E-Companion

Appendix EC.1: Expected Latency Calculations

In this section, we discuss the calculation of expected latency, l_r, for a given a priori route r, visiting a combination of deterministic and stochastic customers. Recall that an a priori route with at least one stochastic customer may correspond to multiple visit realizations due to multiple potential request realizations of its stochastic customers. Therefore, l_r represents the expected latency with respect to all the request realizations, and consequently, visit realizations of r. That is, $l_r = \sum_{v \in \mathcal{V}_r} \sum_{q \in \mathcal{Q}_v} \mathbb{EL}(q)$. We calculate the latency associated with each visit realization by determining the customer-specific latency $l_{c,q}$. Note that the expected disutility for a customer c on route r is directly derived from its expected latency, using the function $g_c(l_{c,q})$ for each customer $c \in r$ and each request realization $q \in \mathcal{Q}_v$ and visit realization $\mathcal{V}_r$.

To describe our calculation of the expected latency for a given a priori route, we take route $r = [0-1-2-3-4-0']$ depicted in Figure EC.1 as an illustrative example which includes all potential relative positions of deterministic and stochastic customers in an a priori route. In what follows, we will enumerate all request realizations of r and provide an analytical approach to calculate the expected latency associated with each request realization.

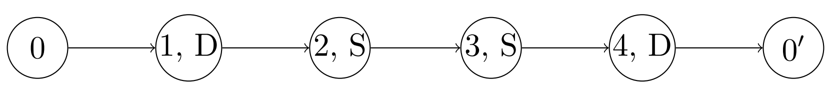

Figure EC.1 0 - Depot, S - stochastic customer, D - deterministic customer

Customer 1 has requested service before the beginning of planning period h (i.e. customer 1 is a deterministic customer), and therefore, will be serviced at time $t_{0,1}$. This is based on the rule that a deterministic customer is serviced (and therefore traveled to) as soon as the technician leaves the depot or is released from the preceding task. Therefore, the expected latency with respect to customer 1 is $t_{0,1}$, and in what follows, we focus on request realizations of the two stochastic customers (customers 2 and 3) and their implications in terms of visit realizations and expected latency. Also, to keep the representations as generic as possible, we develop the formulations based on a generic wait-to time scheme (and not necessarily the maximum wait-to time strategy). If stochastic customer c is not scheduled, or if it is scheduled but its request is received after being skipped, then ξ_c represents the earliest time that customer c can be served in the next time period. In our implementation, we set ξ_c equal to the start of the next time period plus $t_{0,c}$, which serves as a lower bound on the actual start of service for customer c. In that same vein, in Model (1), parameter γ_c, the penalty for not scheduling a customer c could be determined by the difference between ξ_c and the expected request time of customer c.

1. Consider request realization q_1 resulting in visit realization $v_1 = [0-1-②-3-4-0']$. This scenario corresponds to when customer 2 does not request a service by its latest request time $\phi_{v_1}(2)$ and customer 3 requests service before $\phi_{v_1}(2)$. Therefore the service agent must wait until time $\phi_{v_1}(2)$ and can then immediately depart for customer 3. The expected latency of this request realization is

$$\mathbb{EL}(q_1) += F_3(\phi_{v_1}(2)) \int_{\phi_{v_1}(2)}^{\xi_2} f_2(\tau_2)\,[\xi_2 - \tau_2]\,d\tau_2 \qquad \text{(Customer 2)}$$

$$+ [1 - F_2(\phi_{v_1}(2))] \int_{0}^{\phi_{v_1}(2)} f_3(\tau_3)\,[\phi_{v_1}(2) + t_{1,3} - \tau_3]\,d\tau_3 \qquad \text{(Customer 3)}$$

$$+ [1 - F_2(\phi_{v_1}(2))]\,F_3(\phi_{v_1}(2))\,[\phi_{v_1}(2) + t_{1,3} + t_{3,4}]\,. \qquad \text{(Customer 4)}$$

The above calculation is divided by lines into the portion of the expected latency for the request realization q_1 corresponding to each customer on the a priori route.

Customer 2 is stochastic, and on this request realization, it does not request service before its latest request time ($\phi_{v_1}(2)$). The expected latency of skipped customer 2 is the expected value of the difference between the time that customer 2 is assumed to be visited after the current planning period (ξ_2) and customer 2's request time (τ_2) over the range of possible values for that request time, i.e., time interval $]\phi_{v_1}(2), \xi_2]$: $\int_{\phi_{v_1}(2)}^{\xi_2} f_2(\tau_2)\,[\xi_2 - \tau_2]\,d\tau_2$. Further, in this request realization, customer 3 is stochastic and requests service before $\phi_{v_1}(2)$. Therefore, the expected latency of a skipped customer 2 is scaled by the probability of the request timing of customer 3 ($F_3(\phi_{v_1}(2))$).

The expected latency of customer 3 is scaled by the probability that customer 2 is skipped ($1 - F_2(\phi_{v_1}(2))$). The expected latency of customer 3 for this request realization is the expected difference between the known visit time of customer 3 ($\phi_{v_1}(2) + t_{1,3}$) and customer 3's request time (τ_3) over the range of possible values for that request time, i.e., interval $[0, \phi_{v_1}(2)]$.

Finally, the expected latency of customer 4 for this request realization is its known service time ($\phi_{v_1}(2) + t_{1,3} + t_{3,4}$). As for customer 1, the expected latency of customer 4 is scaled by the probability that customer 2 is skipped and customer 3 is visited.

2. Next, consider request realization q_2 leading to visit realization $v_1 = [0 - 1 - ② - 3 - 4 - 0']$. In this scenario, customer 3 requests service after the latest request time of customer 2, $\phi_{v_1}(2)$, but before its own latest request time, $\phi_{v_1}(3)$. Therefore, the service agent waits (at customer 1) until customer 3 requests service and immediately departs for customer 3, such that the latency of customer 3 is $t_{1,3}$. Due to this request realization, the visit time to customer 4 is now stochastic. The expected latency of this request realization is

$$\mathbb{EL}(q_2) += [F_3(\phi_{v_1}(3)) - F_3(\phi_{v_1}(2))] \int_{\phi_{v_1}(2)}^{\xi_2} f_2(\tau_2)\,[\xi_2 - \tau_2]\,d\tau_2 \qquad \text{(Customer 2)}$$

$$+ [1 - F_2(\phi_{v_1}(2))]\,[F_3(\phi_{v_1}(3) - F_3(\phi_{v_1}(2))]\,t_{1,3} \qquad \text{(Customer 3)}$$

$$+ [1 - F_2(\phi_{v_1}(2))] \int_{\phi_{v_1}(2)}^{\phi_{v_1}(3)} f_3(\tau_3)\,[\tau_3 + t_{1,3} + t_{3,4}]\,d\tau_3. \qquad \text{(Customer 4)}$$

3. Next, consider a request realization q_3 resulting in a visit realization $v_2 = [0 - 1 - 2 - 3 - 4 - 0']$, where no stochastic customer is skipped. In this request realization, customer 2 requests service before the service agent release time of customer 1, $\theta_1 = t_{0,1}$, and customer 3 requests service before the service agent release time of customer 2, θ_2. Therefore the service agent travels the a priori route with no waiting times. The expected latency of this request realization is

$$\mathbb{EL}(q_3) += \int_0^{\tau_2} \int_0^{t_{0,1}} f_2(\tau_2) f_3(\tau_3)\,[t_{0,1} + t_{1,2} - \tau_2]\,d\tau_2 d\tau_3 \qquad \text{(Customer 2)}$$

$$+ \int_0^{\tau_2} \int_0^{t_{0,1}} f_2(\tau_2) f_3(\tau_3)\,[t_{0,1} + t_{1,2} + t_{2,3} - \tau_3]\,d\tau_2 d\tau_3 \qquad \text{(Customer 3)}$$

$$+ \int_0^{\tau_2} \int_0^{t_{0,1}} f_2(\tau_2) f_3(\tau_3)\,[t_{0,1} + t_{1,2} + t_{2,3} + t_{3,4}]\,d\tau_2 d\tau_3. \qquad \text{(Customer 4)}$$

4. Next, consider a request realization q_4 leading to visit realization $v_2 = [0 - 1 - 2 - 3 - 4 - 0']$, where customer 2 requests service after the service agent release time of customer 1, $\theta_1 = t_{0,1}$, and customer 3 requests service before the service agent release time of

customer 2, θ_2. Therefore the service agent waits at customer 1 until customer 2 requests service, immediately departs for customer 2, and when service is completed at customer 2 immediately departs for customer 3. The expected latency of this request realization is

$$\mathbb{E}\mathbb{L}(q_4) += \int_0^{\tau_2}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)t_{1,2} \qquad \text{(Customer 2)}$$

$$+ \int_0^{\tau_2}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)\left[\tau_2 + t_{1,2} + t_{2,3} - \tau_3\right] d\tau_2 d\tau_3 \qquad \text{(Customer 3)}$$

$$+ \int_0^{\tau_2}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)\left[\tau_2 + t_{1,2} + t_{2,3} + t_{3,4}\right]. \qquad \text{(Customer 4)}$$

5. Next, consider a request realization q_5 resulting in visit realization $v_2 = [0-1-2-3-4-0']$, where customer 2 requests service before the service agent release time of customer 1, $\theta_1 = t_{0,1}$, and customer 3 requests service after the service agent release time of customer 2, θ_2. Therefore, the service agent departs for customer 2 immediately after completing service at 1, then waits at customer 2 until customer 3 requests service and departs immediately. The expected latency of this request realization is

$$\mathbb{E}\mathbb{L}(q_5) += \left[F_3(\phi_{v_2}(3)) - F_3(t_{0,1})\right]\int_0^{t_{0,1}} f_2(\tau_2)\left[t_{0,1} + t_{1,2} - \tau_2\right] d\tau_2 \qquad \text{(Customer 2)}$$

$$+ F_2(t_{0,1})\left[F_3(\phi_{v_2}(3)) - F_3(t_{0,1})\right] t_{2,3} \qquad \text{(Customer 3)}$$

$$+ F_2(t_{0,1})\int_{t_{0,1}+t_{1,2}}^{\phi_{v_2}(3)} f_3(\tau_3)\left[\tau_3 + t_{2,3} + t_{3,4}\right] d\tau_3. \qquad \text{(Customer 4)}$$

6. Next, consider a request realization q_6 leading to visit realization $v_2 = [0-1-2-3-4-0']$, where customer 2 requests service after the service agent release time of customer 1, θ_1, and customer 3 requests service after the service agent release time of customer 2, θ_2. Therefore the service agent waits at customers 1 and 2 until customers 2 and 3 request service, respectively, immediately departing when the requests are received. The expected latency of this request realization is

$$\mathbb{E}\mathbb{L}(q_6) += \int_{\tau_2}^{\phi_{v_2}(3)}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)t_{1,2} d\tau_2 d\tau_3 \qquad \text{(Customer 2)}$$

$$+ \int_{\tau_2}^{\phi_{v_2}(3)}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)t_{2,3} d\tau_2 d\tau_3 \qquad \text{(Customer 3)}$$

$$+ \int_{\tau_2}^{\phi_{v_2}(3)}\int_{t_{0,1}}^{\phi_{v_2}(2)} f_2(\tau_2)f_3(\tau_3)\left[\tau_3 + t_{2,3} + t_{3,4}\right] d\tau_2 d\tau_3. \qquad \text{(Customer 4)}$$

7. Next, consider a request realization q_7 resulting in visit realization $v_3 = [0-1-2-③-4-0']$, where customer 2 requests service before the service agent release time of customer 1, θ_1, and customer 3 is skipped. Therefore, the service agent travels immediately from customer 1 to customer 2, then waits until $\phi_{v_3}(3)$ to depart for customer 4. The expected latency of this request realization is

$$\begin{aligned}
\mathbb{E}\mathbb{L}(q_7) {+}{=} &\left[1-F_3(\phi_{v_3}(3))\right]\int_0^{t_{0,1}}\left[t_{0,1}+t_{1,2}-\tau_2\right]f_2(\tau_2)d\tau_2 && \text{(Customer 2)}\\
&+F_2(t_{0,1})\int_{\phi_{v_3}(3)}^{\xi_3}\left[\xi_3-\tau_3\right]f_3(\tau_3)d\tau_3 && \text{(Customer 3)}\\
&+\left[1-F_3(\phi_{v_3}(3))\right]F_2\left(t_{0,1}\right)\left[\phi_{v_3}(3)+t_{2,4}\right]. && \text{(Customer 4)}
\end{aligned}$$

8. Next, consider a q_8 request realization leading to visit realization $v_3 = [0-1-2-③-4-0']$, where customer 2 requests service after the service agent release time of customer 1, θ_1, and customer 3 is skipped. Therefore, the service agent waits at customer 1 until customer 2 requests service, leaves when the request is received, then waits at customer 2 until $\phi_{v_3}(3)$ to depart for customer 4. The expected latency of this request realization is

$$\begin{aligned}
\mathbb{E}\mathbb{L}(q_8) {+}{=} &\left[1-F_3(\phi_{v_3}(3))\right]\left[F_2(\phi_{v_3}(3))-F_2(t_{0,1})\right]t_{1,2} && \text{(Customer 2)}\\
&+\left[F_2(\phi_{v_3}(3))-F_2(t_{0,1})\right]\int_{\phi_{v_3}(3)}^{\xi_3}\left[\xi_3-\tau_3\right]f_3(\tau_3)d\tau_3 && \text{(Customer 3)}\\
&+\left[1-F_3(\phi_{v_3}(3))\right]\left[\phi_{v_3}(3)+t_{2,4}\right]. && \text{(Customer 4)}
\end{aligned}$$