	(2.0)	5037.49	5446.51	(7.5)
	sq7	128	4	6869.14	6680.95	6865.55	(2.7)	6680.46	7211.35	(7.4)
	sq10	144	3	10586.51	10242.90	10586.90	(3.2)	10242.70	11102.30	(7.7)
	sq8	160	5	8600.60	8321.34	8587.48	(3.1)	8319.63	9005.93	(7.6)
	sq11	192	4	14135.80	13496.00	14101.60	(4.3)	13495.10	14928.50	(9.6)
	sq12	240	5	17739.64	16884.70	17634.50	(4.3)	16884.70	18581.20	(9.1)

Table EC.13 Comparison with/without heuristic for the MDTSP on instances from the literature.

				BK			BC			BC - no heur.		
type	instance	$\|C\|$	$\|D\|$	LB	UB	time	LB	UB	time	LB	UB	time
BGS	100-5-1	100	5	1569	1569	117	1569	1569	9	1569	1569	5
	100-5-2	100	5	1705	1705	1	1705	1705	2	1705	1705	1
	100-5-3	100	5	1553	1553	58	1553	1553	14	1553	1553	8
	100-10-1	100	10	1524	1524	20	1524	1524	6	1524	1524	5
	100-10-2	100	10	1702	1702	2	1702	1702	3	1702	1702	1
	100-10-3	100	10	1530	1530	124	1530	1530	13	1530	1530	9
	100-20-1	100	20	1509	1509	5	1509	1509	18	1509	1509	5
	100-20-2	100	20	1695	1695	9	1695	1695	4	1695	1695	4
	100-20-3	100	20	1494	1494	14	1494	1494	10	1494	1494	16
	200-10-1	200	10	2266	2266	5997	2266	2266	60	2266	2266	40
	200-20-1	200	20	2250	2250	1110	2250	2250	53	2250	2250	466
	200-40-1	200	40	2211	2211	8134	2211	2211	244	2211	2211	205
	300-10-1	300	10	2675	2826	-	2700	2700	219	2700	2700	692
	300-20-1	300	20	2666	2870	-	2687	2687	320	2687	2687	397
	300-30-1	300	30	2664	2717	-	2679	2679	155	2679	2679	452
	300-40-1	300	40	2662	2693	-	2676	2676	397	2676	2676	879
	300-60-1	300	60	2650	2729	-	2665	2665	625	2665	2665	6525
BM	100-5-1	100	5	37943	37943	13	37943	37943	17	37943	37943	5
	100-5-2	100	5	30832	30832	2	30832	30832	2	30832	30832	1
	100-5-3	100	5	32850	32850	8	32850	32850	5	32850	32850	3
	100-10-1	100	10	39608	39608	8	39608	39608	4	39608	39608	2
	100-10-2	100	10	30641	30641	9	30641	30641	5	30641	30641	3
	100-10-3	100	10	32521	32521	1	32521	32521	3	32521	32521	2
	200-10-1	200	10	53047	53047	123	53047	53047	47	53047	53047	50
	200-10-2	200	10	39663	39663	305	39663	39663	137	39663	39663	150
	200-10-3	200	10	45289	45289	83	45289	45289	70	45289	45289	39

Table EC.14 Comparison with/without heuristic for the SDVRP with limited fleet and rounded costs, part 1/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
S101D1	100	716.00	716.00	3125	716.00	716.00	2919	716.00	716.00	5598
S101D2	100	1337.10	1366.00	-	1276.00	1374.00	(7.1)	1274.00	-	(nan)
S101D3	100	1832.20	1864.00	-	1773.00	1895.00	(6.4)	1770.00	-	(nan)
S101D5	100	2737.10	2770.00	-	2676.00	2823.00	(5.2)	2675.00	-	(nan)
S51D1	50	458.00	458.00	0	458.00	458.00	11	458.00	458.00	1
S51D2	50	703.00	703.00	800	703.00	703.00	266	703.00	703.00	276
S51D3	50	935.17	942.00	-	942.00	942.00	1019	942.00	942.00	1136
S51D4	50	1551.00	1551.00	2102	1551.00	1551.00	1174	1551.00	1551.00	1999
S51D5	50	1325.34	1328.00	-	1318.00	1328.00	(0.8)	1312.00	1328.00	(1.2)
S51D6	50	2153.00	2153.00	4619	2148.00	2153.00	(0.2)	2141.00	2164.00	(1.1)
S76D1	75	592.00	592.00	30	592.00	592.00	73	592.00	592.00	193
S76D2	75	1061.10	1082.00	-	1044.00	1089.00	(4.1)	1038.00	-	(nan)
S76D3	75	1395.90	1420.00	-	1375.00	1434.00	(4.1)	1370.00	1476.00	(7.2)
S76D4	75	2046.10	2073.00	-	2029.00	2097.00	(3.2)	2023.00	2109.00	(4.1)
SD1	8	228.00	228.00	0	228.00	228.00	1	228.00	228.00	0
SD10	64	2688.00	2688.00	2577	2677.00	2688.00	(0.4)	2680.00	2688.00	(0.3)
SD11	80	13280.00	13280.00	5241	13280.00	13280.00	107	13280.00	13280.00	881
SD12	80	7102.36	7315.00	-	7178.00	7216.00	(0.5)	7173.00	7232.00	(0.8)
SD13	96	9933.21	-	-	10023.00	10188.00	(1.6)	10021.00	10121.00	(1.0)
SD14	120	-	-	-	10493.00	10889.00	(3.6)	10493.00	-	(nan)
SD15	144	-	-	-	14738.00	15230.00	(3.2)	14737.00	-	(nan)
SD16	144	-	-	-	3384.00	3496.00	(3.2)	3359.00	-	(nan)
SD17	160	-	-	-	26198.00	26875.00	(2.5)	26198.00	-	(nan)
SD18	160	-	-	-	13833.00	14534.00	(4.8)	13834.00	-	(nan)
SD19	192	-	-	-	19465.00	20481.00	(5.0)	19465.00	-	(nan)
SD2	16	708.00	708.00	0	708.00	708.00	1	708.00	708.00	0
SD20	240	-	-	-	38725.80	41046.00	(5.7)	38733.60	-	(nan)
SD21	288	-	-	-	10887.60	11655.00	(6.6)	10886.10	-	(nan)
SD3	16	432.00	432.00	0	432.00	432.00	1	432.00	432.00	0
SD4	24	630.00	630.00	0	630.00	630.00	1	630.00	630.00	0
SD5	32	1392.00	1392.00	11	1392.00	1392.00	5	1392.00	1392.00	2
SD6	32	832.00	832.00	1	832.00	832.00	3	832.00	832.00	2
SD7	40	3640.00	3640.00	2	3640.00	3640.00	3	3640.00	3640.00	1
SD8	48	5068.00	5068.00	19	5068.00	5068.00	8	5068.00	5068.00	5
SD9	48	2046.00	2046.00	126	2046.00	2046.00	80	2046.00	2046.00	167
eil22	21	375.00	375.00	0	375.00	375.00	2	375.00	375.00	0
eil23	22	569.00	569.00	0	569.00	569.00	1	569.00	569.00	0
eil30	29	510.00	510.00	14	510.00	510.00	7	510.00	510.00	3
eil33	32	835.00	835.00	2	835.00	835.00	5	835.00	835.00	1
eil51	50	521.00	521.00	36	521.00	521.00	26	521.00	521.00	9
eilA101	100	799.80	814.00	-	807.00	818.00	(1.3)	804.00	817.00	(1.6)
eilA76	75	807.60	818.00	-	800.00	829.00	(3.5)	794.00	936.00	(15.2)
eilB101	100	1040.60	1061.00	-	1011.00	1071.00	(5.6)	1013.00	-	(nan)
eilB76	75	981.40	1002.00	-	958.00	1012.00	(5.3)	958.00	1020.00	(6.1)
eilC76	75	717.80	733.00	-	724.00	733.00	(1.2)	721.00	734.00	(1.8)
eilD76	75	666.10	682.00	-	679.00	679.00	1670	674.00	682.00	(1.2)

Table EC.15 Comparison with/without heuristic for the SDVRP with limited fleet and rounded costs, part 2/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
p01_1030	50	753.00	753.00	6098	753.00	753.00	570	753.00	753.00	578
p01_1050	50	993.38	998.00	-	993.00	998.00	(0.5)	990.00	998.00	(0.8)
p01_1090	50	1480.00	1480.00	5368	1480.00	1480.00	6017	1477.00	1480.00	(0.2)
p01_110	50	458.00	458.00	1	458.00	458.00	16	458.00	458.00	1
p01_3070	50	1473.00	1473.00	5355	1460.00	1479.00	(1.3)	1453.00	1478.00	(1.7)
p01_7090	50	2137.98	2142.00	-	2133.00	2143.00	(0.5)	2132.00	2145.00	(0.6)
p02_1030	75	1059.86	1157.00	-	1060.00	1117.00	(5.1)	1054.00	1155.00	(8.7)
p02_1050	75	1449.11	-	-	1451.00	1498.00	(3.1)	1448.00	1538.00	(5.9)
p02_1090	75	2241.79	-	-	2244.00	2305.00	(2.6)	2241.00	2330.00	(3.8)
p02_110	75	612.00	612.00	4284	612.00	612.00	2706	612.00	612.00	4766
p02_3070	75	2166.59	2431.00	-	2154.00	2224.00	(3.1)	2151.00	-	(nan)
p02_7090	75	3155.61	-	-	3157.00	3252.00	(2.9)	3164.00	3395.00	(6.8)
p03_1030	100	1379.00	-	-	1352.00	1450.00	(6.8)	1352.00	-	(nan)
p03_1050	100	1914.80	-	-	1890.00	2013.00	(6.1)	1885.00	-	(nan)
p03_1090	100	2994.36	-	-	2964.00	3098.00	(4.3)	2968.00	-	(nan)
p03_110	100	740.76	760.00	-	747.00	749.00	(0.3)	744.00	751.00	(0.9)
p03_3070	100	2894.75	-	-	2855.00	3006.00	(5.0)	2854.00	-	(nan)
p03_7090	100	4269.98	-	-	4249.00	4385.00	(3.1)	4250.00	-	(nan)
p04_1030	150	-	-	-	1843.00	2041.00	(9.7)	1844.00	-	(nan)
p04_1050	150	-	-	-	2646.00	2856.00	(7.4)	2648.00	-	(nan)
p04_1090	150	-	-	-	4289.00	4573.00	(6.2)	4290.00	-	(nan)
p04_110	150	-	-	-	890.00	926.00	(3.9)	891.00	-	(nan)
p04_3070	150	-	-	-	4098.00	4433.00	(7.6)	4099.00	-	(nan)
p04_7090	150	-	-	-	6125.00	6447.00	(5.0)	6127.00	-	(nan)
p05_1030	199	-	-	-	2226.00	2475.00	(10.1)	2226.00	-	(nan)
p05_1050	199	-	-	-	3179.00	3545.00	(10.3)	3179.00	-	(nan)
p05_1090	199	-	-	-	5182.00	5602.00	(7.5)	5183.00	-	(nan)
p05_110	199	-	-	-	1015.00	1089.00	(6.8)	1016.00	-	(nan)
p05_3070	199	-	-	-	5069.00	5520.00	(8.2)	5070.00	-	(nan)
p05_7090	199	-	-	-	7832.00	8213.00	(4.6)	7833.00	-	(nan)
p10_1030	199	-	-	-	2226.00	2475.00	(10.1)	2226.00	-	(nan)
p10_1050	199	-	-	-	3179.00	3545.00	(10.3)	3179.00	-	(nan)
p10_1090	199	-	-	-	5183.00	5602.00	(7.5)	5183.00	-	(nan)
p10_110	199	-	-	-	1015.00	1089.00	(6.8)	1016.00	-	(nan)
p10_3070	199	-	-	-	5069.00	5520.00	(8.2)	5070.00	-	(nan)
p10_7090	199	-	-	-	7832.00	8213.00	(4.6)	7833.00	-	(nan)
p11_1030	120	-	-	-	2788.00	3009.00	(7.3)	2789.00	-	(nan)
p11_1050	120	-	-	-	4074.00	4290.00	(5.0)	4075.00	-	(nan)
p11_1090	120	-	-	-	6665.00	6930.00	(3.8)	6689.00	-	(nan)
p11_110	120	-	-	-	1014.00	1032.00	(1.7)	1010.00	-	(nan)
p11_3070	120	-	-	-	6415.00	6726.00	(4.6)	6451.00	-	(nan)
p11_7090	120	-	-	-	9924.00	10205.00	(2.8)	9953.00	-	(nan)

Table EC.16 Comparison with/without heuristic for the SDVRP with limited fleet and non-rounded costs, part 1/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
S101D1	100	726.59	726.59	2636	726.59	726.59	2796	726.59	726.59	3209
S101D2	100	1358.90	1378.43	-	1298.71	1387.93	(6.4)	1297.08	-	(nan)
S101D3	100	1853.10	1874.81	-	1786.92	1924.85	(7.2)	1793.26	-	(nan)
S101D5	100	2767.60	2791.22	-	2712.98	2866.23	(5.3)	2712.64	-	(nan)
S51D1	50	459.50	459.50	0	459.50	459.50	27	459.50	459.50	1
S51D2	50	708.42	708.42	368	708.42	708.42	146	708.42	708.42	304
S51D3	50	940.82	947.97	-	947.97	947.97	1398	947.97	947.97	1432
S51D4	50	1553.47	1560.88	-	1560.88	1560.88	695	1560.88	1560.88	689
S51D5	50	1326.61	1333.67	-	1330.24	1333.67	(0.3)	1328.11	1333.67	(0.4)
S51D6	50	2165.64	2169.10	-	2169.10	2169.10	2233	2169.10	2169.10	3837
S76D1	75	598.94	598.94	164	598.94	598.94	57	598.94	598.94	89
S76D2	75	1071.30	1087.40	-	1050.85	1102.03	(4.6)	1045.34	1137.21	(8.1)
S76D3	75	1407.54	1427.86	-	1387.57	1438.24	(3.5)	1383.55	1471.30	(6.0)
S76D4	75	2059.80	2079.76	-	2049.88	2099.94	(2.4)	2043.77	2151.85	(5.0)
SD1	8	228.28	228.28	0	228.28	228.28	1	228.28	228.28	0
SD10	64	2684.88	2684.88	70	2670.17	2684.89	(0.5)	2668.60	2684.88	(0.6)
SD11	80	13280.00	13280.00	5331	13280.00	13280.00	134	13280.00	13280.00	49
SD12	80	7175.80	7213.61	-	7153.52	7230.52	(1.1)	7113.25	7506.42	(5.2)
SD13	96	10053.60	10110.58	-	10015.90	10110.60	(0.9)	10004.30	-	(nan)
SD14	120	10588.20	10725.38	-	10499.60	10837.70	(3.1)	10499.60	-	(nan)
SD15	144	14908.50	15129.68	-	14755.80	15379.30	(4.1)	14755.50	-	(nan)
SD16	144	3381.30	3381.30	6	3381.08	3456.45	(2.2)	3381.10	3524.34	(4.1)
SD17	160	26317.20	26533.39	-	26236.40	26997.60	(2.8)	26236.40	-	(nan)
SD18	160	14029.20	14283.51	-	13813.20	14619.30	(5.5)	13813.10	-	(nan)
SD19	192	19707.20	20191.20	-	19424.90	20653.70	(5.9)	19424.90	-	(nan)
SD2	16	708.28	708.28	0	708.28	708.28	1	708.28	708.28	0
SD20	240	39252.80	39840.00	-	38724.40	40320.80	(4.0)	38728.40	-	(nan)
SD21	288	11271.10	11271.10	205	10974.80	11579.10	(5.2)	10974.80	-	(nan)
SD3	16	430.58	430.58	0	430.58	430.58	2	430.58	430.58	0
SD4	24	631.05	631.05	0	631.05	631.05	3	631.05	631.05	0
SD5	32	1390.57	1390.57	13	1390.57	1390.57	4	1390.57	1390.57	1
SD6	32	831.24	831.24	0	831.24	831.24	4	831.24	831.24	2
SD7	40	3640.00	3640.00	17	3640.00	3640.00	3	3640.00	3640.00	1
SD8	48	5068.28	5068.28	67	5068.28	5068.28	12	5068.28	5068.28	5
SD9	48	2044.20	2044.20	11	2044.20	2044.20	101	2044.20	2044.20	129
eil22	21	375.28	375.28	0	375.28	375.28	2	375.28	375.28	0
eil23	22	568.56	568.56	0	568.56	568.56	1	568.56	568.56	0
eil30	29	512.72	512.72	8	512.72	512.72	9	512.72	512.72	3
eil33	32	837.06	837.06	2	837.06	837.06	8	837.06	837.06	7
eil51	50	524.61	524.61	17	524.61	524.61	22	524.61	524.61	8
eilA101	100	810.06	826.14	-	816.94	831.25	(1.7)	812.38	849.31	(4.3)
eilA76	75	809.67	823.89	-	806.91	827.23	(2.5)	802.76	838.68	(4.3)
eilB101	100	1055.40	1076.26	-	1023.93	1088.32	(5.9)	1023.68	-	(nan)
eilB76	75	985.42	1009.04	-	966.46	1019.12	(5.2)	960.11	1036.20	(7.3)
eilC76	75	724.62	738.67	-	729.07	740.31	(1.5)	728.47	738.67	(1.4)
eilD76	75	677.32	687.60	-	682.67	686.70	(0.6)	680.43	689.57	(1.3)

Table EC.17 Comparison with/without heuristic for the SDVRP with limited fleet and non-rounded costs, part 2/2

		BK			BC			BC - no heur.		
instance	$\lvert C\rvert$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
p01_1030	50	756.71	756.71	915	756.71	756.71	343	756.71	756.71	401
p01_1050	50	996.93	1005.75	-	999.21	1005.75	(0.7)	993.44	1008.11	(1.5)
p01_1090	50	1485.00	1487.18	-	1487.18	1487.18	1326	1487.18	1487.18	1480
p01_110	50	459.50	459.50	0	459.50	459.50	11	459.50	459.50	2
p01_3070	50	1474.10	1481.71	-	1472.36	1481.71	(0.6)	1471.21	1481.71	(0.7)
p01_7090	50	2149.05	2156.14	-	2152.60	2156.14	(0.2)	2148.69	2156.14	(0.3)
p02_1030	75	1093.56	1122.91	-	1064.84	1125.36	(5.4)	1061.91	1123.82	(5.5)
p02_1050	75	1483.17	1509.79	-	1464.75	1508.10	(2.9)	1462.68	1505.44	(2.8)
p02_1090	75	2270.44	2372.22	-	2263.33	2327.94	(2.8)	2262.91	-	(nan)
p02_110	75	616.58	617.85	-	617.85	617.85	4680	617.85	617.85	3426
p02_3070	75	2192.25	2235.61	-	2170.06	2239.89	(3.1)	2167.81	2265.71	(4.3)
p02_7090	75	3192.10	3253.71	-	3179.68	3262.83	(2.5)	3177.07	3376.84	(5.9)
p03_1030	100	1435.23	1491.82	-	1368.80	1480.01	(7.5)	1367.25	-	(nan)
p03_1050	100	1971.43	2018.09	-	1911.33	2038.20	(6.2)	1908.34	-	(nan)
p03_1090	100	3043.27	3136.29	-	3008.80	3119.57	(3.6)	3005.79	-	(nan)
p03_110	100	753.12	762.40	-	756.25	760.00	(0.5)	757.09	760.19	(0.4)
p03_3070	100	2945.76	3044.92	-	2898.70	3030.84	(4.4)	2900.25	-	(nan)
p03_7090	100	4316.42	4452.55	-	4302.24	4417.50	(2.6)	4303.26	-	(nan)
p04_1030	150	1986.79	2109.45	-	1886.15	2052.27	(8.1)	1886.95	-	(nan)
p04_1050	150	2811.64	2956.18	-	2708.02	2909.28	(6.9)	2708.02	-	(nan)
p04_1090	150	4474.18	4708.11	-	4389.91	4622.66	(5.0)	4389.55	-	(nan)
p04_110	150	896.03	1065.94	-	899.53	926.94	(3.0)	897.78	-	(nan)
p04_3070	150	4269.77	4637.69	-	4174.84	4464.12	(6.5)	4174.45	-	(nan)
p04_7090	150	6287.09	6529.63	-	6232.93	6481.48	(3.8)	6229.21	-	(nan)
p05_1030	199	2423.64	2632.22	-	2259.42	2524.65	(10.5)	2259.42	-	(nan)
p05_1050	199	3420.17	3548.13	-	3205.67	3542.18	(9.5)	3209.69	-	(nan)
p05_1090	199	5425.69	5894.69	-	5240.85	5656.48	(7.3)	5240.97	-	(nan)
p05_110	199	1042.37	1188.45	-	1029.40	1112.60	(7.5)	1030.01	-	(nan)
p05_3070	199	5306.11	5669.69	-	5138.50	5557.55	(7.5)	5138.81	-	(nan)
p05_7090	199	8062.24	8400.74	-	7936.82	8323.17	(4.6)	7938.66	-	(nan)
p10_1030	199	-	-	-	2259.42	2524.65	(10.5)	2259.42	-	(nan)
p10_1050	199	-	-	-	3205.67	3542.18	(9.5)	3209.69	-	(nan)
p10_1090	199	-	-	-	5240.85	5656.48	(7.3)	5240.97	-	(nan)
p10_110	199	-	-	-	1029.62	1112.60	(7.5)	1030.01	-	(nan)
p10_3070	199	-	-	-	5138.50	5557.55	(7.5)	5138.50	-	(nan)
p10_7090	199	-	-	-	7937.69	8323.17	(4.6)	7937.51	-	(nan)
p11_1030	120	2879.63	2988.61	-	2845.77	3012.26	(5.5)	2847.04	-	(nan)
p11_1050	120	4162.99	4308.17	-	4167.34	4350.64	(4.2)	4166.99	-	(nan)
p11_1090	120	6808.07	7020.87	-	6732.76	7034.31	(4.3)	6702.79	-	(nan)
p11_110	120	1023.37	1063.73	-	1027.54	1043.57	(1.5)	1022.48	-	(nan)
p11_3070	120	6584.11	6860.65	-	6537.36	6748.64	(3.1)	6528.81	-	(nan)
p11_7090	120	10111.11	10456.19	-	10065.00	10322.50	(2.5)	10082.50	-	(nan)

Table EC.18 Comparison with/without heuristic for the SDVRP with unlimited fleet and rounded costs, part 1/2

		BK			BC			BC - no heur.		
instance	$\lvert C \rvert$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
S101D1	100	714.87	716.00	-	716.00	716.00	4374	716.00	716.00	4161
S101D2	100	1301.93	1366.00	-	1277.00	1374.00	(7.1)	1277.00	1450.00	(11.9)
S101D3	100	1803.51	1864.00	-	1772.00	1890.00	(6.2)	1769.00	-	(nan)
S101D5	100	2709.48	2770.00	-	2668.00	2804.00	(4.9)	2665.00	2867.00	(7.0)
S51D1	50	458.00	458.00	20	458.00	458.00	14	458.00	458.00	2
S51D2	50	703.00	703.00	674	703.00	703.00	198	703.00	703.00	265
S51D3	50	933.07	943.00	-	942.00	942.00	944	942.00	942.00	5328
S51D4	50	1547.44	1551.00	-	1551.00	1551.00	2266	1551.00	1551.00	2272
S51D5	50	1326.73	1328.00	-	1316.00	1328.00	(0.9)	1315.00	1328.00	(1.0)
S51D6	50	2153.00	2153.00	4110	2147.00	2166.00	(0.9)	2153.00	2153.00	4537
S76D1	75	592.00	592.00	134	592.00	592.00	101	592.00	592.00	147
S76D2	75	1040.67	1082.00	-	1039.00	1092.00	(4.9)	1038.00	1092.00	(4.9)
S76D3	75	1379.57	1420.00	-	1375.00	1434.00	(4.1)	1370.00	1442.00	(5.0)
S76D4	75	2034.70	2073.00	-	2030.00	2096.00	(3.1)	2026.00	2205.00	(8.1)
SD1	8	228.00	228.00	0	228.00	228.00	0	228.00	228.00	0
SD10	64	2688.00	2688.00	2317	2670.00	2698.00	(1.0)	2672.00	2697.00	(0.9)
SD11	80	13280.00	13280.00	999	13280.00	13280.00	140	13280.00	13280.00	83
SD12	80	7133.81	7279.00	-	7168.00	7221.00	(0.7)	7160.00	7314.00	(2.1)
SD13	96	9992.94	10112.00	-	10033.00	10232.00	(1.9)	10028.00	10442.00	(4.0)
SD14	120	-	-	-	10485.00	10819.00	(3.1)	10482.00	11296.00	(7.2)
SD15	144	-	-	-	14776.00	15159.00	(2.5)	14775.00	-	(nan)
SD16	144	-	-	-	3348.00	3491.00	(4.1)	3347.00	3599.00	(7.0)
SD17	160	-	-	-	26269.00	26883.00	(2.3)	26269.00	-	(nan)
SD18	160	-	-	-	13826.00	14507.00	(4.7)	13827.00	-	(nan)
SD19	192	-	-	-	19418.00	20408.00	(4.9)	19418.00	-	(nan)
SD2	16	708.00	708.00	0	708.00	708.00	1	708.00	708.00	0
SD20	240	-	-	-	38697.70	40657.00	(4.8)	38670.10	-	(nan)
SD21	288	-	-	-	10956.00	11669.00	(6.1)	10959.00	12110.00	(9.5)
SD3	16	432.00	432.00	0	432.00	432.00	2	432.00	432.00	0
SD4	24	630.00	630.00	1	630.00	630.00	3	630.00	630.00	0
SD5	32	1392.00	1392.00	15	1392.00	1392.00	4	1392.00	1392.00	3
SD6	32	832.00	832.00	3	832.00	832.00	4	832.00	832.00	1
SD7	40	3640.00	3640.00	26	3640.00	3640.00	4	3640.00	3640.00	3
SD8	48	5068.00	5068.00	33	5068.00	5068.00	11	5068.00	5068.00	4
SD9	48	2046.00	2046.00	174	2046.00	2046.00	126	2046.00	2046.00	131
eil22	21	375.00	375.00	1	375.00	375.00	2	375.00	375.00	0
eil23	22	569.00	569.00	2	569.00	569.00	1	569.00	569.00	0
eil30	29	503.00	503.00	1	503.00	503.00	3	503.00	503.00	0
eil33	32	835.00	835.00	12	835.00	835.00	5	835.00	835.00	1
eil51	50	521.00	521.00	61	521.00	521.00	20	521.00	521.00	12
eilA101	100	792.40	814.00	-	806.00	817.00	(1.3)	802.00	839.00	(4.4)
eilA76	75	792.71	818.00	-	799.00	823.00	(2.9)	795.00	846.00	(6.0)
eilB101	100	1017.77	1061.00	-	1012.00	1071.00	(5.5)	1013.00	-	(nan)
eilB76	75	957.60	1002.00	-	956.00	1015.00	(5.8)	955.00	1021.00	(6.5)
eilC76	75	714.24	733.00	-	721.00	734.00	(1.8)	723.00	733.00	(1.4)
eilD76	75	667.93	682.00	-	679.00	679.00	5746	672.00	682.00	(1.5)

Table EC.19 Comparison with/without heuristic for the SDVRP with unlimited fleet and rounded costs, part 2/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
p01_1030	50	752.00	753.00	-	753.00	753.00	415	753.00	753.00	841
p01_1050	50	991.36	998.00	-	998.00	998.00	6710	991.00	998.00	(0.7)
p01_1090	50	1480.00	1480.00	3725	1480.00	1480.00	3655	1472.00	1481.00	(0.6)
p01_110	50	458.00	458.00	10	458.00	458.00	17	458.00	458.00	2
p01_3070	50	1473.00	1473.00	6953	1458.00	1479.00	(1.4)	1454.00	1479.00	(1.7)
p01_7090	50	2134.87	2142.00	-	2133.00	2142.00	(0.4)	2134.00	2142.00	(0.4)
p02_1030	75	1062.36	1172.00	-	1061.00	1108.00	(4.2)	1058.00	1153.00	(8.2)
p02_1050	75	1456.12	1557.00	-	1451.00	1499.00	(3.2)	1452.00	1515.00	(4.2)
p02_1090	75	2258.51	2304.00	-	2246.00	2305.00	(2.6)	2242.00	2297.00	(2.4)
p02_110	75	609.19	612.00	-	612.00	612.00	3254	609.00	613.00	(0.7)
p02_3070	75	2177.37	2253.00	-	2153.00	2234.00	(3.6)	2155.00	2289.00	(5.9)
p02_7090	75	3171.95	3233.00	-	3163.00	3230.00	(2.1)	3161.00	3363.00	(6.0)
p03_1030	100	1379.18	1542.00	-	1357.00	1456.00	(6.8)	1355.00	1502.00	(9.8)
p03_1050	100	1925.21	2053.00	-	1889.00	2014.00	(6.2)	1890.00	-	(nan)
p03_1090	100	3005.48	3152.00	-	2965.00	3098.00	(4.3)	2966.00	3274.00	(9.4)
p03_110	100	737.64	762.00	-	744.00	750.00	(0.8)	743.00	751.00	(1.1)
p03_3070	100	2909.22	3049.00	-	2860.00	3001.00	(4.7)	2855.00	3177.00	(10.1)
p03_7090	100	4251.65	4404.00	-	4245.00	4368.00	(2.8)	4247.00	4563.00	(6.9)
p04_1030	150	-	-	-	1846.00	2034.00	(9.2)	1847.00	-	(nan)
p04_1050	150	-	-	-	2639.00	2860.00	(7.7)	2641.00	-	(nan)
p04_1090	150	-	-	-	4299.00	4573.00	(6.0)	4300.00	-	(nan)
p04_110	150	-	-	-	889.00	947.00	(6.1)	887.00	-	(nan)
p04_3070	150	-	-	-	4062.00	4416.00	(8.0)	4063.00	-	(nan)
p04_7090	150	-	-	-	6113.00	6420.00	(4.8)	6102.00	6563.00	(7.0)
p05_1030	199	-	-	-	2194.51	2480.00	(11.5)	2194.51	-	(nan)
p05_1050	199	-	-	-	3178.00	3510.00	(9.5)	3179.00	-	(nan)
p05_1090	199	-	-	-	5167.00	5579.00	(7.4)	5167.00	-	(nan)
p05_110	199	-	-	-	1015.00	1126.00	(9.9)	1012.81	-	(nan)
p05_3070	199	-	-	-	5041.00	5484.00	(8.1)	5042.00	-	(nan)
p05_7090	199	-	-	-	7807.00	8206.00	(4.9)	7809.00	8478.00	(7.9)
p10_1030	199	-	-	-	2194.51	2480.00	(11.5)	2194.51	-	(nan)
p10_1050	199	-	-	-	3178.00	3510.00	(9.5)	3179.00	-	(nan)
p10_1090	199	-	-	-	5167.00	5579.00	(7.4)	5167.00	-	(nan)
p10_110	199	-	-	-	1015.00	1126.00	(9.9)	1015.00	-	(nan)
p10_3070	199	-	-	-	5041.00	5484.00	(8.1)	5041.00	-	(nan)
p10_7090	199	-	-	-	7807.00	8206.00	(4.9)	7809.00	8478.00	(7.9)
p11_1030	120	-	-	-	2791.00	2976.00	(6.2)	2791.00	-	(nan)
p11_1050	120	-	-	-	4089.00	4296.00	(4.8)	4092.00	-	(nan)
p11_1090	120	-	-	-	6673.00	6982.00	(4.4)	6675.00	7506.00	(11.1)
p11_110	120	-	-	-	1013.00	1032.00	(1.8)	1009.00	-	(nan)
p11_3070	120	-	-	-	6436.00	6663.00	(3.4)	6428.00	-	(nan)
p11_7090	120	-	-	-	9922.00	10193.00	(2.7)	9919.00	10602.00	(6.4)

Table EC.20 Comparison with/without heuristic for the SDVRP with unlimited fleet and non-rounded costs, part 1/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
S101D1	100	716.92	726.59	-	726.59	726.59	3318	726.59	726.59	1839
S101D2	100	1356.78	1378.43	-	1296.13	1394.67	(7.1)	1294.54	-	(nan)
S101D3	100	1845.07	1874.81	-	1784.59	1921.54	(7.1)	1787.46	1952.65	(8.5)
S101D5	100	2758.21	2791.22	-	2687.48	2816.49	(4.6)	2700.65	2864.95	(5.7)
S51D1	50	459.50	459.50	13	459.50	459.50	17	459.50	459.50	3
S51D2	50	708.42	708.42	2376	708.42	708.42	248	708.42	708.42	261
S51D3	50	947.97	947.97	20738	947.97	947.97	1486	947.97	947.97	1719
S51D4	50	1560.88	1560.88	3827	1560.88	1560.88	3860	1560.88	1560.88	1605
S51D5	50	1333.67	1333.67	6310	1329.62	1333.67	(0.3)	1327.61	1333.67	(0.5)
S51D6	50	2169.10	2169.10	16755	2169.10	2169.10	6414	2169.10	2169.10	5457
S76D1	75	598.94	598.94	563	598.94	598.94	88	598.94	598.94	352
S76D2	75	1066.88	1087.40	-	1044.45	1098.92	(5.0)	1041.00	1130.75	(7.9)
S76D3	75	1406.85	1427.86	-	1385.89	1438.24	(3.6)	1382.38	1457.72	(5.2)
S76D4	75	2053.66	2079.76	-	2046.84	2088.79	(2.0)	2047.92	2095.51	(2.3)
SD1	8	228.28	228.28	0	228.28	228.28	0	228.28	228.28	0
SD10	64	2684.86	2684.88	1903	2667.57	2684.88	(0.6)	2661.18	2692.70	(1.2)
SD11	80	13280.00	13280.00	1340	13280.00	13280.00	122	13280.00	13280.00	998
SD12	80	7135.27	7270.87	-	7159.98	7242.17	(1.1)	7159.92	7221.58	(0.9)
SD13	96	9992.74	10110.58	-	10035.20	10110.60	(0.7)	10026.50	10110.60	(0.8)
SD14	120	10502.76	10754.70	-	10487.90	10849.10	(3.3)	10484.10	11062.20	(5.2)
SD15	144	14787.05	15151.10	-	14755.40	15342.60	(3.8)	14755.30	-	(nan)
SD16	144	3379.33	3379.33	9772	3349.78	3467.84	(3.4)	3379.34	3482.86	(3.0)
SD17	160	26166.80	26547.44	-	26226.00	26798.50	(2.1)	26226.00	-	(nan)
SD18	160	13892.74	14334.03	-	13808.00	14438.30	(4.4)	13806.20	-	(nan)
SD19	192	19584.84	20191.20	-	19457.50	20562.10	(5.4)	19457.50	-	(nan)
SD2	16	708.28	708.28	0	708.28	708.28	1	708.28	708.28	0
SD20	240	38901.37	39840.00	-	38734.90	40375.30	(4.1)	38742.30	-	(nan)
SD21	288	11254.83	11271.10	-	10971.40	11592.60	(5.4)	10975.80	-	(nan)
SD3	16	430.58	430.58	0	430.58	430.58	2	430.58	430.58	0
SD4	24	631.05	631.05	1	631.05	631.05	4	631.05	631.05	0
SD5	32	1390.57	1390.57	51	1390.57	1390.57	5	1390.57	1390.57	3
SD6	32	831.24	831.24	4	831.24	831.24	4	831.24	831.24	1
SD7	40	3640.00	3640.00	55	3640.00	3640.00	4	3640.00	3640.00	2
SD8	48	5068.28	5068.28	72	5068.28	5068.28	7	5068.28	5068.28	9
SD9	48	2044.20	2044.20	267	2044.20	2044.20	114	2044.20	2044.20	88
eil22	21	375.28	375.28	6	375.28	375.28	2	375.28	375.28	0
eil23	22	568.56	568.56	1	568.56	568.56	1	568.56	568.56	0
eil30	29	505.01	505.01	2	505.01	505.01	3	505.01	505.01	0
eil33	32	837.06	837.06	46	837.06	837.06	7	837.06	837.06	4
eil51	50	524.61	524.61	95	524.61	524.61	16	524.61	524.61	25
eilA101	100	804.40	826.14	-	815.00	835.79	(2.5)	814.03	831.85	(2.1)
eilA76	75	809.58	823.89	-	805.25	828.75	(2.8)	801.80	829.31	(3.3)
eilB101	100	1055.59	1076.26	-	1023.72	1088.19	(5.9)	1023.38	-	(nan)
eilB76	75	984.13	1009.04	-	962.02	1015.83	(5.3)	961.93	1039.84	(7.5)
eilC76	75	722.76	738.67	-	728.73	738.67	(1.3)	727.68	739.19	(1.6)
eilD76	75	674.17	687.60	-	681.31	690.17	(1.3)	681.44	687.60	(0.9)

Table EC.21 Comparison with/without heuristic for the SDVRP with unlimited fleet and non-rounded costs, part 2/2

		BK			BC			BC - no heur.		
instance	$\|C\|$	LB	UB	time	LB	UB	time (gap)	LB	UB	time (gap)
p01_1030	50	756.70	756.71	20494	756.71	756.71	281	756.71	756.71	771
p01_1050	50	1005.75	1005.75	23257	998.04	1005.75	(0.8)	998.39	1005.75	(0.7)
p01_1090	50	1487.18	1487.18	7155	1487.18	1487.18	1388	1487.18	1487.18	2627
p01_110	50	459.50	459.50	8	459.50	459.50	12	459.50	459.50	3
p01_3070	50	1481.71	1481.71	12418	1473.52	1481.71	(0.6)	1473.15	1481.71	(0.6)
p01_7090	50	2155.80	2155.80	42587	2155.80	2155.80	7098	2155.80	2155.80	5994
p02_1030	75	1095.65	1121.82	-	1067.52	1119.73	(4.7)	1065.00	1136.78	(6.3)
p02_1050	75	1482.50	1514.39	-	1461.17	1509.02	(3.2)	1460.07	1523.69	(4.2)
p02_1090	75	2272.05	2318.28	-	2259.26	2329.53	(3.0)	2255.66	2474.10	(8.8)
p02_110	75	612.45	617.85	-	617.85	617.85	5415	615.90	617.85	(0.3)
p02_3070	75	2190.16	2237.19	-	2172.76	2253.51	(3.6)	2169.77	2237.96	(3.0)
p02_7090	75	3192.55	3232.15	-	3181.27	3237.92	(1.7)	3178.80	3276.74	(3.0)
p03_1030	100	1437.78	1477.35	-	1369.50	1480.01	(7.5)	1367.52	1535.92	(11.0)
p03_1050	100	1971.34	2040.92	-	1898.68	2038.20	(6.8)	1897.86	-	(nan)
p03_1090	100	3042.93	3127.06	-	3001.04	3115.42	(3.7)	3007.12	3174.78	(5.3)
p03_110	100	749.42	769.42	-	759.38	760.00	(0.1)	756.71	760.00	(0.4)
p03_3070	100	2945.42	3030.66	-	2896.12	3026.71	(4.3)	2883.15	3150.10	(8.5)
p03_7090	100	4334.44	4417.57	-	4305.20	4436.99	(3.0)	4299.22	4518.45	(4.9)
p04_1030	150	1986.34	2066.46	-	1876.92	2056.87	(8.7)	1876.92	-	(nan)
p04_1050	150	2811.98	2917.80	-	2699.47	2901.97	(7.0)	2699.47	-	(nan)
p04_1090	150	4474.92	4665.87	-	4388.76	4624.60	(5.1)	4389.84	-	(nan)
p04_110	150	895.46	947.14	-	895.02	966.90	(7.4)	896.26	-	(nan)
p04_3070	150	4267.33	4438.76	-	4153.56	4455.14	(6.8)	4159.68	-	(nan)
p04_7090	150	6284.76	6482.19	-	6216.65	6457.37	(3.7)	6218.88	6614.99	(6.0)
p05_1030	199	2423.99	2583.29	-	2254.17	2528.69	(10.9)	2254.17	-	(nan)
p05_1050	199	3420.23	3568.25	-	3244.54	3542.18	(8.4)	3244.54	-	(nan)
p05_1090	199	5422.95	5673.18	-	5257.30	5627.28	(6.6)	5257.79	-	(nan)
p05_110	199	1042.37	1148.27	-	1028.84	1117.58	(7.9)	1029.71	-	(nan)
p05_3070	199	5304.09	5559.29	-	5147.61	5505.91	(6.5)	5147.90	-	(nan)
p05_7090	199	8062.14	8359.65	-	7937.06	8301.59	(4.4)	7937.34	-	(nan)
p10_1030	199	-	-	-	2254.17	2528.69	(10.9)	2254.17	-	(nan)
p10_1050	199	-	-	-	3244.54	3542.18	(8.4)	3244.54	-	(nan)
p10_1090	199	-	-	-	5257.30	5627.28	(6.6)	5257.30	-	(nan)
p10_110	199	-	-	-	1028.68	1117.58	(8.0)	1029.65	-	(nan)
p10_3070	199	-	-	-	5147.73	5505.94	(6.5)	5147.73	-	(nan)
p10_7090	199	-	-	-	7937.06	8301.59	(4.4)	7937.34	-	(nan)
p11_1030	120	2867.79	2983.82	-	2844.93	3017.20	(5.7)	2837.99	-	(nan)
p11_1050	120	4156.68	4259.94	-	4168.03	4350.64	(4.2)	4167.64	-	(nan)
p11_1090	120	6780.19	6995.85	-	6738.52	7173.25	(6.1)	6742.81	7706.19	(12.5)
p11_110	120	1023.39	1055.28	-	1024.49	1049.30	(2.4)	1022.65	-	(nan)
p11_3070	120	6593.28	6822.31	-	6542.85	6711.40	(2.5)	6550.62	7548.56	(13.2)
p11_7090	120	10113.55	10376.94	-	10111.30	10342.90	(2.2)	10109.40	10664.20	(5.2)