Online Appendix: Auction Mechanism Design for Order Allocation and Payment in a Crowdshipping System

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Appendix A: Nomenclature

Table A1 summarizes the main mathematical notations used in the main body. Notations specific to Section 3.2, the numerical experiments (Section 5), or the appendices are omitted for brevity.

Appendix B: Pickup and Delivery Problem Formulation and Bid Construction Algorithm

B.1. The Pickup and Delivery Problem Formulation

This appendix formulates the route-specific Pickup and Delivery Problem (PDP) in Section 3.2, which is also used for the numerical experiments in Section 5. Specifically, the PDP determines the feasibility of a bundle $s \subseteq J$ for a baseline route $l \in L_i$ and computes the shortest detour time $\Delta T_{l,s}$ if the bundle is feasible (Savelsbergh and Sol 1995).

Denote o_j and d_j the pickup and delivery location of order j. Let P_s and D_s denote the sets of pickup and delivery locations for the orders in bundle s, respectively. Recall that o_i and d_i represent the origin and destination of crowd carrier i, and T_l is the direct travel time of baseline route l. Define $V = \{o_i, d_i\} \cup P_s \cup D_s$ as the set of all nodes, t_{vw} as the travel time from node v to node w, $\tau_{i,l}$ as the detour time tolerance, and $Q_{i,l}$ as the physical capacity limit. The load change at node v is denoted by q_v (1 for $v \in P_s$, -1 for $v \in D_s$, and 0 otherwise).

We introduce binary variables $x_{v,w}$ indicating whether arc (v, w) is traversed, continuous variables $y_v \geq 0$ tracking the cumulative arrival time at node v, and integer variables $z_v \in \mathbb{Z}_{\geq 0}$ representing the on-board load after leaving node v. The mathematical formulation of the PDP model is presented as a mixed integer linear program (MILP) (A1).

The objective function (A1a) minimizes the extra detour time incurred by serving bundle s. Constraints (A1b) to (A1e) are standard flow conservation and degree constraints, ensuring a valid routing sequence from o_i to d_i that visits every pickup and delivery node exactly once. Constraints (A1f) to (A1i) manage the temporal logic: they initialize the start time, respect the detour time tolerance $\tau_{i,l}$, eliminate sub-tours by connecting spatial routing to temporal states, and guarantee that every order is picked up before it is delivered. Finally, Constraints (A1j) to (A1l) track the real-time vehicle load along the path to ensure that the physical concurrent capacity $Q_{i,l}$ is strictly respected. Any bundle s that renders MILP (A1) infeasible is identified as infeasible on route l.

Table A1 Summary of main mathematical notations.

Notation	Description
Sets	
I	Set of crowd carriers
J	Set of orders
S_i	Set of feasible bundles for crowd carrier $i \in I$
s	A bundle of orders
$\hat{I}$	Winners of the auction determined by the VCG mechanism
$\tilde{I}$	Winners of the auction determined by the greedy mechanism
Variables / Parameters	
r_j	Fulfillment fee of order j by dedicated delivery services
f_j	Service fare charged from the customer for shipping order j
L_i	The set of baseline routes for crowd carrier i
$c_i(l,s)$	True cost function for carrier i when serving a bundle $s \in S_i$ based on a baseline route $l \in L_i$
$\hat{c}_i(l,s)$	Reported cost function by carrier i when serving a bundle $s \in S_i$ based on a baseline route $l \in L_i$
$c_{i,s}$	True cost of crowd carrier i serving the orders in bundle $s \in S_i$
$b_{i,s}$	Bid of crowd carrier i on their submitted bundle $s \in S_i$
$x_{i,s}$	Whether bundle s is assigned to crowd carrier i
y_j	Whether order j is outsourced and fulfilled by dedicated delivery services
$\mathbf{b}_i$	Bids of crowd carrier i for their bundle set S_i (column vector)
$\mathbf{x}_i$	Allocation result of crowd carrier i (column vector)
$\mathbf{c}_i$	True costs of crowd carrier i for their bundle set S_i (column vector)
$\mathbf{b}$	Bids of all crowd carriers (row vector)
$\mathbf{x}$	Allocation results of all crowd carriers (row vector)
$\mathbf{y}$	Outsourcing decisions of orders (row vector)
g	An allocation rule maps the bidding price vector $\mathbf{b}$ to an outcome $(\mathbf{x}, \mathbf{y})$
p	A payment rule maps the bidding price vector $\mathbf{b}$ to the compensation made to each crowd carrier p_i
$\mathcal{M}$	A mechanism consisting of an allocation rule g and a payment rule p
u_i	Utility derived by crowd carrier i
$(\mathbf{x}^*, \mathbf{y}^*)$	Optimal solution to the winner determination problem (WDP)
$\mathbf{b}_{-i}$	Bid vector excluding crowd carrier i
h_i	A mapping from $\mathbf{b}_{-i}$ to a real number
π	Optimal value of the WDP
π^{-i}	Optimal value of the WDP corresponding to the auction where crowd carrier i is excluded
λ_i	Lagrange multiplier in the primal-dual analysis; Largest reduced cost associated with crowd carrier i in the greedy mechanism
μ_j	Lagrange multiplier
s'	Bundle assigned to crowd carrier i when he/she misreports
i'	Crowd carrier with the second-largest reduced cost corresponding to winner $i \in \tilde{I}$
M	Maximum number of orders per bundle
N	Maximum number of bids per crowd carrier
$\hat{\pi}$	Optimal value of the WDP when bundle size is limited to M

$$\min \quad \sum_{v\in V}\sum_{w\in V} t_{vw}x_{vw} - T_l \tag{A1a}$$

$$\text{s.t.} \quad \sum_{w\in V\setminus\{o_i\}} x_{o_i,w} = 1 \tag{A1b}$$

$$\sum_{v\in V\setminus\{d_i\}} x_{v,d_i} = 1 \tag{A1c}$$

$$\sum_{v\in V} x_{vw} = 1, \quad \forall w \in P_s \cup D_s \tag{A1d}$$

$$\sum_{w\in V} x_{vw} = 1, \quad \forall v \in P_s \cup D_s \tag{A1e}$$

$$y_{o_i} = 0 \tag{A1f}$$

$$y_{d_i} - T_l \le \tau_{i,l} \tag{A1g}$$

$$x_{vw} = 1 \Rightarrow y_w \ge y_v + t_{vw}, \quad \forall v, w \in V \tag{A1h}$$

$$y_{o_j} + t_{o_j,d_j} \le y_{d_j}, \quad \forall j \in s \tag{A1i}$$

$$z_{o_i} = 0 \tag{A1j}$$

$$0 \le z_v \le Q_{i,l}, \quad \forall v \in V \tag{A1k}$$

$$x_{vw} = 1 \Rightarrow z_w = z_v + q_w, \quad \forall v, w \in V \tag{A1l}$$

$$x_{vw} \in \{0,1\}, \quad \forall v, w \in V \tag{A1m}$$

$$y_v \in \mathbb{R}_{\ge 0}, \quad \forall v \in V \tag{A1n}$$

$$z_v \in \mathbb{Z}_{\ge 0}, \quad \forall v \in V \tag{A1o}$$

B.2. A Bid Construction Algorithm

To construct all feasible bundles and their associated bidding prices for a given crowd carrier, we adopt a recursive algorithm with apriori pruning, inspired by Arslan et al. (2019). The algorithm proceeds per baseline route $l \in L_i$ and builds bundles incrementally by size. Starting from single-order bundles, it verifies feasibility for each candidate using a PDP oracle that checks capacity, precedence, and detour constraints. Only bundles that are feasible on the current route are kept as seeds for generating larger bundles in the next level.

Candidate bundles of size $k+1$ are formed by merging two feasible bundles of size k that share exactly $k-1$ orders; this follows the standard apriori principle. A key pruning step eliminates any candidate that contains an infeasible subset of size k, thereby drastically reducing the search space. For each bundle that is found feasible on any route, we record the minimum detour time across all routes that can serve it. The final bid is then calculated as the product of the carrier's value of time α_i and the minimal detour time. The pseudocode of this procedure is given in Algorithm A1.

Appendix C: Proofs Related to the VCG Mechanism

C.1. Proof of Proposition 1 (Strategy-Proofness)

Proof. By definition of strategy-proofness, this proposition is equivalent to the claim that reporting the truthful additional cost $\mathbf{c}_i$ is the dominant strategy for each crowd carrier $i \in I$. Recall that the utility function of crowd carrier

Algorithm A1 Bid Construction Algorithm with Apriori Pruning

Input: $L_i, \{K_{i,l}\}_{l \in L_i}, \{Q_{i,l}\}_{l \in L_i}, \tau_{i,l}, \alpha_i, t_{uv}$

Output: S_i, $\{b_{i,s}\}_{s \in S_i}$

$S_i \leftarrow \emptyset$
$\Delta T_{i,s}^{\min} \leftarrow \{\}$ ▷ min detour time per bundle
for each $l \in L_i$ **do**
    $C_1 \leftarrow \big\{\{j\} \mid j \in J\big\}$, $k \leftarrow 1$
    while $C_k \neq \emptyset \wedge k \leq K_{i,l}$ **do**
        $\mathcal{F}_k \leftarrow \emptyset$
        for each $s \in C_k$ **do**
            $[\text{feasible}, \Delta T_{l,s}] \leftarrow \text{PDP\_ORACLE}(s, l, Q_{i,l}, \tau_{i,l}, t_{uv})$
            if feasible **then**
                $\mathcal{F}_k \leftarrow \mathcal{F}_k \cup \{s\}$
                if $s \notin S_i$ **then**
                    $S_i \leftarrow S_i \cup \{s\}$, $\Delta T_{i,s}^{\min} \leftarrow \Delta T_{l,s}$
                else
                    $\Delta T_{i,s}^{\min} \leftarrow \min(\Delta T_{i,s}^{\min}, \Delta T_{l,s})$
                end if
            end if
        end for
        if $k < K_{i,l}$ **then**
            $C_{k+1} \leftarrow \big\{s_1 \cup s_2 \mid s_1, s_2 \in \mathcal{F}_k,\ |s_1 \cup s_2| = k+1\big\}$
            for each $s' \in C_{k+1}$ **do**
                if $\exists s \subset s'$ with $|s| = k$ and $s \notin \mathcal{F}_k$ **then**
                    $C_{k+1} \leftarrow C_{k+1} \setminus \{s'\}$ ▷ Apriori pruning
                end if
            end for
        end if
        $k \leftarrow k + 1$
    end while
end for
for each $s \in S_i$ **do**
    $b_{i,s} \leftarrow \alpha_i \cdot \Delta T_{i,s}^{\min}$
end for

i is represented as $u_i(\mathbf{b}) = p_i(\mathbf{b}) - \mathbf{c}_i^T \mathbf{x}_i(\mathbf{b})$. Given the assumption that each crowd carrier aims to maximize their utility, we want to show $u_i(\mathbf{c}_i, \mathbf{b}_{-i}) \geq u_i(\mathbf{b})$ holds for any fixed $\mathbf{b}_{-i}$. Note that $\mathbf{b}_{-i}$ may not necessarily equal $\mathbf{c}_{-i}$ here.

Consider (x', y') as the optimal solution of the cost minimization problem corresponding to the bidding price vector $(\mathbf{c}_i, \mathbf{b}_{-i})$, i.e., $(x', y') = g^G(\mathbf{c}_i, \mathbf{b}_{-i})$. In the Groves mechanism, where the allocation rule aims to minimize the total cost, we have the relationship:

$$\begin{aligned} & \mathbf{c}_i^T \mathbf{x}_i' + \sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k' + \sum_{j \in J} r_j y_j' \\ \leq\ & \mathbf{c}_i^T \mathbf{x}_i^* + \sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k^* + \sum_{j \in J} r_j y_j^* \end{aligned} \tag{A2}$$

Then by definition of the utility function and payment rule we have

$$u_i^G(\mathbf{c}_i, \mathbf{b}_{-i}) = p_i^G(\mathbf{c}_i, \mathbf{b}_{-i}) - \mathbf{c}_i^T \mathbf{x}_i' \tag{A3a}$$

$$= \left[h_i(\mathbf{b}_{-i}) - \sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k' - \sum_{j \in J} r_j y_j' \right] - \mathbf{c}_i^T \mathbf{x}_i' \tag{A3b}$$

$$= h_i(\mathbf{b}_{-i}) - \left[\mathbf{c}_i^T \mathbf{x}_i' + \sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k' + \sum_{j \in J} r_j y_j' \right] \tag{A3c}$$

$$\geq h_i(\mathbf{b}_{-i}) - \left[\mathbf{c}_i^T \mathbf{x}_i^* + \sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k^* + \sum_{j \in J} r_j y_j^* \right] \tag{A3d}$$

$$= \left[h_i(\mathbf{b}_{-i}) - \left(\sum_{k \in I \setminus \{i\}} \mathbf{b}_k^T \mathbf{x}_k^* + \sum_{j \in J} r_j y_j^* \right) \right] - \mathbf{c}_i^T \mathbf{x}_i^* \tag{A3e}$$

$$= p_i^G(\mathbf{b}) - \mathbf{c}_i^T \mathbf{x}_i^* \tag{A3f}$$

$$= u_i^G(\mathbf{b}) \tag{A3g}$$

The inequality comes from the fact that $(\mathbf{x}', \mathbf{y}')$ is the minimizer of WDP when the bidding price vector is $(\mathbf{c}_i, \mathbf{b}_{-i})$. To exploit the minimizer property, we separate i from I when doing the summations in the objective function. Thus, truthfully reporting the additional cost is a dominant strategy since a crowd carrier cannot achieve higher utility than telling the truth under any circumstances. □

C.2. Proof of Proposition 2 (Individual Rationality)

Proof. This property is established based on strategy-proofness. If all crowd carriers truthfully report their costs at the dominant strategy equilibrium, i.e., $\mathbf{b}_i = \mathbf{c}_i$, then by definition of the utility function we obtain

$$u_i^{VCG}(\mathbf{c}_i, \mathbf{b}_{-i}) = p_i^{VCG}(\mathbf{b}) - \mathbf{c}_i^T \mathbf{x}_i^* \tag{A4a}$$

$$= \left[\pi^{-i} - \pi + \mathbf{b}_i^T \mathbf{x}_i^* \right] - \mathbf{c}_i^T \mathbf{x}_i^* \tag{A4b}$$

$$= \pi^{-i} - \pi + \mathbf{b}_i^T \mathbf{x}_i^* - \mathbf{c}_i^T \mathbf{x}_i^* \tag{A4c}$$

$$= \pi^{-i} - \pi \tag{A4d}$$

$$\geq 0 \tag{A4e}$$

Given that the allocation problem is a minimization integer program, we can infer from feasible region monotonicity that if crowd carrier i is excluded from participating in the auction, the optimal objective function will not deteriorate compared to when he/she is included. This observation leads to the final inequality. □

C.3. Proof of Proposition 3 (Payment Non-Negativity)

Proof. Crowd carrier i's received payment equals $p_i^{VCG}(\mathbf{b}) = \pi^{-i} - \pi + \mathbf{b}_i^T\mathbf{x}_i^*$. By the minimization problem property and feasible set monotonicity, we know that $\pi \leq \pi^{-i}$. This, combined with $\mathbf{b}_i^T\mathbf{x}_i^* \geq 0$, leads to $p_i^{VCG}(\mathbf{b}) \geq 0$. Therefore, $p_i^{VCG}(\mathbf{b})$ cannot be negative.

In fact, there are two possible results of $\mathbf{x}_i^*$: (1) If $\mathbf{x}_i^* = \mathbf{0}$, it implies that crowd carrier i will not be assigned a bundle in the optimal solution. Consequently, the optimal solution remains unchanged even if he/she is absent, leading to $\pi = \pi^{-i}$ and $p_i^{VCG}(\mathbf{b}) = 0$. (2) If $\mathbf{x}_i^* \neq \mathbf{0}$, this indicates crowd carrier i is allocated a bundle such that $\sum_{s \in S_i} x_{i,s}^* = 1$, he/she will receive a bonus of $(\pi^{-i} - \pi)$ in addition to the bidding price of the assigned bundle. □

C.4. Proof of Proposition 4 (Budget Balance)

We introduce a lemma below before establishing budget balance of the VCG mechanism applied in the crowdshipping system.

LEMMA 1. *For any crowd carrier i who wins with s_i in the auction ($s_i = \emptyset$ if crowd carrier i does not win), we have $\pi^{-i} \leq \pi - \mathbf{b}_i^T\mathbf{x}_i^* + \sum_{j \in s_i} r_j$.*

Proof of Lemma 1. We start the proof from the right hand side of the inequality. There are two possibilities:

- If crowd carrier i is not in the winner set, we have $\mathbf{b}_i^T\mathbf{x}_i^* = \sum_{j \in s_i} r_j = 0$ and $\pi^{-i} = \pi$, and thus the inequality holds.
- If crowd carrier i is in the winner set, then $\pi - \mathbf{b}_i^T\mathbf{x}_i^*$ represents the total system cost excluding i under the optimal

solution when i participates in the auction. However, if he/she is absent, the orders that would have been served by him, denoted by s_i, will remain unfulfilled, which violates the constraint (2c). To ensure feasibility of the WDP (2) when i is not involved, the orders in s_i will be fulfilled by dedicated delivery services with incurred cost $\sum_{j \in s_i} r_j$. Hence, the right hand side presents a feasible solution to WDP (2) without carrier i, and the inequality is satisfied based on the definition of π^{-i}.

This completes the proof. □

Proof of Proposition 4. We first prove the property of weak budget balance when $f_j = r_j$. This entails proving that the payments to the winning crowd carriers and the outsourcing fees for dedicated delivery services do not surpass the service fares received from the customers. The platform's profit is calculated as the total revenue collected minus the total payment for crowd-sourced and dedicated delivery services:

$$\begin{aligned}
&\sum_{j\in J} r_j - \sum_{i\in I} p_i^{VCG}(\mathbf{b}) - \sum_{j\in J} r_j y_j^* \\
&= \sum_{j\in J} r_j - \sum_{i\in I} \left(\pi^{-i} - \pi + \mathbf{b}_i^T \mathbf{x}_i^*\right) - \sum_{j\in J} r_j y_j^* && \text{(A5a)}\\
&= \sum_{j\in J} r_j - \sum_{i\in I} \left(\pi^{-i} - \pi\right) - \sum_{i\in I} \mathbf{b}_i^T \mathbf{x}_i^* - \sum_{j\in J} r_j y_j^* && \text{(A5b)}\\
&= \sum_{j\in J} r_j - \sum_{i\in I} \left(\pi^{-i} - \pi\right) - \pi && \text{(A5c)}\\
&= \sum_{j\in J} r_j - \sum_{i\in I} \pi^{-i} + (|I| - 1)\,\pi && \text{(A5d)}\\
&\geq \sum_{j\in J} r_j - \sum_{i\in I} \left(\pi - \mathbf{b}_i^T \mathbf{x}_i^* + \sum_{j\in s_i} r_j\right) + (|I| - 1)\,\pi && \text{(A5e)}\\
&= \sum_{j\in J} r_j - \sum_{i\in I} \left(-\mathbf{b}_i^T \mathbf{x}_i^* + \sum_{j\in s_i} r_j\right) - \pi && \text{(A5f)}\\
&= \sum_{j\in J} r_j - \sum_{i\in I} \sum_{j\in s_i} r_j - \sum_{j\in J} r_j y_j^* && \text{(A5g)}\\
&= 0 && \text{(A5h)}
\end{aligned}$$

The inequality comes from Lemma 1. The case of strong budget balance is straightforward as it is a Pareto-improvement compared with the case of $f_j = r_j$. This completes the proof. □

Appendix D: Proofs Related to the Greedy Mechanism

D.1. Proof of Proposition 5 (Approximate Strategy-Proofness)

Proof. Considering a specific bidder i with the bids of other bidders fixed at $\mathbf{b}_{-i}$, we compare the utility derived when bidder i tells the truth and when they misreport their additional cost. The ex post regret is $\sup_{\mathbf{b}_i} \left(u_i(\mathbf{b}_i, \mathbf{b}_{-i}) - u_i(\mathbf{c}_i, \mathbf{b}_{-i})\right)$. Let $u_i = u_i(\mathbf{c}_i, \mathbf{b}_{-i})$ and $u_i' = u_i(\mathbf{b}_i, \mathbf{b}_{-i})$ represent the resulting utilities in the truthful and misreporting scenarios, respectively. Define $\Delta u_i = u_i' - u_i$ to quantify the utility difference.

Because monotonicity is difficult to establish directly in this multi-bid XOR setting, we prove this proposition in an end-to-end fashion. From the outcomes, we analyze each of the following four cases.

Case 1: $\varnothing \to \varnothing$ *Regardless of whether crowd carrier i tells the truth or not, they do not win the auction.*

The utility of a losing bidder is 0, therefore $u_i = u_i' = 0$. In this case, $\Delta u_i = 0$, meaning that the utility of crowd carrier i remains at 0 if they switch from truth telling to misreporting. Thus the ex post regret of crowd carrier i is 0.

Case 2: $s \to \varnothing$ *When the truthful bids are revealed, crowd carrier i is assigned bundle s, whereas no bundle is assigned to them when they misreport.*

Let i' be the bidder with the second-largest reduced cost with respect to bidder i when they tell the truth and win. If crowd carrier i is assigned bundle s when they reveal their true valuation, the derived utility equals $u_i = p_i - c_{i,s} = b_{i,s} + \lambda_i - \lambda_{i'} - c_{i,s} = \lambda_i - \lambda_{i'} \geq 0$. The inequality is based on the definition of i' in Algorithm 2. However, when they do not tell the truth and thus loses the auction, the utility gained is 0. Therefore, in this case, $\Delta u_i \leq 0$, meaning the utility of crowd carrier i will not increase if they switch from truth-telling to misreporting. In this case, the ex post regret of crowd carrier i is still 0.

Case 3: $\varnothing \to s'$ *When crowd carrier i tells the truth, they do not win any bundle. Misreporting results in bundle s' being assigned to them.*

Let i'' be the bidder with the second-largest reduced cost with respect to bidder i when they misreport and win. If crowd carrier i misreports their bids and is assigned bundle s' in loop t' of the While loop, then the winners with assigned bundles in any loop before t' must be the same since the strategies of other bidders are fixed. Moreover, s' doesn't conflict with the assigned bundles before t'. Thus we have $\lambda_i \le \lambda_{i''}$, or equivalently, $\sum_{j\in s'} r_j - b_{i,s'} = \sum_{j\in s'} r_j - c_{i,s'} \le \lambda_{i''}$. Otherwise, crowd carrier i should be the winner (rather than i'') in loop t' when telling the truth, which contradicts the fact that crowd carrier i tells the truth but does not win any bundle. If bundle s' is assigned to crowd carrier i when they misreport, their payment becomes $p'_i = \sum_{j\in s'} r_j - \lambda_{i''}$, and their utility becomes $u'_i = \sum_{j\in s'} r_j - \lambda_{i''} - c_{i,s'} \le 0$. Therefore, in this case, $\Delta u_i \le 0$ holds as well, indicating that the ex post regret of crowd carrier i is still 0.

Case 4: $s \to s'$ *When the truthful bids are revealed, crowd carrier i is assigned bundle s. Misreporting results in bundle s' being assigned to them (s' and s may or may not be the same).*

Suppose crowd carrier i wins at loop t under truthful bidding and at loop t' under misreporting, respectively. The corresponding crowd carrier with the second-largest reduced cost is denoted by i' and i'', respectively. Let us now discuss each of the following three possibilities:

- $t' = t$. Given that s is the original assigned bundle for crowd carrier i when they tell the truth, we can deduce that $\sum_{j\in s} r_j - c_{i,s} \ge \sum_{j\in s'} r_j - c_{i,s'}$. Moreover, the winners in loops before t remain unchanged since each crowd carrier can win at most one bid, resulting in $i'' = i'$ and $\lambda_{i'} = \lambda_{i''}$. This leads to $u_i = p_i - c_{i,s} = \sum_{j\in s} r_j - \lambda_{i'} - c_{i,s} = \sum_{j\in s} r_j - \lambda_{i''} - c_{i,s} \ge \sum_{j\in s'} r_j - \lambda_{i''} - c_{i,s'} = p'_i - c_{i,s'} = u'_i$. Consequently, $\Delta u_i \le 0$, indicating that misreporting will not increase the utility derived by crowd carrier i.
- $t' < t$. In **Case 2**, we established that crowd carrier i obtains a non-negative utility from the assigned bundle s when telling the truth, i.e., $u_i \ge 0$. When they underbid and win bundle s' in loop t' before t, the winners in loops before t' must remain the same. This also implies that bundle s' should be eliminated in or after loop t' in the algorithm when crowd carrier i tells the truth. In addition, i'' must emerge as the winner in loop t' when crowd carrier i tells the truth, indicating that $\sum_{j\in s'} r_j - c_{i,s'} \le \lambda_{i''}$. Therefore, the derived utility $u'_i = \sum_{j\in s'} r_j - c_{i,s'} - \lambda_{i''} \le 0$ when crowd carrier underbids on some bundle(s) and wins bundle s', which yields $\Delta u_i \le 0$.
- $t' > t$. This happens when a crowd carrier overbids on some bundle(s). When crowd carrier i tells the truth and is assigned bundle s, we know $\lambda_i = \sum_{j\in s} r_j - b_{i,s} = \sum_{j\in s} r_j - c_{i,s}$. Then we have $\Delta u_i = u'_i - u_i = \sum_{j\in s'} r_j - \lambda_{i''} - c_{i,s'} - \left(\sum_{j\in s} r_j - \lambda_{i'} - c_{i,s}\right) \le \sum_{j\in s'} r_j - c_{i,s'} - \sum_{j\in s} r_j + \lambda_{i'} + c_{i,s} \le \sum_{j\in s'} r_j - c_{i,s'} - \sum_{j\in s} r_j + \lambda_i + c_{i,s} = \sum_{j\in s'} r_j - c_{i,s'}$ by virtue of $\lambda_{i''} \ge 0$ and $\lambda_i \ge \lambda_{i'}$. Thus in this scenario $\Delta u_i = u'_i - u_i \le \sum_{j\in s'} r_j - c_{i,s'}$.

Therefore, in this case the ex post regret is $\max\left\{\sum_{j\in s'} r_j - c_{i,s'}, 0\right\}$.

Table A2 The results of truthful bidding and misreporting for crowd carrier i.

Possible cases	Result of truth telling	Result of misreporting	Ex post regret
Case 1	not assigned	not assigned	0
Case 2	win bundle s	not assigned	0
Case 3	not assigned	win bundle s'	0
Case 4	win bundle s	win bundle s'	$\max\{\sum_{j\in s'} r_j - c_{i,s'}, 0\}$

Table A2 summarizes the results of all 4 possible cases. Overall, the ex post regret of a crowd carrier i equals $\max\{\sum_{j\in s'} r_j - c_{i,s'}, 0\}$, where s' is the bundle assigned to crowd carrier i when they misreport. A positive ex post regret may only be induced when a crowd carrier overbids on some bundle(s). □

D.2. Proof of Proposition 6 (Individual Rationality)

Proof. If a crowd carrier i is not part of the winning set, their derived utility is 0 since $p_i = 0$ and $\mathbf{x}_i = 0$.

Now suppose that crowd carrier i wins bundle s_i in some iteration of Algorithm 2. Let $\lambda_{i'}$ denote the second-largest current reduced cost used in the payment rule for crowd carrier i, with $\lambda_{i'} = 0$ if no such carrier remains. Since Algorithm 2 selects the carrier with the largest current reduced cost in each iteration, we have $\lambda_i \geq \lambda_i'$. Under truthful bidding, $b_{i,s_i} = c_{i,s_i}$. Therefore, the utility of crowd carrier i is

$$u_i = p_i - c_{i,s_i} = b_{i,s_i} + \lambda_i - \lambda_i' - c_{i,s_i} = \lambda_i - \lambda_i' \geq 0.$$

Thus, every truthful crowd carrier obtains non-negative utility under the greedy mechanism. □

D.3. Proof of Proposition 7 (Payment Non-Negativity)

Proof. If a crowd carrier i is not assigned any of their desired bundles, their payment is 0. However, if they successfully win bundle s_i in a given loop, their payment p_i is calculated as $b_{i,s_i} + \lambda_i - \lambda_{i'}$, which is at least b_{i,s_i} due to $\lambda_i \geq \lambda_{i'}$. Since the bid $b_{i,s_i} \geq 0$, the payment is non-negative. □

D.4. Proof of Proposition 8 (Budget Balance)

Proof. Analogous to Proposition 4, we initially establish weak budget balance under the condition $f_j = r_j$, with the strong budget balance case naturally following from Pareto improvement. It is evident that in this algorithm, each order is fulfilled only once, either by crowdshipping or by dedicated delivery services. Let the orders served by crowd carriers be denoted by J_C, and those fulfilled by dedicated delivery services as J_D, such that $J_C \cup J_D = J$ and $J_C \cap J_D = \emptyset$.

Based on Algorithm 2, let $\tilde{I}$ represent the set of winning crowd carriers, and s_i denote the assigned bundle for $i \in \tilde{I}$. The platform's profit, which is the total revenue collected from customers minus the total payment for crowd-sourced and dedicated delivery services, can be expressed as:

$$\sum_{j\in J} r_j - \sum_{i\in\tilde{I}} p_i - \sum_{j\in J_D} r_j = \sum_{j\in J_C} r_j - \sum_{i\in\tilde{I}} p_i \tag{A6a}$$

$$= \sum_{j\in J_C} r_j - \sum_{i\in\tilde{I}} \left(b_{i,s_i} + \lambda_i - \lambda_{i'}\right) \tag{A6b}$$

$$= \sum_{j\in J_C} r_j - \sum_{i\in\tilde{I}} \left[\sum_{j\in s_i} r_j - \lambda_{i'}\right] \tag{A6c}$$

$$= \sum_{j\in J_C} r_j - \sum_{i\in\tilde{I}} \sum_{j\in s_i} r_j + \sum_{i\in\tilde{I}} \lambda_{i'} \tag{A6d}$$

$$= \sum_{i\in\tilde{I}} \lambda_{i'} \geq 0 \tag{A6e}$$

The non-negativity of $\lambda_{i'}$ is directly derived from Algorithm 2. □

D.5. Proof of Proposition 9 (Optimality Bound)

To streamline the proof, we reframe IP (2) by leveraging the connection between $x_{i,s}$ and y_j in constraints (2c) to eliminate y_j and derive a simplified formulation solely involving $x_{i,s}$.

LEMMA 2. *The integer program (2) is equivalent to the following integer program:*

$$\min_{x_{i,s}} \quad \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i}\left(b_{i,s} - \sum_{j\in s} r_j\right) x_{i,s} \tag{A7a}$$

$$\text{s.t.} \quad \sum_{s\in S_i} x_{i,s} \le 1 \qquad \forall i \in I \tag{A7b}$$

$$\sum_{i\in I}\sum_{s\ni j, s\in S_i} x_{i,s} \le 1 \qquad \forall j \in J \tag{A7c}$$

$$x_{i,s} \in \{0,1\} \qquad \forall i \in I, s \in S_i \tag{A7d}$$

where the only decision variable is $x_{i,s}$.

Proof of Lemma 2. The proof is mainly in two parts as follows. First we demonstrate the equivalence of the objective function, followed by proving that the constraints are also equivalent.

Replacing y_j by $x_{i,s}$ from (2c) in the objective function, we obtain:

$$\begin{aligned} &\sum_{i\in I}\sum_{s\in S_i} b_{i,s}x_{i,s} + \sum_{j\in J} r_j\left(1 - \sum_{i\in I}\sum_{s\in S_i} x_{i,s}\delta_s^j\right) \\ =& \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i} b_{i,s}x_{i,s} - \sum_{j\in J} r_j \sum_{i\in I}\sum_{s\in S_i} x_{i,s}\delta_s^j \end{aligned} \tag{A8a}$$

$$= \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i} b_{i,s}x_{i,s} - \sum_{i\in I}\sum_{s\in S_i}\sum_{j\in J} r_j x_{i,s}\delta_s^j \tag{A8b}$$

$$= \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i}\left(b_{i,s} - \sum_{j\in J} r_j\delta_s^j\right) x_{i,s} \tag{A8c}$$

$$= \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i}\left(b_{i,s} - \sum_{j\in s} r_j\right) x_{i,s} \tag{A8d}$$

Similarly, to represent y_j as a binary value, we have

$$\begin{aligned} &1 - \sum_{i\in I}\sum_{s\in S_i} x_{i,s}\delta_s^j \in \{0,1\} \Longleftrightarrow \sum_{i\in I}\sum_{s\in S_i} x_{i,s}\delta_s^j \in \{0,1\} \\ \Longleftrightarrow& \sum_{i\in I}\sum_{s\ni j, s\in S_i} x_{i,s} \in \{0,1\} \Longleftrightarrow \sum_{i\in I}\sum_{s\ni j, s\in S_i} x_{i,s} \le 1 \end{aligned} \tag{A9}$$

The final equivalence stems from $x_{i,s} \in \{0,1\}, \forall i \in I, s \in S_i$. □

Proof of Proposition 9. By Lemma 2, the equivalent objective function is transformed into:

$$\begin{aligned} &\min_{x_{i,s}} \sum_{j\in J} r_j + \sum_{i\in I}\sum_{s\in S_i}\left(b_{i,s} - \sum_{j\in s} r_j\right) x_{i,s} \\ \Longleftrightarrow& \sum_{j\in J} r_j - \max_{x_{i,s}} \sum_{i\in I}\sum_{s\in S_i}\left(\sum_{j\in s} r_j - b_{i,s}\right) x_{i,s} \end{aligned} \tag{A10}$$

Here, let $V_{i,s} = \sum_{j \in s} r_j - b_{i,s}$ represent the "value" of a bundle s for $s \in S_i$ and $i \in I$. Then the following integer program can be examined first:

$$\max_{x_{i,s}} \quad \sum_{i \in I} \sum_{s \in S_i} V_{i,s} x_{i,s} \tag{A11a}$$

$$\text{s.t.} \quad \sum_{s \in S_i} x_{i,s} \le 1 \qquad \forall i \in I \tag{A11b}$$

$$\sum_{i \in I} \sum_{s \ni j, s \in S_i} x_{i,s} \le 1 \qquad \forall j \in J \tag{A11c}$$

$$x_{i,s} \in \{0,1\} \qquad \forall i \in I, s \in S_i \tag{A11d}$$

This problem can be categorized as a specific instance of the set packing problem and is thus NP-hard. (Karp 1972, De Vries and Vohra 2003, Cramton, Shoham, and Steinberg 2006). In fact, if the size of each s has an upper limit $M \ge 3$, the conclusion still holds.

Let OPT represent the optimal value of the objective function in (A11) and ALG denotes the objective value of (A11) calculated using Algorithm 2. We examine two cases below: one without bidding restriction and the other with restriction on bundle sizes.

Without restriction In the absence of constraints on the bundle size, i.e., $|s|$, the worst-case scenario occurs when the greedy algorithm terminates prematurely after one step, thereby missing out on potential further gains.

The optimal solution picks the winner set $\hat{I}$ from the crowd carriers, where each winner $i \in \hat{I}$ has a "value" of V_{i,s_i} associated with the winning bundle s_i. In contrast, the greedy algorithm only selects one crowd carrier with a "value" of $\epsilon + \max_{i \in \hat{I}} V_{i,s_i}$, where ϵ represents a positive value. This choice results in the exclusion of the optimal crowd carriers in $\hat{I}$ from being selected. Therefore, in the worst-case scenario, we have:

$$\begin{aligned} \frac{OPT}{ALG} = \frac{\sum_{i \in \hat{I}} V_{i,s_i}}{\max_{i \in \hat{I}} V_{i,s_i} + \epsilon} &\le \frac{\sum_{i \in \hat{I}} V_{i,s_i}}{\max_{i \in \hat{I}} V_{i,s_i}} \\ &\le \frac{|\hat{I}| \max_{i \in \hat{I}} V_{i,s_i}}{\max_{i \in \hat{I}} V_{i,s_i}} = |\hat{I}| \end{aligned} \tag{A12}$$

The winners must be a subset of the whole crowd carriers and thus $|\hat{I}| \le |I|$. We also have $|\hat{I}| \le |J|$ since each crowd carrier can be assigned with at most one order $j \in J$. Therefore, $\frac{OPT}{ALG} \le \min\{|I|, |J|\}$. Recover the objective value of WDP (2) we obtain an upper bound $\sum_{j \in J} r_j - \frac{\sum_{j \in J} r_j - \pi}{\min\{|I|,|J|\}}$ of WDP (2) where π is optimal value of WDP (2).

With restriction If the bundle size is constrained to not exceed M (i.e., $|s| \le M, \forall s$), we focus on the bundles included in the algorithmic solution but not in the optimal solution.

For these bundles, there must be some conflicted bundles in the optimal solution. Otherwise, the optimal solution can be further improved. The remaining bundles are the common part of optimal solution and algorithmic solution. We denote the corresponding crowd carriers as set $\bar{I}$.

For any crowd carrier $i \in \tilde{I} \setminus \bar{I}$ and the corresponding assigned bundle s_i, there are at most $M + 1$ bundles in the optimal solution conflicting with s_i, i.e., M bundles with common orders of s_i and 1 bundle also from crowd carrier i. Through greedy selection, each "value" of these conflicting bundles corresponding to s_i must be at most as large as V_{i,s_i} where $i \in \tilde{I}$ and $s \in S_i$. Thus:

$$\sum_{i\in\tilde{I}\setminus\bar{I}} V_{i,s_i} \geq \sum_{i\in\hat{I}\setminus\bar{I}} \frac{V_{i,s_i}}{M+1} \Longrightarrow \sum_{i\in\tilde{I}} V_{i,s_i} \geq \sum_{i\in\hat{I}} \frac{V_{i,s_i}}{M+1} \Longrightarrow ALG \geq \frac{OPT}{M+1} \tag{A13}$$

Similarly, recover the objective value of WDP (2) we obtain an upper bound $\sum_{j\in J} r_j - \frac{\sum_{j\in J} r_j - \hat{\pi}}{M+1}$ of WDP (2) where $\hat{\pi}$ is optimal value of WDP (2) when bundle sizes are restricted to at most M. Intuitively, the worst-case scenario occurs when each assigned bundle in the algorithmic solution prevents exactly $M+1$ "optimal" bundles.

This completes the proof. □

D.6. Proof of Proposition 10 (Time Complexity)

Proof. In the initialization stage (lines 2–6), the time complexity is $O\big(|I|\cdot\max_{i\in I}|S_i|\big)$. In the main **while** loop (lines 8–26), there are at most $|J|$ iterations and we analyze an iteration step by step. Line 9 finds the winner i^* with maximum λ_i in $O(|I|)$. Line 10 finds the optimal bundle for winner i^* in $O(|S_{i^*}|)$. Line 11 finds the second highest λ_i in $O(|I|-1)$. Lines 15–18 update the bids for the remaining carriers and the assignment state of orders in $O\big(|I|\cdot\max_{i\in I}|S_i|\big)$. Lines 20–25 recompute λ_i for the remaining carriers, also in $O\big(|I|\cdot\max_{i\in I}|S_i|\big)$. Therefore, combining all of these, the overall time complexity is

$$\begin{aligned} &O\left(|J|\cdot\left(|I|+|S_{i^*}|+(|I|-1)+|I|\cdot\max_{i\in I}|S_i|\right)\right)\\ &=O\left(|I|\cdot|J|\cdot\max_{i\in I}|S_i|\right)\end{aligned} \tag{A14}$$

This completes the proof. □

Appendix E: A Toy Example for Illustration

We present a concrete example to illustrate and compare the two mechanisms developed. This example is small yet meaningful to interpret the approximation of the VCG mechanism to the greedy mechanism.

Consider an environment where there are two crowd carriers $I=\{1,2\}$ and two orders $J=\{1,2\}$. The outsourcing fee for order 1 and order 2 is $r_1=9$ and $r_2=8$, respectively. Crowd carrier 1 (truthfully) submits two bids on bundle $\{1\}$ and bundle $\{2\}$, respectively, with $b_{1,\{1\}}=c_{1,\{1\}}=3$ and $b_{1,\{2\}}=c_{1,\{2\}}=4$. Crowd carrier 2 (truthfully) submits one bid on bundle $\{1\}$ with $b_{2,\{1\}}=c_{2,\{1\}}=4$.

Under the VCG mechanism, the optimal allocation is $x^*_{1,\{1\}}=0$, $x^*_{1,\{2\}}=1$ and $x^*_{2,\{1\}}=1$ with the minimum total cost equal to $\pi=b_{1,\{2\}}+b_{2,\{1\}}=8$. By definition we have $\pi^{-1}=12$ and $\pi^{-2}=11$. The payment made to each crowd carrier is $p_1^{VCG}=4+12-8=8$ and $p_2^{VCG}=4+11-8=7$, respectively.

Under the greedy mechanism, the allocation is $x_{1,\{1\}}=1$, $x_{1,\{2\}}=0$ and $x_{2,\{1\}}=0$. The total system cost equals $3+8=11$. By definition we have $\lambda_1=6$ and $\lambda_2=5$. The payment made to each crowd carrier is $p_1^{greedy}=3+6-5=4$ and $p_2^{greedy}=0$, respectively.

E.1. On the Upper Bound

The greedy mechanism cannot find the optimal allocation even when each bundle only contains one order in the toy example. This is because it relies on local information while each crowd carrier is allowed to submit multiple bids. However, the greedy mechanism provides an upper bound for the optimality value, which equals $9+8-\frac{9+8-8}{2}=12.5$ in this example. Obviously the total cost generated by the greedy mechanism does not exceed this ceiling ($11 \leq 12.5$), as indicated in Proposition 9.

E.2. On the Approximate Strategy-Proofness

It is easy to check the strategy-proofness of the VCG mechanism. We focus on examining the approximate strategy-proofness of the greedy mechanism here. The utility derived by crowd carrier 1 and 2 under truthful bidding is $u_1^{greedy}=4-3=1$ and $u_2^{greedy}=0$, respectively. Under the greedy mechanism, crowd carrier 1 may have an incentive to overbid for a higher utility. Specifically, if $b'_{1,\{1\}}=6$ then $\lambda'_1=4$, and the new allocation will be $x'_{1,\{1\}}=0, x'_{1,\{2\}}=1$ and $x'_{2,\{1\}}=1$. The payment made to crowd carriers is $p_1^{greedy\prime}=4+4-0=8$ and $p_2^{greedy\prime}=4+5-4=5$. Thus $u_1^{greedy\prime}=8-4=4$ and $u_2^{greedy\prime}=5-4=1$, which means crowd carrier 1 derives a higher utility by overbidding on some of their bundles than by telling the truth. In addition, the utility gain from misreporting ($4-1=3$) does not exceed their ex post regret ($\max\{8-4,0\}=4$), which verifies Proposition 5. The positive ex post regret reveals that misreporting yields true cost savings for the assigned bundle (i.e., $r_2 > c_{1,\{2\}}$), suggesting that crowd carriers' incentive to misreport may reduce total system cost.

Appendix F: Pricing Outcomes under Bidding Restrictions and at Different Problem scales

F.1. Pricing Outcomes under Different Bidding Restrictions

Table A3 reports the pricing outcomes of the two mechanisms under different bidding restrictions. Across all scenarios, the VCG mechanism consistently pays higher crowd carrier compensation than the greedy mechanism, consistent with the strategy-proofness property of VCG. Although higher compensation does not necessarily translate into lower platform profit (since profit also accounts for outsourcing fees paid to dedicated delivery services), the trade-off becomes evident when the two mechanisms achieve comparable total system costs. For instance, when $M=N=1$, total system costs are identical, and the difference in profit (greedy 630.2 vs. VCG 350.2) directly mirrors the difference in compensation (367.8 vs. 87.8). In such cases, the savings on crowd carrier payments under the greedy mechanism directly translate into higher platform profit.

F.2. Pricing Outcomes at Different Problem Scales

Table A4 reports crowd carrier compensation and platform profit under both mechanisms. Across all scenarios, the greedy mechanism consistently pays lower crowd carrier compensation while retaining higher platform profit. For a fixed number of crowd carriers, as the number of orders increases, VCG compensation rises sharply whereas greedy compensation grows only incrementally. This divergence reflects a fundamental difference in pricing structures: VCG payments are tied to each winner's marginal contribution under the global optimality, which escalates with demand intensity, while the greedy mechanism sets payments based on local thresholds that remain relatively insensitive to mounting demand pressure. Conversely, for a fixed number of orders, more crowd carriers induce only a modest rise in VCG compensation. This is because a larger supply pool dilutes the marginal contribution of each individual crowd carrier under the VCG outcome.

Table A3 Pricing outcomes of the two mechanisms under different bidding restrictions

Scenarios		Crowd Carrier Compensation ($)		Platform Profit ($)	
M	N	VCG mechanism	Greedy mechanism	VCG mechanism	Greedy mechanism
1	1	367.8	87.8	350.2	630.2
1	5	885.4	139.1	558.6	1087.0
1	10	1130.5	142.3	412.3	1136.3
1	50	1223.8	145.5	343.1	1147.4
1	∞	1224.7	146.4	342.9	1151.1
3	1	421.2	153.2	448.8	696.8
3	5	738.7	204.1	729.4	1110.0
3	10	908.2	224.4	789.9	1233.7
3	50	1230.3	248.6	758.3	1409.9
3	∞	1444.6	267.8	650.5	1459.1
5	1	446.5	181.5	461.5	702.5
5	5	725.4	250.8	756.6	1095.2
5	10	893.4	274.4	810.7	1207.6
5	50	1230.8	309.5	769.9	1402.7
5	∞	1422.4	321.8	692.9	1472.4

Note: ∞ stands for “No Limit”.

Table A4 Pricing outcomes of the two mechanisms under different problem scales

Scenarios		Crowd Carrier Compensation ($)		Platform Profit ($)	
$\|I\|$	$\|J\|$	VCG mechanism	Greedy mechanism	VCG mechanism	Greedy mechanism
50	50	625.9	202.0	206.4	520.0
50	100	947.5	272.0	314.5	830.0
50	150	1206.7	313.5	359.3	1046.5
50	200	1462.0	318.1	412.0	1325.9
100	50	813.6	226.2	503.6	856.3
100	100	1422.4	321.8	692.9	1472.4
100	150	2006.9	374.6	767.9	1949.6
100	200	2334.6	395.3	878.0	2301.0
150	50	961.8	280.5	733.1	1143.1
150	100	1685.9	362.8	1039.4	1873.8
150	150	2399.0	421.6	1174.5	2520.7
150	200	2885.6	456.7	1333.8	3032.3
200	50	1034.8	283.3	946.4	1365.0
200	100	1894.6	395.9	1327.8	2240.0
200	150	2614.3	449.6	1608.2	2981.4
200	200	3402.4	493.0	1684.1	3660.2

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