Online Appendix for P2P Crowdshipping as an Omnichannel Retail Strategy

Appendix A: Profit Functions

B2C Delivery Model

Based on the discussions in Section 3, we first define the profit contribution for the low-type and high-type consumer segments at location z. The low-type consumers shop in store; the high-type consumers shop in store if they reside in $z \in [0, f/(2t_H))$, and online if they reside in $z \in [f/(2t_H), S]$.

$$\pi_{Lb}(p,z) = (p+a)\delta_L q_L^s(p,z) = \delta_L(p+a)(1-\theta-p-2t_L z)$$

$$\pi_{Hb}(p,z) = \begin{cases} (p+a)\delta_H q_L^s(p,z) & \text{if } z < f/(2t_H) \\ \left(p+f-\frac{2t_b z}{m_b}-\gamma_b(p)\right)\delta_L q_H^s(p,z) & \text{otherwise} \end{cases}$$

$$= \begin{cases} \delta_H(p+a)(1-p-2t_H z) & \text{if } z < f/(2t_H) \\ \delta_H\left(p+f-\frac{2t_b}{m_b}z\right)(1-p-f) - t_b k(m_b)\sqrt{\delta_H(1-p-f)} & \text{otherwise.} \end{cases}$$

The profit of the store, as a function of price p, can be obtained by integrating over the catchment area of the store. By symmetry, it is sufficient to consider the upper-left quadrant of the catchment area (see Figure 1(b)). Note that at distance $(z, z+dz)$ from the store, i.e., the shaded area in all four quadrants, there are $4\delta_L z dz$ low-type and $4\delta_H z dz$ high-type consumers, respectively. For notational brevity, we omit the constant factor 4 and express the total profit as follows:

$$\begin{aligned}\Pi_b(p) &= \int_0^{f/(2t_H)} \pi_{Lb}(p,z) z dz + \int_0^{f/(2t_H)} \pi_{Hb}(p,z) z dz + \int_{f/(2t_H)}^{S} \pi_{Lb}(p,z) z dz + \int_{f/(2t_H)}^{S} \pi_{Hb}(p,z) z dz \\ &= \delta_L(p+a)\left[(1-\theta-p)\frac{S^2}{2} - \frac{2t_L S^3}{3}\right] + \delta_H(p+a)\left[\frac{(1-p)}{2}\left(\frac{f}{2t_H}\right)^2 - \frac{2t_H}{3}\left(\frac{f}{2t_H}\right)^3\right] \\ &\quad + \frac{\delta_H}{2}\left[(p+f)(1-p-f) - t_b k(m_b)\sqrt{\delta_H(1-p-f)}\right]\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] \\ &\quad - \frac{2\delta_H t_b}{3m_b}(1-p-f)\left(S^3 - \left(\frac{f}{2t_H}\right)^3\right).\end{aligned}$$

The store selects price p to maximize the profit function above as follows.

LEMMA 4. *The optimal price for the B2C delivery model, p_b^*, is the solution to:*

$$\begin{aligned}&\frac{\delta_L}{2}(1-\theta-a)S^2 - \frac{2\delta_L}{3}t_L S^3 + \frac{\delta_H}{2}\left(1-a-\frac{2f}{3}\right)\left(\frac{f}{2t_H}\right)^2 + \frac{\delta_H}{2}(1-2f)S^2 + \frac{2\delta_H t_b}{3m_b}\left(S^3 - \left(\frac{f}{2t_H}\right)^3\right) \\ &= (\delta_L+\delta_H)pS^2 - \frac{t_b k(m)\sqrt{\delta_H}}{4\sqrt{1-p-f}}\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] \qquad (13)\end{aligned}$$

which is unique when $(\delta_L + \delta_H)(1-p-f) \geq \frac{t_b k(m)\sqrt{\delta_H}}{8\sqrt{1-p-f}}$.

Note that, since the high- and low-type segments respond to changes in p differently, the optimal price, denoted by p_b^*, is a compromise value that does not necessarily maximize profit for either segment. The interactions between the different segments and their responses to pricing drive many of our results to be discussed in subsequent sections.

Besides the firm's profit, it is of interest to formulate the consumer surplus for each segment (at location z) as functions of price p as follows:

$$\kappa_{Lb}(p,z) = \frac{\delta_L}{2}(1-\theta-p-2t_L z)^2$$

$$\kappa_{Hb}(p,z) = \begin{cases} \frac{\delta_H}{2}(1-p-2t_H z)^2 & \text{if } z < f/(2t_H) \\ \frac{\delta_H}{2}(1-p-f)^2 & \text{otherwise.} \end{cases}$$

P2P Crowdshipping Model

Cost-Based Remuneration Similarly to the B2C delivery model, we can then formulate the profit contribution of each of the consumer segments at location z as follows for the P2P crowdshipping model with cost-based remuneration:

$$\pi_{Lc}(p,z) = (p+a)\delta_L q_L^s(p,z) = \delta_L(p+a)(1-\theta-p-2t_L z)$$

$$\pi_{Hc}(p,z) = \begin{cases} (p+a)\delta_H q_L^s(p,z) & \text{if } z < f/(2t_H) \\ (p+f-\gamma(p,z))\delta_L q_H^s(p,z) & \text{otherwise} \end{cases}$$

$$= \begin{cases} \delta_H(p+a)(1-p-2t_H z) & \text{if } z < f/(2t_H) \\ \delta_H(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)} & \text{otherwise.} \end{cases}$$

Note that we attribute the remuneration amount to the online consumers in $\pi_{Hc}(p,z)$ as effective delivery costs. The retailer's overall profit is given by:

$$\begin{aligned} \Pi_c(p) &= \int_0^{f/(2t_H)} \pi_{Lc}(p,z) z dz + \int_0^{f/(2t_H)} \pi_{Hc}(p,z) z dz + \int_{f/(2t_H)}^{S} \pi_{Lc}(p,z) z dz + \int_{f/(2t_H)}^{S} \pi_{Hc}(p,z) z dz \\ &= \Pi_b(p) + \Delta_c(p). \end{aligned}$$

where:

$$\Delta_c(p) = \frac{2\delta_H t_b}{3m_b}(1-p-f)\left(S^3 - \left(\frac{f}{2t_H}\right)^3\right) + \frac{\delta_H}{2}(t_b k(m_b) - t_L k(m))\sqrt{\delta_H(1-p-f)}\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] \quad (14)$$

Besides the firm's profit, we also assess the consumer surplus under the P2P crowdshipping model. Note that, since consumers gain zero participation surplus under the cost-based remuneration scheme, the consumer surplus for each segment is identical to that under B2C delivery, given the same price p. That is, $\kappa_{ic}(p,z) = \kappa_{id}(p,z)$ for $i = L, H$.

Cross Subsidy Similarly to the previous cases, we first formulate the profit contribution for each location z given price, and then integrate this over the catchment area to obtain the overall profit function. For notational brevity, we first perform a variable substitution and define $u(p,z) = mr(p,z) - \gamma(p,z)$, which can be interpreted as the participation surplus (remuneration less delivery costs incurred) enjoyed by the in-store consumer performing a delivery gig. This allows the participation constraint to be rewritten as $u(p,z) \geq 0$, where $u(p,z) = 0$ holds under the aforementioned cost-based remuneration scheme. We shall now explore whether and how the retailer can benefit from allowing $u(p,z) > 0$.

LEMMA 5. *For given p and z, when the participation constraint holds, $\alpha(p,z)$ and $q_L^s(p,z)$ are strictly decreasing and strictly increasing in $u(p,z)$, respectively.*

Lemma 5 suggests that, as the participation surplus $u(p,z)$ increases, more low-type consumers are incentivized to come to the store, and thus the probability of being offered a delivery gig gets diluted.

For a location $z < f/(2t_H)$, there is a density of $\delta_L q_L^s(p,z) + \delta_H q_H^s(p,z)$ consumers shopping in store. As each of them contributes a profit margin of $p+a$, the profit contribution of location z is given as follows, with a slight abuse of notation:

$$\pi(p,r,z) = (p+r)(\delta_L q_L^s(p,z) + \delta_H q_H^s(p,z)).$$

For $z \in [f/(2t_H), S]$, there are $\delta_L q_L^s(p,z)$ consumers shopping in store and $\delta_H q_H^o(p,z)$ shopping online. Each online shopper contributes a margin of $p+f$. Each in-store customer contributes a margin of $p+a$, less the expected remuneration payment of

$$\alpha(p,z)r(p,z)m = \alpha(p,z)u(p,z) + \alpha(p,z)\frac{t_L k(m)m}{\sqrt{\delta_H(1-p-f)}}.$$

Therefore, the profit contribution of location z is given by:

$$\left(p + a - \alpha(p,z)u(p,z) - \alpha(p,z)\frac{t_L k(m)m}{\sqrt{\delta_H(1-p-f)}}\right)\delta_L q_L^s(p,z) + \delta_H(p+f)q_H^s(p,z)$$
$$= (p+a-\alpha(p,z)u(p,z))\delta_L q_L^s(p,z) + \delta_H\left(p+f-\frac{t_L k(m)}{\sqrt{\delta_H(1-p-f)}}\right)q_H^s(p,z).$$

The above equality holds because $\alpha(p,z)m\delta_L q_L^s(p,z) = \delta_H q_H^o(p,z)$. Note that, as a consequence of our local homogeneity approximation, the variable $u(p,z)$ only affects the profit contribution at the specific location z, but not elsewhere. That is, given p, $u(p,z)$ can be optimized separately for each z. The optimal values are then given by Proposition 1.

For tractability, and to draw a contrast with the cost-based remuneration case, we shall subsequently focus on the case where $u(p,z) = \left(p - \frac{1-\theta-a-2t_L z}{2}\right)/\alpha(p,z)$ for all $z \in [f/(2t_H), S]$. From Proposition 1, this is feasible and optimal as long as it satisfies the participation constraint for all $z \in [f/(2t_H), S]$, which happens when $p \geq \frac{1-\theta-a-t_L f/t_H}{2}$. In that case, the profit contributions of the low- and high-type segments at location z can be written as follows:

$$\pi_{Ls}(p,z) = \begin{cases} (p+a)\delta_L q_L^s(p,z) & \text{if } z < f/(2t_H) \\ (p+a-\alpha(p,z)u(p,z))q_L^s(p,z) & \text{otherwise} \end{cases}$$
$$= \begin{cases} \delta_L(p+a)(1-\theta-p-2t_L z) & \text{if } z < f/(2t_H) \\ \frac{\delta_L}{4}(1-\theta+a-2t_L z)^2 & \text{otherwise} \end{cases}$$
$$\pi_{Hs}(p,z) = \begin{cases} (p+a)\delta_H q_L^s(p,z) & \text{if } z < f/(2t_H) \\ (p+f-\gamma(p,z))\delta_L q_H^s(p,z) & \text{otherwise} \end{cases}$$
$$= \begin{cases} \delta_H(p+a)(1-p-2t_H z) & \text{if } z < f/(2t_H) \\ \delta_H(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)} & \text{otherwise.} \end{cases}$$

Then, the retailer's overall profit is given by:

$$\Pi_s(p) = \int_0^{f/(2t_H)} \pi_{Ls}(p,z) z\,dz + \int_0^{f/(2t_H)} \pi_{Hs}(p,z) z\,dz + \int_{f/(2t_H)}^{S} \pi_{Ls}(p,z) z\,dz + \int_{f/(2t_H)}^{S} \pi_{Hs}(p,z) z\,dz$$
$$= \Pi_c(p) + \Delta_s(p).$$

$$\text{where } \Delta_s(p) = \frac{\delta_L}{8}(1-\theta-a-2p)^2\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] - \frac{\delta_L t_L}{3}(1-\theta-a-2p)\left[S^3 - \left(\frac{f}{2t_H}\right)^3\right]$$
$$+\frac{\delta_L}{4}t_L^2\left[S^4 - \left(\frac{f}{2t_H}\right)^4\right].$$

We further formulate the consumer surplus under cross subsidy. Similar to the cost-based remuneration case, we note that the consumer surplus for the high-type segment at any location is identical to that under B2C delivery, given the same price p, i.e., $\kappa_{Hs}(p,z) = \kappa_{Hb}(p,z)$. However, for the low-type in $z \geq f/(2t_H)$, the consumer surplus is different:

$$\kappa_{Ls}(p,z) = \begin{cases} \frac{\delta_L}{2}(1-\theta-p-2t_L z)^2 & \text{if } z < f/(2t_H) \\ \frac{\delta_L}{2}\left(1-\theta-2t_L z + \frac{1-\theta-a-2t_L z}{2}\right)^2 & \text{otherwise.} \end{cases}$$
$$= = \begin{cases} \frac{\delta_L}{2}(1-\theta-p-2t_L z)^2 & \text{if } z < f/(2t_H) \\ \frac{\delta_L}{4}(1-\theta+a-2t_L z)^2 & \text{otherwise.} \end{cases}$$

Appendix B: Proofs of Analytical Results

Proof of Lemma 1

The derivative of the right hand side of (4) with respect to $\hat{\alpha}(p,z)$:

$$-\frac{(1-p-f)\delta_H/\delta_L}{\left(1-\theta-p-2t_L z+\hat{\alpha}(p,z)(r(p,z)m-\frac{t_L km}{\sqrt{\delta_H(1-p-f)}}\right)^2}\left(r(p,z)m-\frac{t_L k(m)m}{\sqrt{\delta_H(1-p-f)}}\right)\leq 0$$

due to the participation constraint. Thus, the right hand side of (4) decreases in $\hat{\alpha}(p,z)$, while the left hand side increases linearly in $\alpha(p,z)$. This implies that, with $\alpha(p,z)=\hat{\alpha}(p,z)$, the two sides equate at a unique point.

Proof of Lemma 2

The difference between the two profit functions is given by:

$$\Delta_c(p)=\frac{2\delta_H t_b}{3m_b}(1-p-f)\left[S^3-\left(\frac{f}{2t_H}\right)^3\right]+\frac{\delta_H}{2}(t_b k(m_b)-t_L k(m))\sqrt{\delta_H(1-p-f)}\left[S^2-\left(\frac{f}{2t_H}\right)^2\right].$$

This is non-negative if:

$$t_L k(m)-t_b k(m_b)\leq\frac{4t_b}{3m_b\sqrt{\delta_H}}\sqrt{1-p-f}\left[S^3-\left(\frac{f}{2t_H}\right)^3\right]/\left[S^2-\left(\frac{f}{2t_H}\right)^2\right],$$

which can be rewritten as (8).

Consider the derivative of $\Delta_c(p)$

$$\Delta_c'(p)=-\frac{2\delta_H t_b}{3m_b}\left(S^3-\left(\frac{f}{2t_H}\right)^3\right)+(t_L k(m)-t_b k(m_b))\frac{\delta_H^{3/2}}{4\sqrt{1-p-f}}\left[S^2-\left(\frac{f}{2t_H}\right)^2\right],$$

which is non-positive if:

$$t_L k(m)-t_b k(m_b)\leq\frac{8t_b}{3m_b\sqrt{\delta_H}}\sqrt{1-p-f}\left(S^3-\left(\frac{f}{2t_H}\right)^3\right)/\left[S^2-\left(\frac{f}{2t_H}\right)^2\right].$$

Note that this is implied by (8) (which is equivalent to halving the right hand side of the last inequality).

Proof of Proposition 2

If (8) holds, Lemma 2 implies that $\Pi_c(p)\geq\Pi_b(p)$ for any p. Let p_c^* and p_b^* denote the optimal prices under the P2P crowdshipping (with cost-based remuneration) and B2C delivery models. The above implies that $\Pi_c(p_c^*)\geq\Pi_c(p_b^*)\geq\Pi_b(p_b^*)$, i.e., the retailer's optimal profit is higher under the crowdshipping model.

To show that $p_c^* \leq p_b^*$, note that p_c^* and p_b^* are the solutions to the first order conditions $\Pi_c'(p) = 0$ and $\Pi_b'(p) = 0$, respectively. Since $\Pi_c'(p) - \Pi_b'(p) = \Delta_c'(p)$, Lemma 2 also suggests that $\Pi_b'(p_c^*) \geq \Pi_c'(p_c^*) = 0$ if (8) holds. Since $\Pi_b'(p)$ is decreasing (by the proof of Lemma 4), this implies that $p_b^* \geq p_c^*$.

Finally, recall that $\kappa_{ic}(p) = \kappa_{ib}(p)$ for $i = L, H$, and that these functions increase in p. Then, $p_c^* \leq p_b^*$ implies that $\kappa_{ic}(p_c^*) \geq \kappa_{ib}(p_b^*)$, i.e., the consumer surplus is higher under the P2P crowdshipping model, under each segment and at every location. Aggregating over the segments and the catchment area, the overall consumer surplus is thus higher under the P2P crowdshipping model.

Proof of Proposition 1

Taking the partial derivative of the profit contribution at z with respect to u (supressing (p,z) for brevity),

$$\begin{aligned}
\frac{\partial \pi}{\partial u} &= \delta_L(p + a - \alpha u)\frac{\partial}{\partial u}q_L^s - \delta_L q_L^s \alpha - \delta_L q_L^s u \frac{\partial \hat{\alpha}}{\partial u} \\
&= \delta_L(p + a - \alpha u)\frac{\partial}{\partial u}q_L^s - \delta_L q_L^s \frac{\partial}{\partial u}q_L^s \quad \text{(by (18))} \\
&= \delta_L(p + a - \alpha u - q_L^s)\frac{\partial}{\partial u}q_L^s \\
&= \delta_L(p + a - \alpha u - (1 - p - 2t_L s + \alpha u)\frac{\partial}{\partial u}q_L^s \\
&= \delta_L(2p - 1 + \tau + 2t_L z - 2\alpha u)\frac{\partial}{\partial u}q_L^s.
\end{aligned}$$

From Lemma 5, $\frac{\partial}{\partial u}q_L^s > 0$. Therefore, the first order condition yields $u = \left(p - \frac{1-\theta-a-2t_L z}{2}\right)/\alpha$. When u is strictly smaller (larger) than this quantity, $\frac{\partial \pi}{\partial u} > 0$ (< 0). This suggests that the optimal u is given by the larger between this quantity and zero.

Proof of Lemma 3

Considering $\Delta_s(p)$ as a function in (p, θ), we may take the partial derivatives as follows:

$$\begin{aligned}
\frac{\partial}{\partial p}\Delta_s &= -\int_{f/(2t_H)}^{S} \frac{d}{dp}\pi_{Lc}(p,z)zdz \quad \text{(by Leibniz's rule)} \\
&= -\int_{f/(2t_H)}^{S} \delta_L[1 - \theta - a - 2t_L z - 2p]zdz \\
\frac{\partial}{\partial \theta}\Delta_s &= \delta_L \frac{\partial}{\partial \theta}\int_{f/(2t_H)}^{S} \left[\frac{1}{2}(1 - \theta + a - 2t_L z)^2 - (p + a)(1 - \theta - p - 2t_L z)\right] dz \\
&= \delta_L \int_{f/(2t_H)}^{S} \left[-\frac{1}{2}(1 - \theta - a - 2t_L z) + p\right] zdz.
\end{aligned}$$

which are both positive, if $p \geq \frac{1-\theta-a-t_L f/t_H}{2}$. To show supermodularity, take the cross derivative:

$$\frac{\partial^2}{\partial p \partial \theta}\Delta_s = \delta_L \int_{f/(2t_H)}^{S} z dz = \delta_L \left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] \geq 0.$$

Proof of Proposition 3

The difference between the profit functions, at price p, is given as follows when $t_L = t_b$:

$$\begin{aligned}\Pi_s(p) - \Pi_b(p) &= \Delta_s(p) + \Delta_c(p) \\ &= \int_{f/(2t_H)}^{S} \left[\frac{\delta_L}{4}(1-\theta+a-2t_L z)^2 - \pi_{Lc}(p,z)\right] z dz + \int_{f/(2t_H)}^{S} \frac{2\delta_H t_L z}{m_b}(1-p-f) z dz.\end{aligned}$$

The first term is positive, since $\frac{\delta_L}{4}(1-\theta+a-2t_L z)^2 = \max_p \pi_{Lc}(p,z)$ (recall Observation 2). The second term is also positive, as discussed in the proof of Proposition 2. Thus, $\Pi_s(p) \geq \Pi_b(p)$ for any p, which further implies that $\Pi_s(p_s^*) \geq \Pi_b(p_b^*)$, i.e., the profit under P2P crowdshipping exceeds that under B2C delivery, when the prices for both cases are respectively optimized.

Taking the difference in the derivatives of the profit functions:

$$\begin{aligned}\Pi_s'(p) - \Pi_b'(p) &= -\int_{f/(2t_H)}^{S} \frac{d}{dp}\pi_{Lc}(p,z) z dz + \int_{f/(2t_H)}^{S} \frac{d}{dp}\frac{2\delta_H t_L z}{m_b}(1-p-f) z dz \quad \text{(by Leibniz's rule)} \\ &= -\int_{f/(2t_H)}^{S} \delta_L[1-\theta-a-2p-2t_L z] z dz - \int_{f/(2t_H)}^{S} \frac{2\delta_H t_L z^2}{m_b} dz \\ &= -\int_{f/(2t_H)}^{S} \delta_L \left[1-\theta-a-2p-2t_L z\left(1-\frac{\delta_H}{\delta_L m_b}\right)\right] z dz.\end{aligned}$$

If $p \geq \frac{1}{2}\left[1-\theta-a-\frac{t_L f}{t_H}\left(1-\frac{\delta_H}{\delta_L m_b}\right)\right]$, then the above expression is positive, which implies $\Pi_s'(p) - \Pi_b'(p) \geq 0$. Since $\Pi_b'(p)$ is decreasing, this implies that $p_s^* \geq p_b^*$.

Finally, recall that the consumer surplus expressions are identical under the P2P crowdshipping (with cross subsidy) and B2C delivery models for both low-type and high-type consumers in $z \in [0, f/(2t_H))$ and for high-type consumers in $z \in [f/(2t_H), S]$, and that these expressions are decreasing in p. If $p \geq \frac{1}{2}\left[1-\theta-a-\frac{t_L f}{t_H}\left(1-\frac{\delta_H}{\delta_L m_b}\right)\right]$, the optimal price is lower under the P2P crowdshipping model, which implies that the consumer surplus for each of these segments is higher. Thus, all consumer segments, except for low-type consumers in $z \in [0, f/(2t_H))$, have their consumer surplus reduced under the cross subsidy model.

Proof of Proposition 4

The statement trivially holds for $z \in [0, \frac{f}{2t_H})$. For $z \in [\frac{f}{2t_H}, S]$, we note that

$$\frac{dv_c(p)}{dp} = \frac{-2z\delta_L - \frac{k(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}}{\delta_L(1-\theta-p-2t_L z) + \delta_H(1-p-f)} + (\delta_H + \delta_L)\frac{2z\delta_L(1-\theta-p-2t_L z) + k(m)\sqrt{\delta_H(1-p-f)}}{[\delta_L(1-\theta-p-2t_L z) + \delta_H(1-p-f)]^2}$$

$$
\begin{aligned}
&= \frac{1}{[\delta_L q_L + \delta_H q_H]^2}\left[\left(2z\delta_L\delta_H - \frac{k(m)\delta_L\sqrt{\delta_H}}{2\sqrt{1-p-f}}\right)(q_L^s - q_H^o)\right.\\
&\quad \left. + \frac{k(m)(\delta_H+\delta_L)}{2}\sqrt{\delta_H q_H^o}\right]\\
&= \frac{1}{[\delta_L q_L^s + \delta_H q_H^o]^2}\left[2z\delta_L\delta_H(q_L^s - q_H^o) + \frac{k(m)(\delta_H + 2\delta_L - \delta_L q_L^s/q_H^o)}{2}\sqrt{\delta_H q_H^o}\right]\\
&= \frac{1}{[\delta_L q_L^s + \delta_H q_H^o]^2}\left[2z\delta_L\delta_H(q_L^s - q_H^o) + \frac{k(m)\delta_H\left(1+\frac{2\delta_L}{\delta_H} - \frac{1}{\alpha m}\right)}{2}\sqrt{\delta_H q_H^o}\right].
\end{aligned}
$$

Recall that $q_L^s - q_H^o \geq 0$. Therefore, the last expression is positive if $1 + \frac{2\delta_L}{\delta_H} \geq \frac{1}{\alpha m}$, which is implied by $\alpha m \geq 1$. If this holds, $v_c(p)$ increases in p. From Proposition 2, the optimal price under the P2P crowdshipping model (p_c^*) is lower than that under the B2C delivery model (p_b). Then, it holds that $v_b(p_b^*) \geq v_c(p_b^*) \geq v_c(p_c^*)$.

Proof of Proposition 5

Similar to the proof of Proposition 4, the statement holds trivially for $z \in [0, f/(2t_H))$. For $z \in [f/(2t_H), S]$, we take the derivative of $v_s(p)$:

$$
\begin{aligned}
\frac{dv_s(p)}{dp} &= \frac{-\frac{k(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}}{\frac{\delta_L}{2}(1-\theta+a-2t_L z) + \delta_H(1-p-f)} + \delta_H\frac{z\delta_L(1-\theta+a-2t_L z) + k(m)\sqrt{\delta_H(1-p-f)}}{[\frac{\delta_L}{2}(1-\theta+a-2t_L z) + \delta_H(1-p-f)]^2}\\
&= \frac{1}{(\delta_L q_L^s + \delta_H q_H^o)^2}\left[-\frac{k(m)\sqrt{\delta_H}}{2\sqrt{q_H^s}}(\delta_L q_L^s + \delta_H q_H^o) + 2z\delta_L q_L^s + \frac{k(m)}{2}\sqrt{\delta_H q_H^o}\right]\\
&= \frac{1}{(\delta_L q_L^s + \delta_H q_H^o)^2}\left[\frac{k(m)\sqrt{\delta_H}q_L^s}{2\sqrt{q_H^s}}(\alpha m - 1) + 2z\delta_L q_L^s\right].
\end{aligned}
$$

When $\alpha m \geq 1$, the last expression is positive, i.e., $v_s(p)$ increases in p. From the proof of Proposition 3, the optimal price under the P2P crowdshipping model with cross subsidy (p_s^*) is lower than that under the B2C deliver model (p_b) if $1-\theta+a-2p > 2t_L S$. In this case, it holds that $v_b(p_b^*) \geq v_s(p_b^*) \geq v_s(p_s^*)$.

Proof of Proposition 6

By definition, we can write:

$$
\begin{aligned}
R_c(p,S) &= \frac{\delta_L}{2S^2}(p+a)\left[(1-\theta-p)\frac{S^2}{2} - \frac{2t_L S^3}{3}\right] + \frac{\delta_H}{2S^2}(p+a)\left[\frac{(1-p)}{2}\left(\frac{f}{2t_H}\right)^2 - \frac{2t_H}{3}\left(\frac{f}{2t_H}\right)^3\right]\\
&\quad + \frac{\delta_H}{4S^2}\left[(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)}\right]\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] - \frac{F}{2S^2}\\
&= \frac{\delta_L}{4}(p+a)(1-\theta-p) - \frac{\delta_L}{3}(p+a)t_L S + \frac{\delta_H}{4}(p+a)(1-p)\left(\frac{f}{2t_H}\right)^2\frac{1}{S^2}
\end{aligned}
$$

$$-\frac{\delta_H}{3}(p+a)t_H\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}+\frac{\delta_H}{4}\left[(p+f)(1-p-f)-t_Lk(m)\sqrt{\delta_H(1-p-f)}\right]$$
$$-\frac{\delta_H}{4}\left[(p+f)(1-p-f)-t_Lk(m)\sqrt{\delta_H(1-p-f)}\right]\left(\frac{f}{2t_H}\right)^2\frac{1}{S^2}-\frac{F}{2S^2}$$

$$\begin{aligned}\frac{\partial}{\partial S}R_c(p,S) &= -\frac{\delta_L}{3}(p+a)t_L-\frac{\delta_H}{4S^3}\left[(p+a)(1-p)-\frac{4}{3}(p+a)t_H\left(\frac{f}{2t_H}\right)\right]\left(\frac{f}{2t_H}\right)^2 \\ &\quad +\frac{\delta_H}{4S^3}\left[(p+f)(1-p-f)-t_Lk(m)\sqrt{\delta_H(1-p-f)}\right]\left(\frac{f}{2t_H}\right)^2+\frac{F}{S^3} \\ &= -\frac{\delta_H}{4S^3}\left(\frac{f}{2t_H}\right)^2\left[\left(\frac{4f}{3}-a\right)p+a-\frac{2af}{3}-f(1-f)+t_Lk(m)\sqrt{\delta_H(1-p-f)}\right] \\ &\quad -\frac{\delta_L}{3}(p+a)t_L+\frac{F}{S^3}.\end{aligned}$$

$$\frac{\partial^2}{\partial S^2}\frac{(\Pi_c(p)+F)}{2S^2}=\frac{3\delta_H}{4S^4}\left(\frac{f}{2t_H}\right)^2\left[\left(\frac{4f}{3}-a\right)p+a-\frac{2af}{3}-f(1-f)+t_Lk(m)\sqrt{\delta_H(1-p-f)}\right]-\frac{3F}{S^4}.$$

Given p, the optimal value of S, denoted by $S_c(p)$, can be obtained by solving the first-order condition, which can be rewritten as:

$$(p+a)S_c(p)^3=\frac{3F}{\delta_Lt_L}-\frac{\delta_H}{\delta_Lt_L}\left(\frac{f}{2t_H}\right)^2\left[\left(f-\frac{3a}{4}\right)p-\frac{3a}{4}+\frac{af}{2}-\frac{3f(1-f)}{4}+\frac{3t_Lk(m)}{4}\sqrt{\delta_H(1-p-f)}\right]. \tag{15}$$

Note that if the right hand side of (15) is positive, the second derivative of average profit per unit area is negative (i.e., the average profit function is concave). Implicit differentiation with respect to p on both sides of (15) yields:

$$\begin{aligned}S_c(p)^3+3(p+a)S_c(p)^2\frac{d}{dp}S_c(p) &= -\frac{\delta_H}{\delta_Lt_L}\left(\frac{f}{2t_H}\right)^2\left(f-\frac{3a}{4}-\frac{3t_Lk(m)\sqrt{\delta_H}}{8\sqrt{1-p-f}}\right) \\ 3(p+a)S_c(p)^2\frac{d}{dp}S_c(p) &= -\frac{\delta_H}{8\delta_Lt_L}\left(\frac{f}{2t_H}\right)^2\left(8f-6a-\frac{3t_Lk(m)\sqrt{\delta_H}}{\sqrt{1-p-f}}\right)-S(p)^3.\end{aligned}$$

From the above, $S_c(p)$ is decreasing in p if $8f\geq 6a+\frac{3t_Lk(m)\sqrt{\delta_H}}{\sqrt{1-p-f}}$. To further optimize p, we substitute $S=S_c(p)$ and consider the profit function $R_c(p,S_c(p))$ as a function of p, when the value of S is optimized based on the given p. Then:

$$\begin{aligned}\frac{d}{dp}R_c(p,S_c(p)) &= \frac{\delta_L}{4}(1-\theta-a-2p)-\frac{\delta_L}{3}t_LS_c(p)+\frac{\delta_H}{4}(1-a-2p)\left(\frac{f}{2t_H}\right)^2\frac{1}{S_c(p)^2} \\ &\quad -\frac{\delta_H}{3}t_H\left(\frac{f}{2t_H}\right)^3\frac{1}{S_c(p)^2}+\frac{\delta_H}{4}\left(1-2p-2f+\frac{t_Lk(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}\right)\left[1-\left(\frac{f}{2t_H}\right)^2\frac{1}{S_c(p)^2}\right]\end{aligned}$$

$$
\begin{aligned}
&+ \frac{\partial}{\partial S}\bigg|_{S=S_c(p)} R_c(p,S)\frac{d}{dp}S_c(p)\\
&= -\frac{1}{2}(\delta_L+\delta_H)p + \frac{\delta_L}{4}(1-\theta-a) - \frac{\delta_L}{3}t_L S_c(p) + \frac{\delta_H}{4}\left(\frac{4f}{3}-a\right)\left(\frac{f}{2t_H}\right)^2 \frac{1}{S_c(p)^2}\\
&+\frac{\delta_H}{4}(1-2f) + \frac{t_L k(m)\sqrt{\delta_H}}{8\sqrt{1-p-f}}\left[1-\left(\frac{f}{2t_H}\right)^2\frac{1}{S_c(p)^2}\right] \qquad \text{since } \frac{\partial}{\partial S}\Big|_{S=S_c(p)} R_c(p,S)=0.
\end{aligned}
$$

The first-order condition is given by:

$$
\begin{aligned}
&(\delta_L+\delta_H)p - \frac{t_L k(m)\sqrt{\delta_H}}{4\sqrt{1-p-f}}\left[1-\left(\frac{f}{2t_H}\right)^2\frac{1}{S_c(p)^2}\right]\\
&= \frac{\delta_L}{2}(1-\theta-a) - \frac{2\delta_L}{3}t_L S_c(p) + \frac{\delta_H}{2}\left(\frac{4f}{3}-a\right)\left(\frac{f}{2t_H}\right)^2\frac{1}{S_c(p)^2} + \frac{\delta_H}{2}(1-2f).
\end{aligned}
$$

For the B2C delivery model, we note that the following hold when $t_b = t_L$:

$$
\begin{aligned}
R_b(p,S) &= R_c(p,S) - \frac{\delta_H t_b}{3m_b}(1-p-f)\left(S-\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}\right)\\
\frac{\partial}{\partial S}R_b(p,S) &= \frac{\partial}{\partial S}R_c(p,S) - \frac{\delta_H t_b}{3m_b}(1-p-f)\left(1+\left(\frac{f}{2t_H}\right)^3\frac{1}{2S^3}\right)\\
\frac{\partial}{\partial p}R_b(p,S) &= \frac{\partial}{\partial S}R_c(p,S) + \frac{\delta_H t_b}{3m_b}\left(S-\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}\right).
\end{aligned}
$$

Then, it holds that:

$$
\begin{aligned}
\frac{d}{dp}R_b(p,S_b(p)) &= \frac{\partial}{\partial p}R_c(p,S_b(p)) + \frac{\delta_H t_b}{3m_b}\left(S-\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}\right)\\
&\geq \frac{\partial}{\partial p}R_c(p,S_b(p))\\
&\geq \frac{\partial}{\partial p}R_c(p,S_c(p)) = \frac{d}{dp}R_c(p,S_c(p)).
\end{aligned}
$$

In the above, the second inequality holds because $S_b(p)\leq S_c(p)$ and $\frac{\partial}{\partial p}R_c(p,S)$ is decreasing in S. The former holds because $\frac{\partial}{\partial S}R_b(p,S)\leq\frac{\partial}{\partial S}R_c(p,S)$ for any p. The latter holds as:

$$
\frac{\partial^2}{\partial S\partial p}R_c(p,S) = -\frac{\delta_H}{12}\left(\frac{f}{2t_H}\right)^2\left(\frac{4f}{3}-a-\frac{t_L k(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}\right) - \frac{\delta_L}{3t_L}\leq 0.
$$

Since $\frac{d}{dp}G_b(p,S_b(p))\geq\frac{d}{dp}G_c(p,S_c(p))$, the solution p_b^* to the first-order condition involving the former is larger than that for the latter, p_c^*. Finally, since $\frac{\partial}{\partial S}G_b(p,S)\leq\frac{\partial}{\partial S}G_c(p,S)$, it holds that $S_b(p)\leq S_c(p)$ for any p. Since $S_c(p)$ is decreasing in p, $S_c(p_c^*)\geq S_c(p_b^*)\geq S_b(p_b^*)$.

Proof of Proposition 7

Similarly to the proof of Proposition 6, we first consider:

$$
\begin{aligned}
R_s(p,S) = & \frac{\delta_L}{2S^2}(p+a)\left[\frac{(1-\theta-p)}{2}\left(\frac{f}{2t_H}\right)^2 - \frac{2t_L}{3}\left(\frac{f}{2t_H}\right)^3\right] \\
& + \frac{\delta_H}{2S^2}(p+a)\left[\frac{(1-p)}{2}\left(\frac{f}{2t_H}\right)^2 - \frac{2t_H}{3}\left(\frac{f}{2t_H}\right)^3\right] \\
& + \frac{\delta_L}{16S^2}(1-\theta+a)^2\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right] - \frac{\delta_L}{6S^2}(1-\theta+a)t_L\left[S^3 - \left(\frac{f}{2t_H}\right)^3\right] \\
& + \frac{\delta_L}{8S^2}t_L^2\left[S^4 - \left(\frac{f}{2t_H}\right)^4\right] + \frac{\delta_H}{4S^2}\left[(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)}\right]\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right. \\
& - \frac{F}{2S^2} \\
= & \frac{\delta_L}{4}(p+a)(1-\theta-p)\left(\frac{f}{2t_H}\right)^2\frac{1}{S^2} - \frac{\delta_L}{3}(p+a)t_L\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2} \\
& + \frac{\delta_H}{4}(p+a)(1-p)\left(\frac{f}{2t_H}\right)^2\frac{1}{S^2} - \frac{\delta_H}{3}(p+a)t_H\left(\frac{f}{2t_H}\right)^3\frac{1}{S^2} \\
& + \frac{\delta_L}{16}(1-\theta+a)^2\left[1 - \left(\frac{f}{2t_H}\right)^2\frac{1}{S^2}\right] - \frac{\delta_L}{6}(1-\theta+a)t_L\left[S - \left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}\right] \\
& + \frac{\delta_L}{8}t_L^2\left[S^2 - \left(\frac{f}{2t_H}\right)^4\frac{1}{S^2}\right] + \frac{\delta_H}{4}\left[(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)}\right] \\
& - \frac{\delta_H}{4}\left[(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)}\right]\left(\frac{f}{2t_H}\right)^2\frac{1}{S^2} - \frac{F}{2S^2} \\
\frac{\partial}{\partial S}R_c(p,S) = & -\frac{\delta_L}{4S^3}\left[(p+a)(1-\theta-p) - \frac{4}{3}(p+a)t_L\left(\frac{f}{2t_H}\right)\right]\left(\frac{f}{2t_H}\right)^2 \\
& - \frac{\delta_H}{4S^3}\left[(p+a)(1-p) - \frac{4}{3}(p+a)t_H\left(\frac{f}{2t_H}\right)\right]\left(\frac{f}{2t_H}\right)^2 \\
& + \frac{\delta_L}{8S^3}(1-\theta+a)^2\left(\frac{f}{2t_H}\right)^2 - \frac{\delta_L}{6}(1-\theta+a)t_L - \frac{\delta_L}{3S^3}(1+a)t_L\left(\frac{f}{2t_H}\right)^3 + \frac{\delta_L}{4}t_L S^2 \\
& + \frac{\delta_L t_L^2}{4S^3}\left(\frac{f}{2t_H}\right)^4 + \frac{\delta_H}{4S^3}\left[(p+f)(1-p-f) - t_L k(m)\sqrt{\delta_H(1-p-f)}\right]\left(\frac{f}{2t_H}\right)^2 + \frac{F}{S^3} \\
= & -\frac{\delta_H}{4S^3}\left(\frac{f}{2t_H}\right)^2\left[\left(\frac{4f}{3} - a\right)p + a - \frac{2af}{3} - f(1-f) + t_L k(m)\sqrt{\delta_H(1-p-f)}\right] \\
& + \frac{\delta_L}{4S^3}\left(\frac{f}{2t_H}\right)^2\left[\frac{1}{2}(1-\theta-a-p)^2 + \frac{p^2}{2} + a(1-\theta) + \frac{2t_L f}{3t_H}(1-\theta-p)\right] - \frac{\delta_L t_L}{6}(1+a)t_L \\
& + \frac{\delta_L}{4}t_L^2 S + \frac{F}{S^3}.
\end{aligned}
$$

Given p, the optimal value of S, denoted by $S_s(p)$, can be obtained by solving the first-order condition, which can be rewritten as:

$$\frac{(1+a)}{2}S_s(p)^3 - \frac{3t_L}{4}S_s(p)^4 = \frac{3F}{\delta_L t_L} - \frac{\delta_H}{\delta_L t_L}\left(\frac{f}{2t_H}\right)^2\left[\left(f-\frac{3a}{4}\right)p - \frac{3a}{4} + \frac{af}{2} - \frac{3f(1-f)}{4} + \frac{3t_L k(m)}{4}\sqrt{\delta_H(1-p-f)}\right] + \frac{1}{t_L}\left(\frac{f}{2t_H}\right)^2\left[\frac{3}{8}(1-a-2p)^2 + \frac{t_L f}{2t_H}\right] \quad (16)$$

Note that $2t_L S < 1$ (because otherwise there no low-type consumers would be willing to travel to the store). This implies that the left hand side of (16) is strictly increasing in S, and thus the solution is unique.

Furthermore,

$$\frac{\partial^2}{\partial S \partial p}R_s(p,S) = -\frac{\delta_H}{12}\left(\frac{f}{2t_H}\right)^2\left(\frac{4f}{3} - a - \frac{t_L k(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}\right) - \frac{\delta_L}{2S^3}\left(\frac{f}{2t_H}\right)^2\left[1-a-2p-\frac{4}{3}t_L\left(\frac{f}{2t_H}\right)\right] \le 0.$$

For the B2C delivery model, we note that the following hold when $t_b = t_L$:

$$\begin{aligned}
\frac{\partial}{\partial S}R_b(p,S) &= \frac{\partial}{\partial S}R_s(p,S) - \frac{\delta_H t_b}{3m_b}(1-p-f)\left(1+\left(\frac{f}{2t_H}\right)^3\frac{1}{2S^3}\right) \\
&\quad - \frac{\delta_L}{4S^3}\left(\frac{f}{2t_H}\right)^2\left[\frac{1}{2}(1-a-2p)^2 + \frac{2t_L f}{3t_H}\right] - \frac{\delta_L}{4}t_L^2 S + \frac{t_L\delta_L}{6}(1-a-2p) \\
\frac{\partial}{\partial p}R_b(p,S) &= \frac{\partial}{\partial p}R_s(p,S) + \frac{\delta_H t_b}{3m_b}\left(S - \left(\frac{f}{2t_H}\right)^3\frac{1}{S^2}\right) + \frac{\delta_L}{2S^2}(1+a-2p)\left(\frac{f}{2t_H}\right) - \frac{\delta_L t_L}{2S^2}\left(\frac{f}{2t_H}\right)^2
\end{aligned}$$

If $1+a-2p \le \frac{3}{2}t_L S + 2a$, it can be seen that $\frac{\partial}{\partial S}R_b(p,S) \le \frac{\partial}{\partial S}R_s(p,S)$. Following the argument in the proof of Proposition 3, it also holds that $\frac{\partial}{\partial p}R_b(p,S) \ge \frac{\partial}{\partial p}R_s(p,S)$ if $1+a-2p \ge t_L f/(2t_H)$. Then, using a similar argument as in the proof of Proposition 6:

$$\begin{aligned}
\frac{d}{dp}R_b(p,S_b(p)) &\ge \frac{\partial}{\partial p}R_s(p,S_b(p)) \\
&\ge \frac{\partial}{\partial p}R_s(p,S_s(p)) = \frac{d}{dp}R_s(p,S_s(p)),
\end{aligned}$$

if $t_L f/(2t_H) \le 1+a-2p \le \frac{3}{2}t_L S + 2a$. In the above, the second inequality holds because $S_b(p) \le S_s(p)$ and $\frac{\partial}{\partial p}R_s(p,S)$ is decreasing in S. The former holds because $\frac{\partial}{\partial S}R_b(p,S) \le \frac{\partial}{\partial S}R_c(p,S)$ for any p. Then, following the same argument as in Proposition 6, $S_s(p_s^*) \ge S_s(p_b^*) \ge S_b(p_b^*)$.

Proof of Lemma 4 Taking derivative of the profit function,

$$\frac{d}{dp}\Pi_b(p) = \delta_L\left[(1-\theta-p)\frac{S^2}{2} - \frac{2t_L S^3}{3}\right] - \delta_L(p+a)\frac{S^2}{2} + \delta_H\left[\frac{(1-p)}{2}\left(\frac{f}{2t_H}\right)^2 - \frac{2t_H}{3}\left(\frac{f}{2t_H}\right)^3\right]$$

$$-\delta_H \frac{(p+a)}{2}\left(\frac{f}{2t_H}\right)^2 + \frac{1}{2}\left[\delta_H(1-2f-2p) + \frac{t_b k(m)\sqrt{\delta_H}}{2\sqrt{1-p-f}}\right]\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right]$$
$$+\frac{2\delta_H t_b}{3m_b}\left(S^3 - \left(\frac{f}{2t_H}\right)^3\right).$$

Setting this equal to zero, the first order condition yields (13). To check when this gives the unique optimal solution, we further take the second derivative:

$$\frac{d^2}{dp^2}\Pi_b(p) = -(\delta_L+\delta_H)S^2 + \frac{t_b k(m)\sqrt{\delta_H}}{8(1-p-f)^{\frac{3}{2}}}\left[S^2 - \left(\frac{f}{2t_H}\right)^2\right].$$

For the second derivative to be negative, it is sufficient that

$$\delta_L + \delta_H \geq \frac{t_b k(m)\sqrt{\delta_H}}{8(1-p-f)^{\frac{3}{2}}} \quad \text{or} \quad (\delta_L+\delta_H)(1-p-f) \geq \frac{t_b k(m)\sqrt{\delta_H}}{8\sqrt{1-p-f}}.$$

Proof of Lemma 5

Substituting the definition of u in (4) and evaluating at $\hat{\alpha} = \alpha$ (suppressing (p,z) for brevity) yields:

$$\alpha = \frac{(1-p-f)\delta_H/\delta_L}{1-\theta-p-2t_L z+\alpha u}.$$

Differentiating both sides with respect to u gives:

$$\begin{aligned}
\frac{\partial \alpha}{\partial u} &= -\frac{u(1-p-f)\delta_H/\delta_L}{(1-\theta-p-2t_L z+\alpha u)^2}\frac{\partial \alpha}{\partial u} - \frac{\alpha(1-p-f)\delta_H/\delta_L}{(1-p-2t_L z+\alpha u)^2} \\
&= -\frac{\alpha u}{q_L^s}\frac{\partial \alpha}{\partial u} - \frac{\alpha^2}{q_L^s} \\
\left(1+\frac{\alpha u}{q_L^s}\right)\frac{\partial \alpha}{\partial u} &= -\frac{\alpha^2}{q_L^s} \qquad (17) \\
\frac{\partial \alpha}{\partial u} &= -\frac{\alpha}{q_L^s/\alpha + u}.
\end{aligned}$$

This implies that $\frac{\partial \alpha}{\partial u} < 0$, i.e., $\alpha(p,z)$ is strictly decreasing in u.

Next, recall that $q_L^s = 1-p-2t_L z+\alpha u$. Therefore:

$$\begin{aligned}
\frac{\partial}{\partial u} q_L^s &= \alpha + u\frac{\partial \hat{\alpha}}{\partial u} \\
&= -\frac{q_L^s}{\alpha}\frac{\partial \hat{\alpha}}{\partial u} \quad \text{(by (17))} \qquad (18) \\
&= \frac{\alpha}{1+\alpha u/q_L^s}.
\end{aligned}$$

This suggests that $\frac{\partial}{\partial u} q_L^s > 0$, i.e., $q_L^s(p,z)$ is strictly increasing in u.

Appendix C: Parameter Values in Numerical Illustration

In the numerical study reported in Section 3.3, we set the parameter values based on public information regarding Walmart's retail network in the San Francisco Bay Area. In particular, Walmart operates 21 stores in the counties of Alameda, Contra Costa and Santa Clara (note that Walmart has no stores in the Marin, San Francisco and San Mateo counties) [5]. We note that the purpose of this exercise is to provide a quantitative example of the various effects and trade-offs involved as a numerical illustration; we do not draw direct comparison to Walmart's store network, since other factors such as intensifying competition with competitors will play a significant role which we do not capture in our model. Table 2 provides a summary of the notation.

First, the combined land area of the Alameda (739 sq. miles), Contra Costa (716) and Santa Clara counties (1,304) is 2,759 sq. miles [4]. Thus, each of the 21 stores covers an average catchment area of 131 sq miles, leading to a value of $S = \sqrt{131/2} = 8.1$ miles. In 2022, there are 581,683, 405,106 and 646,847 households residing in the counties, respectively, giving a total of 1.63 million households [4]. We consider households up to the 80-th percentile in the household income distribution as the potential consumers. Thus, the total density of consumers is $\delta_L + \delta_H = 1.63mil \times 0.8/2759 = 0.472$ ('000) consumers per sq. mile. For simplicity in our numerical illustration, we consider the low-type potential consumers to correspond to the bottom 40% of households in the San Francisco Bay Area by household income, and the high type to correspond to the 40-th to 80-th percentile. This suggests that $\delta_L = \delta_H = 0.236$ ('000 consumers per sq. mile).

It is estimated that consumers spend \$55 per visit to Walmart. We assume that this corresponds to the normalized price of $p = 0.5$ (the optimal price ignoring travel and shipping costs), thus the maximum valuation of one unit corresponds to a dollar value of \$110. We consider the travel costs t_L and t_H to consist of both the fuel cost plus the value of time. For fuel cost, we consider the gas price of \$3.5 per gallon and a fuel economy of 30 miles per gallon, which yields \$0.117 per mile. For low-type consumers, the average household annual income (2017) is \$36,000 [2] (\$30,022 after tax [3]), which translates to \$10.42 per person-hour (assuming 1.5 persons per household, 20 working days per month and 8 working hours per day), and \$0.521 per mile (assuming a commuting and shopping speed of 20 miles per hour). Adding the fuel cost, the travel cost for low-type consumers is \$0.638 per mile, or 0.0058 in the converted (zero to one) valuation scale. For high-type consumers, we consider the average income of the 40-th to 80-th percentiles (\$117,000 pre-tax and \$82,065 after tax), which translates to \$1.54 per mile including fuel cost, or 0.014 in the converted scale. Furthermore, It is estimated that additional in-store traffic generates about $20 - 25\%$

extra revenue in addition to consumers' planned purchase (e.g., [6]). Thus, we consider the store traffic value to be worth $a = 0.2$ to the retailer. Finally, we consider shipping costs per online order to be $f = 0.1$.

For the store density analysis in Section 5, we consider the fixed cost F as estimated as follows. From the 2017 financial reports, Walmart's total depreciation and amortization expense was $10.95 billion, while its gross income amounted $143.57 billion [1]. In Section 3.3, the profit per store according to our model is in the range of 3.5 to 3.9 units. Considering the same fixed (depreciation and amortization) cost to gross income ratio, we consider the fixed cost to be $F = 0.25$ units.

Appendix D: Effect of Number of Stops in Vehicle Routing Formula

To prescribe a good solution on a vehicle routing problem on a planar shape, Daganzo [12] consider a heuristic that divides the region into rectangular subregions of width l and length l' ($l' \geq l$). Each rectangular subregions contains an average of $m = \hat{\delta} l l'$ stops to be visited on a single tour. The problem of designing good tours under this heuristic is reduced to selecting the best shape of the rectangle, characterized by the ratio $\beta = l/l'$ while keeping the area (thus m) constant.

Using the derivation from Daganzo [11], the average detour distance to visit each point in the rectangle is given by $\phi(\hat{\delta} l^2)/\sqrt{\hat{\delta}}$, where $\phi(x) = \frac{\sqrt{x}}{6} + \frac{2}{\sqrt{x}}$ for the ℓ_1 metric. Let the center of the rectanglar subregion to be located z miles from the store; which implies that the near end of the subregion is $z - l'/2$ miles from the store. The average distance (per each of the m points) traveled by the vehicle to serve all customers in the rectangular subregion is given by:

$$\begin{aligned}
d/m &= 2(z - l'/2) + m\phi(\hat{\delta} l^2)/\sqrt{\delta} \\
&= \frac{2}{m}(z - l'/2) + \frac{\sqrt{m\beta}}{6\sqrt{\hat{\delta}}} + \frac{2}{\sqrt{\hat{\delta} m\beta}}, \qquad \text{since } m\beta = ml/l' = \hat{\delta} l^2 \\
&= \frac{2}{m} z + \left(\frac{\sqrt{m\beta}}{6} + \frac{1}{\sqrt{m\beta}} \right) \hat{\delta}^{-1/2}, \qquad \text{since } \frac{l'}{m} = \frac{1}{\hat{\delta} l} = \frac{1}{\sqrt{\hat{\delta} m\beta}}.
\end{aligned}$$

The above is minimized by setting $m\beta = 6$. However, this is only achievable for $m \leq 6$ due to the constraint $\beta \geq 1$. Therefore, for $m < 6$, the best option is to set $\beta = 1$, and thus it holds that the average delivery distance per stop

$$\frac{d}{m} = \frac{2z}{m} + \frac{k(m)}{\sqrt{\hat{\delta}}}, \text{ where } k(m) = \begin{cases} \frac{\sqrt{m}}{6} + \frac{1}{\sqrt{m}} & \text{for } m < 6 \\ 0.82 & \text{for } m \geq 6. \end{cases}$$

The values are summarized in Table 1 in the main text.

Table 2 Summary of Notation

Notation	Meaning	Value in Numerical Illustration
t_L, t_H	Travel cost for low-type and high-type consumers	0.0058, 0.014
t_b	Travel cost per mile for B2C delivery vehicle	0.0058
δ_L, δ_H	Spatial density of low-type and high-type consumers ('000 per sq. mile)	0.236
v	Consumers' valuation for product, with $[0,1]$ range scaled to represent \$0 to \$110	Uniform$[0, 1-\theta]$ for low-type, Uniform$[0,1]$ for high-type consumers
θ	Parameter capturing correlation between travel cost and valuation	0 to 0.1
a	Extra exogenous revenue for in-store sales	0.2
z	Travel distance from consumer's home to store (miles)	0 to S
S	Radius of store's catchment area (miles)	8.1
m, m_b	Number of delivery stops per tour under P2P and B2C models	3, 12
$k(m)$	Economies of scale parameter for a delivery tour with m stops	See Table 1
f	Delivery fee for online orders	0.1
p	Price for product	Decision variable
$r(p,z)$	Remuneration for P2P deliveries as function of location z and price p	Endogenous
$\gamma(p), \gamma_b(p)$	Delivery cost for P2P and B2c models as function of price p	Endogenous
$q_i^s(p,z), q_i^o(p,z)$	Demand (fraction) of type-i ($\in \{H, L\}$) consumers residing at location z who purchase under P2P model, as function of price p	Endogenous
$q_{ib}^s(p,z), q_{ib}^o(p,z)$	Demand (fraction) of type-i ($\in \{H, L\}$) consumers residing at location z who purchase under B2C model, as function of price p	Endogenous
$\alpha(p,z)$	Probability of an in-store consumer being offered a P2P delivery gig, as function of location z and price p	Endogenous
$\hat{\alpha}(p,z)$	Consumers' rational expectation on $\alpha(p,z)$	Endogenous
$\Pi_b(p)$	Aggregate profit function for store under B2C model	Endogenous
$\Pi_c(p)$	Aggregate profit function for store under P2P model with cost-based remuneration	Endogenous
$\Pi_s(p)$	Aggregate profit function for store under P2P model with cross subsidy	Endogenous
$v_j(p,z)$	Vehicle-miles traveled per unit sales under B2C ($j=b$), and P2P model with cost-based remuneration ($j=c$) and cross subsidy ($j=s$)	Endogenous
F	Fixed operating cost for opening a store (in Section 6)	0.25

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