Appendix

A. Integrating the aggregation approaches into the EDES IP

First, under an aggregation scheme, the set $\mathcal{P}_t$ should be replaced by a superset of all possible *aggregated* PDS vectors determined by the aggregation scheme (in Section 4.1). We call this set $\mathcal{Q}_t$ (recall that aggregation $\mathcal{U}$ maps $\mathcal{P}$ onto $\mathcal{Q}$). Note that the objective function and constraints (9b), (9c), and (9f) will not change when using a different aggregation, but we need to modify constraints (9d) and (9e) depending on the scheme used (the latter only requires changes in the left hand side of the constraint to represent the components of the aggregated vector). The specific modifications to be made are different for different aggregation schemes.

A.1. Slacks

In this aggregation scheme, there are as many components in the vectors as possible slack values dictated by the instance parameters. Thus, constraints (9e) become:

$$\bar{R}_{s,t} = \sum_{\mathbf{q} \in \mathcal{Q}_t} q_{s,t} z_{\mathbf{q}}, \qquad \forall s = -\rho, \ldots, T-1-(\rho+1), \tag{10}$$

where $q_{s,t}$ represents the component of the enumerated slack vectors containing the total number of pallets whose slack is equal to s.

Furthermore, we can modify constraint (9d). To do that, we use $\bar{l}$ (the promised delivery decision epoch discussed in Section 4.1) which we can define as follows:

$$\bar{l} = \begin{cases} l + T\lceil \frac{t+1}{T} \rceil, & \text{if } l < [t+1], \\ l + T\lfloor \frac{t+1}{T} \rfloor, & \text{if } l \geq [t+1]. \end{cases} \tag{11}$$

Using this, we can re-write constraints (9d) as follows:

$$\bar{R}_{s,t} = \sum_{i,j,d,l} x_{jit}^{dl} \mathbb{1}_{\left\{ \Delta^{out}\left(t+1,i,d,\rho\right)=0 \;\wedge\; \left[\bar{l} - \left[(t+1) + \delta_{[t+1]}(i,d) \right] \right] = s \right\}}, \qquad \forall s = -\rho, \ldots, T-1-(\rho+1). \tag{12}$$

A.2. TS + C (L)

In this aggregation scheme, the first component represents the total slack, while the second component is the lookahead congestion measure. Thus, constraints (9e) are replaced by two constraints (one for each component):

$$\bar{R}_t^{TS} = \sum_{\mathbf{q} \in \mathcal{Q}_t} q_t^{TS} z_{\mathbf{q}}, \tag{13}$$

$$\bar{R}_t^{C} = \sum_{\mathbf{q} \in \mathcal{Q}_t} q_t^{C} z_{\mathbf{q}}, \tag{14}$$

where q_t^{TS} and q_t^C represent the total slack and congestion components of the enumerated slack vectors, respectively.

Next, we need to compute $\kappa_{id}^{t^*}$ for each (i,d) pair (defined as in Section 4.1), i.e., for each terminal-destination pair, and from the day $[t]$, we look ahead to find the first upcoming day with positive outgoing capacity on the alts of that (i,d) pair, and record its capacity. Note that t^* depends on the (i,d) pair we are considering, but we suppress that in the notation. Then, we can add the following two constraints (we can keep constraints (9d) to capture $\bar{R}^{dl}_{it}$):

$$\bar{R}^{TS}_t = \sum_{s=-\rho}^{T-1-(\rho+1)} \sum_{i,j,d,l} x^{dl}_{jit} \mathbb{1}_{\left\{\Delta^{out}\left(t+1,i,d,\rho\right)=0 \;\wedge\; \left[\bar{l}-\left[(t+1)+\delta_{[t+1]}(i,d)\right]\right]=\;s\right\}}, \tag{15}$$

$$\bar{R}^{C}_t = \sum_{i,d} \max\{\sum_{l} \bar{R}^{dl}_{it} - \kappa^{t^*}_{id}, 0\}. \tag{16}$$

Constraints (16) can then be linearized for incorporation into the IP.

A.3. TS + C (PD) variants

In this aggregation scheme, the first component represents the total slack, while the second component is the pallet-days congestion measure. Thus, similar to TS + C (L), we will have constraints (13) and (14) in place of constraints (9e).

Given a state S_t, capturing the pallet-days measure for the PDS (time period $t+1$ and beyond) can be done by the following approach. Let M be a large enough number of time periods such that we ensure that we can empty the current supply of pallets at any location i for any terminal-destination pair (i,d) in M periods. Then, define auxiliary variables f^k_{id} to represent the *deficit*, i.e., the number of leftover pallets, in time period $t+k$ where $k \geq 1$. Note that this approach computes this leftover for each (i,d) pair independently, and thus, does not capture interactions between pallets with different (i,d) pairs. Then, we can add the following set of constraints (we can keep constraints (9d) to capture $\bar{R}^{dl}_{it}$):

$$\bar{R}^{TS}_t = \sum_{s=-\rho}^{T-1-(\rho+1)} \sum_{i,j,d,l} x^{dl}_{jit} \mathbb{1}_{\left\{\Delta^{out}\left(t+1,i,d,\rho\right)=0 \;\wedge\; \left[\bar{l}-\left[(t+1)+\delta_{[t+1]}(i,d)\right]\right]=\;s\right\}}, \tag{17}$$

$$f^1_{id} = \max\{\sum_{l} \bar{R}^{dl}_{it} - \kappa^{t+1}_{id}, 0\}, \qquad \forall i,d, \tag{18}$$

$$f^k_{id} = \max\{f^{k-1}_{id} - \kappa^{t+k}_{id}, 0\}, \qquad \forall i,d,\; k=2,\dots,M, \tag{19}$$

$$\bar{R}^{C}_t = \sum_{i,d} \sum_{k=1}^{M} f^k_{i,d}. \tag{20}$$

Constraints (18) and (19) can then be linearized for incorporation into the IP.

Finally, for Fixed Scaling, constraint (20) becomes:

$$\bar{R}^{C}_t = \frac{1}{30} \sum_{i,d} \sum_{k=1}^{M} f^k_{i,d}, \tag{21}$$

whereas for Varying Scaling, it becomes:

$$\bar{R}^{C}_t = \frac{1}{m_t} \sum_{i,d} \sum_{k=1}^{M} f^k_{i,d}, \tag{22}$$

where m_t represents the number of (i,d) pairs for which we may find a positive number of pallets in the PDS of decision epoch t.

B. Pseudocode for PDS-IP-Bounding

In this section, we present the pseudocode for the PDS-IP-Bounding algorithm presented in Section 4.3.

Algorithm 1 PDS-IP-Bounding

Input: $\mathcal{P}_t$.
1: Initialize: $\hat{v}_t^{best} = -\infty$, $x_t^{best} = \emptyset$, $p_t^{best} = \emptyset$.
2: Determine c^{UB} by solving Model (9) myopically.
3: $PDSList \leftarrow$ Sort vectors in $\mathcal{P}_t$ in non-increasing order of their lookup table values.
4: **while** $PDSList \neq \emptyset$ **do**
5: $\quad p \leftarrow$ First vector in $PDSList$.
6: $\quad$ Remove p from $PDSList$.
7: $\quad$ Solve PDS IP for vector p.
8: $\quad$ **if** p is compatible **then**
9: $\quad\quad \hat{v}_t^p \leftarrow$ Objective value for EDES IP for vector p.
10: $\quad\quad$ **if** $\hat{v}_t^p = \gamma \bar{V}^{\pi_{t,N}}(p) + c^{UB}$ **then** ▷ Checking for optimality.
11: $\quad\quad\quad \hat{v}_t^{best} \leftarrow \hat{v}_t^p$.
12: $\quad\quad\quad$ Update decision vector, x_t^{best}, and PDS vector, p_t^{best}.
13: $\quad\quad\quad$ Optimal solution found. Go to line 24.
14: $\quad\quad$ **end if**
15: $\quad\quad$ **if** $\hat{v}_t^p > \hat{v}_t^{best}$ **then**
16: $\quad\quad\quad \hat{v}_t^{best} \leftarrow \hat{v}_t^p$.
17: $\quad\quad\quad$ Update decision vector, x_t^{best}, and PDS vector, p_t^{best}.
18: $\quad\quad$ **end if**
19: $\quad\quad$ Remove all PDS vectors with lookup table values less than or equal to $\frac{\hat{v}_t^p - c^{UB}}{\gamma}$ from $PDSList$.
20: $\quad$ **else**
21: $\quad\quad$ Go to line 4.
22: $\quad$ **end if**
23: **end while**
24: **return** Optimal solution: $(\hat{v}_t^{best}, x_t^{best}, p_t^{best})$.

C. Sensitivity analysis for the scaling parameter of the TS+C (PD Fix) aggregation approach

In this section, we perform a sensitivity analysis of the scaling parameter used to group the PDS vectors into partitions in the TS+C (PD Fix). Recall that the default value used in our experiments was 30. We test values of 5, 15, 50, and 100 and analyze the impact of the choice of this parameter on the runtime of both the EDES IP and PDS-IP-Bounding. We perform this experiment only for the four 12544 instances, i.e., 12544-U-5, 12544-U-10, 12544-T-5, and 12544-T-10. The results are present in Table 8. Column names are similar to those above with the addition of the new column "Scaling Parameter" which indicates the value of the parameter (we include the results above for the value of 30 for convenience). The values indicated in bold in the "TS+C (PD Fix) Value" column show the best value obtained using TS+C (PD Fix) for that instance, while we indicate the faster algorithm between solving the EDES-IP directly and PDS-IP-Bounding using bold typeface, as well.

From this table, we make the following two observations:

1. The effect of the scaling parameter on the runtimes: a smaller value of this parameter results in decision subproblems with noticeably longer runtimes compared to larger choices. This is due to the granular aggregation resulting from smaller partitions, which leads to a much larger superset size. The superset size causes EDES IP to be slower due to the time required to load the MIP and solve it, which in turn, gives PDS-IP-Bounding an edge in terms of runtime, and provides more of the computational advantages discussed above. On the other hand, larger values of the scaling parameter favors solving the EDES IP directly.

Table 8 Sensitivity analysis for the scaling parameter of the TS+C (PD Fix) aggregation scheme

Instance	Myopic Value	Scaling Parameter	TS+C (PD Fix) Value	EDES Time (s)	PDS-IP Time (s)	SF	Superset size
12544-U-5	214.47	5	303.00	**3.07**	7.41	0.41	57,294.34
		15	**313.31**	**1.13**	1.51	0.75	20,348.42
		30	311.79	**0.85**	3.23	0.26	10,744.64
		50	298.37	**0.53**	2.34	0.23	7,090.79
		100	299.23	**0.34**	1.90	0.18	4,227.97
12544-U-10	502.76	5	461.30	11.50	**6.91**	1.66	241290.14
		15	676.28	5.77	**3.55**	1.63	81,717.89
		30	648.14	**3.03**	4.39	0.69	42,759.37
		50	**712.90**	**1.38**	2.23	0.62	26,381.96
		100	701.74	**0.91**	2.41	0.38	14,833.79
12544-T-5	213.47	5	323.51	2.46	**1.73**	1.42	55,243.77
		15	302.71	**1.04**	1.98	0.53	19,728.84
		30	340.61	**0.88**	2.13	0.41	10,581.60
		50	**353.52**	**0.54**	1.40	0.39	6,813.56
		100	324.51	**0.44**	1.41	0.31	4,138.68
12544-T-10	886.10	5	506.73	14.40	**11.37**	1.27	30,0406.86
		15	617.18	**4.42**	5.86	0.75	97,086.59
		30	863.36	**2.66**	5.07	0.52	48,760.97
		50	**946.87**	**1.28**	2.39	0.54	30,742.14
		100	758.02	**0.92**	2.63	0.35	16,506.20

2. The effect of the scaling parameter on the policy values: initially, increasing the scaling value improves the value of the policy obtained, but increasing that value too much results in a worse policy. On the one hand, aggregating too little results in a large lookup table with cells that will never be visited (or will not be visited enough) which, in turn, yields poor policies; on the other, aggregating too much can result in the loss of accuracy/definition (see Powell (2011)). Therefore, we observe that a choice of the scaling parameter that is suitable to the instance can be beneficial. As seen for the two instances 12544-U-10 and 12544-T-10, we are able to obtain a policy whose values are much better than those of the myopic policy by being slightly more aggressive in scaling/partitioning the PDS vectors. Due to their much larger PDS space (the pallet-days measure has many more possible values when the number of pallets in the system is increased), this is not surprising.

Therefore, this shows that there is value in solving the EDES IP directly provided the superset size is manageable which can be achieved either via: (1) a smart enumeration of the superset, and/or (2) slightly more aggressive aggregation schemes. It also shows that adjusting the value of the scaling parameter in TS+C (PD Fix) relative to the size of the instance is beneficial.

D. Runtimes for testing iterations

In this section, we summarize and compare the runtimes for PDS-IP-Bounding and for solving the EDES IP directly for testing iterations only. The discussion for this table is presented in Section 5.3.

Table 9 EDES IP vs. PDS-IP-Bounding runtime comparison (testing)

Instance	Slacks			TS+C (L)		
	EDES IP Time (s)	PDS-IP Time (s)	SF	EDES IP Time (s)	PDS-IP Time (s)	SF
04333 - U - 5	1.46	0.22	6.49	0.17	0.11	1.46
06333 - U - 5	10.79	0.41	26.62	0.47	0.22	2.12
08433 - U - 5	14.74	1.44	10.27	0.54	0.17	3.10
10433 - U - 5	-	-	-	1.34	0.33	4.08
04333 - U - 10	-	-	-	0.67	0.26	2.59
04333 - T - 5	1.29	0.18	7.10	0.17	0.14	1.20
06333 - T - 5	11.08	0.48	22.99	0.32	0.12	2.72
08433 - T - 5	12.65	0.54	23.26	0.68	0.36	1.88
10433 - T - 5	-	-		0.93	0.27	3.51
04333 - T - 10	-	-		0.52	0.16	3.25
12544 - U - 5	-	-		2.20	0.28	7.91
06333 - U - 10	-	-		1.38	0.16	8.44
08433 - U - 10	-	-		2.80	0.31	9.16
10433 - U - 10	-	-		6.05	0.49	12.42
12544 - U - 10	-	-		11.36	1.25	9.06
12544 - T - 5	-	-		2.23	0.43	5.24
06333 - T - 10	-	-		1.81	0.17	10.51
08433 - T - 10	-	-		2.47	0.17	14.16
10433 - T - 10	-	-		3.39	0.34	9.91
12544 - T - 10	-	-		11.54	0.79	14.68
Avg. SF (Slacks)	-	-	16.12	-	-	2.08
Avg. SF	-	-	-	-	-	6.37

Instance	TS+C (PD Var)			TS+C (PD Fix)		
	EDES IP Time (s)	PDS-IP Time (s)	SF	EDES IP Time (s)	PDS-IP Time (s)	SF
04333 - U - 5	0.13	0.51	0.25	0.10	0.11	0.93
06333 - U - 5	0.21	0.37	0.59	0.20	0.16	1.19
08433 - U - 5	0.29	0.44	0.66	0.28	0.58	0.48
10433 - U - 5	0.27	0.30	0.92	0.30	0.77	0.39
04333 - U - 10	0.34	0.35	0.97	0.45	0.27	1.63
04333 - T - 5	0.12	0.24	0.49	0.09	0.15	0.59
06333 - T - 5	0.24	0.28	0.84	0.18	0.22	0.82
08433 - T - 5	0.25	0.45	0.56	0.40	0.66	0.60
10433 - T - 5	0.25	0.45	0.56	0.40	0.66	0.60
04333 - T - 10	0.35	1.25	0.28	0.19	0.47	0.39
12544 - U - 5	0.52	4.61	0.11	0.83	0.77	1.08
06333 - U - 10	0.70	0.36	1.96	0.51	0.32	1.60
08433 - U - 10	0.83	0.96	0.87	0.52	0.52	1.00
10433 - U - 10	0.71	0.73	0.97	0.70	0.57	1.23
12544 - U - 10	1.65	5.78	0.29	2.11	3.39	0.62
12544 - T - 5	0.79	1.01	0.78	0.98	1.57	0.62
06333 - T - 10	0.85	0.83	1.02	0.46	0.46	0.99
08433 - T - 10	0.61	0.39	1.55	0.65	0.37	1.75
10433 - T - 10	1.28	0.97	1.33	1.25	0.57	2.19
12544 - T - 10	1.69	3.39	0.50	3.14	3.46	0.91
Avg. SF (Slacks)	-	-	0.57	-	-	0.77
Avg. SF	-	-	0.78	-	-	0.98