

Online Supplement

The Undirected Team Orienteering Arc Routing Problem: Formulations, Valid Inequalities, and Exact Algorithms

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A Inaccessible Traversal Inequalities and Edges in Conflict

We demonstrate the minimum travel time used in inaccessible traversal inequalities and the definition of edges in conflict. Figure 1a illustrates the minimum travel time $l_{0e}^1 = d_{0i} + t_e + d_{j0}$ for edge $e = \{i, j\}$, where the dashed edges represent shortest paths between pairs of vertices. Figure 1b visualizes the possible tours contributing to the minimum travel time $l_{0e}^2 = \min\{d_{0i} + 2t_e + d_{i0}, d_{0j} + 2t_e + d_{j0}\}$ for edge $e = \{i, j\}$. Since the graph is undirected, we can simplify it as $2t_e + 2\min\{d_{0i}, d_{0j}\}$. Starting from the depot 0, a vehicle can travel either from vertex i , covering a distance of d_{0i} , or via vertex j , covering a distance of d_{0j} . After that, the vehicle can traverse edge e twice, and finally, follow the original path (either d_{i0} or d_{j0}) back to the depot. Figure 1c depicts the possible tours contributing to the minimum travel time $L_{\min}(e, e') = \min\{d_{0i} + t_e + d_{j'0} + t_{e'} + d_{i'0}, d_{0i} + t_e + d_{j'j'} + t_{e'} + d_{i'0}, d_{0j} + t_e + d_{i'0} + t_{e'} + d_{j'0}, d_{0j} + t_e + d_{i'j'} + t_{e'} + d_{j'0}\}$ for edges $e = \{i, j\}$ and $e' = \{i', j'\}$. Starting from the depot 0, a vehicle can reach edge e by arriving at either vertex i or j . After traversing edge e , it departs from the opposite vertex and proceeds to edge e' , entering through either vertex i' or j' . The vehicle then traverses edge e' and returns to the depot from opposite vertex of e' .

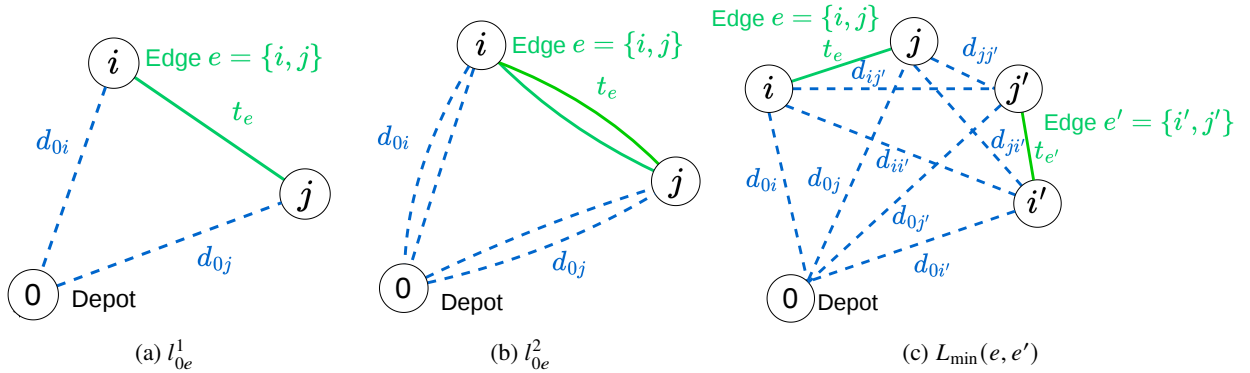


Figure 1: Minimum Travel Time

(a) Tour for Traversing Edge e Once from the Depot; (b) Tours for Traversing Edge e Twice from the Depot; (c) Tours for Traversing Two Edges e and e' Once from the Depot

B MIP Start Heuristic

The pseudocode is presented in Algorithm 1.

To better explain the pseudocode, we introduce more notation. Let τ^k be the travel time of the current tour $k \in K$. Let w_e be the maximum weight defined as the profit r_e of unserved edge $e \in E_D \setminus H^k$ divided by the minimum travel time l_{ie}^1 over all vertices $i \in V_{H^k}$, provided that (i) the edge e can be served within the travel time and the capacity limits based on the current tour's travel time τ^k and cumulative served demand δ^k , and (ii) the number of times the edge has been served σ_e is strictly less than its upper bound u_e , i.e., $w_e = \max_{i \in V_{H^k}} \{ \frac{r_e}{l_{ie}^1} | \tau^k + l_{ie}^1 \leq T_{\max}, \delta^k + q_e \leq Q, \sigma_e + 1 \leq u_e \}$. Let \mathcal{W} be the set of candidate

Algorithm 1: MIP Start Heuristic

Input: the travel time limit T_{\max} , fraction of candidates γ for the candidate list

Output: a candidate solution ψ

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1  $\sigma = (\sigma_e)_{e \in E_D}$ ,  $\sigma \leftarrow \mathbf{0}$ ;  
2 for  $k \in K$  do  
3    $\tau^k \leftarrow 0$ ,  $\delta^k \leftarrow 0$ ,  $H^k \leftarrow \emptyset$ ,  $\mathcal{V}_{H^k} \leftarrow \{0\}$ ,  $\bar{\psi}^k \leftarrow \emptyset$ ;  
4   repeat  
5     SolutionChange  $\leftarrow$  False;  
6     repeat // Search for a candidate  
7        $\mathcal{W} \leftarrow \{w_e \mid e \in E_D \setminus H^k\}$  where  $w_e = \max_{i \in V_{H^k}} \{ \frac{r_e}{l_{ie}^1} \tau^k + l_{ie}^1 \leq T_{\max}, \delta^k + q_e \leq Q, \sigma_e + 1 \leq u_e \}$ , and let  
8          $i_e \in \arg \max_{i \in V_{H^k}} \{ \frac{r_e}{l_{ie}^1} \tau^k + l_{ie}^1 \leq T_{\max}, \delta^k + q_e \leq Q, \sigma_e + 1 \leq u_e \}$ ;  
9         if  $\mathcal{W} \neq \emptyset$  then // Select a candidate  
10           Sort edges from  $E_D \setminus H^k$  in non-increasing order according to  $w \in \mathcal{W}$ ;  
11           Randomly choose one  $e' = \{i', j'\}$  from the top  $\gamma$  fraction of the sequence of  $E_D \setminus H^k$ ;  
12           Update  $H^k \leftarrow H^k \cup \{e'\}$ ,  $\tau^k \leftarrow \tau^k + l_{i_e', e'}^1$ ,  $\delta^k \leftarrow \delta^k + q_{e'}$ ,  $\sigma_{e'} \leftarrow \sigma_{e'} + 1$ ,  $\mathcal{V}_{H^k} \leftarrow \mathcal{V}_{H^k} \cup \{i'\} \cup \{j'\}$ ,  
13              $\bar{\psi}^k \leftarrow \bar{\psi}^k \cup (e', L_{i_e', e'})$ ;  
14             SolutionChange  $\leftarrow$  True;  
15     until  $\mathcal{W} = \emptyset$ ;  
16     if SolutionChange  $\leftarrow$  True then  
17       Solve a URPP of  $H^k$  with  $\bar{\psi}^k$  as initial solution, and let  $\psi^k = (H^k, L^k)$  be a feasible solution obtained from  
18       the URPP and  $T^*(H^k)$  the corresponding objective value;  
19        $\tau^k \leftarrow T^*(H^k)$ ;  
20       for  $e \in L^k$  do // Improvement Heuristic  
21         if  $e = \{i, j\} \in E_D \setminus H^k$  and  $\delta^k + q_e \leq Q$  and  $\sigma_e + 1 \leq u_e$  then  $H^k \leftarrow H^k \cup \{e\}$ ,  $L^k \leftarrow L^k \setminus \{e\}$ ,  
22          $\psi^k \leftarrow (H^k, L^k)$ ,  $\delta^k \leftarrow \delta^k + q_e$ ,  $\sigma_e \leftarrow \sigma_e + 1$ ,  $\mathcal{V}_{H^k} \leftarrow \mathcal{V}_{H^k} \cup \{i\} \cup \{j\}$ ;  
23     until SolutionChange = False;  
24    $\psi = \psi \cup \{\psi^k\}$ ;  
25 Return the candidate UTOARP solution  $\psi$ .
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Instance	$ K = 2$				$ K = 3$				$ K = 4$									
	LBBB		B&P		LBBB		B&P		LBBB		B&P							
	LB	UB	Gap (%)	Time (s)	LB	UB	Gap (%)	Time (s)	LB	UB	Gap (%)	Time (s)						
val1A	175	175	0	1.99	175	0	251	251	0	2.45	251	0	323	323	0	3.82	323	0
val1B	175	175	0	1.69	175	0	252	252	0	2.29	252	0	324	324	0	3.6	324	0
val1C	122	122	0	1.87	122	0	179	179	0	1.94	179	0	233	233	0	3.23	233	0
val2A	125	125	0	0.22	125	0	168	168	0	0.28	168	0	183	183	0	0.27	183	0
val2B	125	125	0	0.25	125	0	168	168	0	0.29	168	0	183	183	0	0.31	183	0
val2C	85	85	0	0.3	85	0	115	115	0	0.28	115	0	125	125	0	0.24	125	0
val3A	143	143	0	0.06	143	-	196	196	0	0.07	196	-	236	236	0	0.16	236	-
val3B	133	133	0	0.11	133	-	182	182	0	0.12	182	-	222	222	0	0.17	222	-
val3C	92	92	0	0.06	92	-	127	127	0	0.07	127	-	154	154	0	0.21	154	-
val4A	165	165	0	3.23	165	0	240	240	0	3.54	240	0	308	308	0	4.86	308	0
val4B	166	166	0	3.32	166	0	241	241	0	3.51	241	0	309	309	0	4.34	309	0
val4C	155	155	0	3.32	155	0	226	226	0	3.77	226	0	291	291	0	4.26	291	0
val4D	125	125	0	3.43	125	0	182	182	0	3.5	182	0	235	235	0	4.06	235	0
val5A	166	166	0	1.68	166	0	227	227	0	1.9	227	0	285	285	0	2.27	285	0
val5B	166	166	0	1.69	166	0	227	227	0	1.93	227	0	285	285	0	2.27	285	0
val5C	166	166	0	1.76	166	0	227	227	0	1.84	227	0	285	285	0	2.29	285	0
val5D	133	133	0	1.78	133	0	183	183	0	1.74	183	0	229	229	0	2.16	229	0
val6A	208	208	0	2.48	208	0	286	286	0	3.94	286	0	355	355	0	6.17	355	0
val6B	208	208	0	2.65	208	0	286	286	0	4.01	286	0	355	355	0	6.2	355	0
val6C	137	137	0	2.48	137	0	192	192	0	3.43	192	0	244	244	0	5.98	244	0
val7A	181	181	0	10.89	181	0	263	263	0	11.85	263	0	342	342	0	17.62	342	0
val7B	181	181	0	10.14	181	0	263	263	0	12.56	263	0	344	344	0	15.36	344	0
val7C	154	154	0	10.05	154	0	223	223	0	12.66	223	0	291	291	0	34.03	291	0
val8A	163	163	0	2.17	163	0	233	233	0	2.83	233	0	300	300	0	3.7	300	0
val8B	164	164	0	2.48	164	0	234	234	0	2.79	234	0	301	301	0	3.35	301	0
val8C	129	129	0	2.21	129	0	186	186	0	2.67	186	0	240	240	0	3.06	240	0
val9A	176	176	0	11.05	176	-	257	257	0	14.94	253	-	333	333	0	25.56	330	-
val9B	175	175	0	16.16	175	-	256	256	0	20.8	253	-	332	332	0	22.09	329	-
val9C	176	176	0	11.07	176	-	257	257	0	15.07	253	-	333	333	0	25.26	330	-
val9D	144	144	0	9.83	144	-	210	210	0	17.14	208	-	273	273	0	28.72	273	0
val10A	170	170	0	13.63	170	0	244	244	0	19.89	244	0	317	317	0	49.56	317	0
val10B	171	171	0	14.54	171	0	245	245	0	23.31	245	0	318	318	0	41.45	318	0
val10C	169	169	0	13.77	169	0	243	243	0	24.38	243	0	316	316	0	34.95	316	0
val10D	154	154	0	14.13	154	0	221	221	0	22.4	221	0	288	288	0	44.06	288	0

Table 1: The First Scenario: Performance of Strengthened LBBB and B&P

weights w_e for unserved edge $e \in E_D \setminus H^k$. Let i_e be the corresponding vertex in V_{H^k} that achieves the maximum feasible weight $\frac{r_e}{l_{ie}}$ of edge e , i.e., $i_e \in \arg \max_{i \in V_{H^k}} \{\frac{r_e}{l_{ie}} | \tau^k + l_{ie}^1 \leq T_{\max}, \delta^k + q_e \leq Q, \sigma_e + 1 \leq u_e\}$. Let $\tilde{\psi}^k = (H^k, \bar{L}^k)$ be a trivial feasible solution for the URPP of H^k , constructed from the subtour (e, L_{ie}) , where edge e is served from vertex i and L_{ie} is the set of edges deadheaded to serve edge e from vertex i . Let γ be the fraction of candidates kept in the candidate set, which is set to 0.25 in our tests.

For each tour $k \in K$, we initialize the vertex set V_{H^k} with only the depot and choose the candidates from all unserved demand edges of this vehicle $e \in E_D \setminus H^k$. For each such edge e , we calculate the weight w_e as defined above, and add it to the candidate set \mathcal{W} and record the corresponding vertex i_e . We then randomly choose the next demand edge $e' \in E_D \setminus H^k$ to visit from the candidate set \mathcal{W} based on the fraction γ , and update all related sets $H^k, \tau^k, V_{H^k}, \delta^k, \sigma$. A feasible solution $\tilde{\psi}^k$ of tour k is constructed by inserting the chosen demand edge e' into the tour at vertex $i_{e'}$, along with the deadhead edges $L_{i_e, e'}$ (i.e., $\tilde{\psi}^k = \tilde{\psi}^k \cup (e', L_{i_e, e'})$). We repeat this procedure until no candidate is found. For the required-edge setting, the above process applies only to required edges E_R in order to increase the likelihood of generating feasible solutions. We then solve a URPP for H^k with $\tilde{\psi}^k$ as an initial feasible solution, which enables us to find the minimum travel time $T^*(H^k)$, and hence gives us the chance to continue the previous search procedure. The feasible solution ψ^k found by the URPP will be improved by seeking edges satisfying the two properties: (i) they are deadheaded but not served in the current tour; (ii) serving them in the current tour does not exceed the capacity limit Q and the upper bound u_e . This improvement step, again, applies only to required edges in the required-edge setting. The algorithm stops when no candidate is found after solving the URPP.

C Comparison to B&P for the Capacitated-Vehicle Setting

In Tables 1 and 2, we report the computational results of the strengthened LBBB approach without an initial feasible solution and the B&P approach from Archetti et al. (2010) for the first and second parameter scenarios, respectively. The B&P approach was tested on an Intel Pentium 4 CPU 1.60 GHz and 256 MB RAM. The code was written in C++ and the exact solver used is CPLEX 9.0. A time limit of 3600 seconds was imposed. For each instance (column *Instance*) and each number of vehicles ($|K| = \{2, 3, 4\}$), we present the lower bound (column *LB*), upper bound (column *UB*), gap in percentage at the termination (column *Gap (%)*), runtime in seconds (column *Time (s)*) for the strengthened LBBB approach, and the lower bound (column *LB*) and the gap in percentage at the termination (column *Gap (%)*) for the B&P approach. Note that in column *Time (s)*, “TL” indicates that LBBB reached the time limit, whereas in column *Gap (%)* for B&P “-” indicates that no valid upper bound was obtained for the corresponding instance.

Instance	K = 2						K = 3						K = 4					
	LBB			B&P			LBB			B&P			LBB			B&P		
	LB	UB	Gap (%)	Time (s)	LB	Gap (%)	LB	UB	Gap (%)	Time (s)	LB	Gap (%)	LB	UB	Gap (%)	Time (s)	LB	Gap (%)
val1A	713	728.00	2.06	TL	713	-	728	728.00	0.00	4.94	728	-	728	728.00	0.00	1.42	728	0
val1B	516	516.00	0.00	582.77	516	1.2	682	682.00	0.00	1255.86	682	2.62	731	731.00	0.00	189.13	731	0
val1C	179	179.00	0.00	2.78	179	0	256	256.00	0.00	6.26	256	0	328	328.00	0.00	18.42	328	0
val2A	585	585.00	0.00	301.12	585	-	605	605.00	0.00	5.88	605	0	605	605.00	0.00	4.36	605	0
val2B	409	409.00	0.00	42.04	409	0	481	481.00	0.00	25.84	481	0	541	541.00	0.00	28.78	541	0
val2C	146	146.00	0.00	2.50	146	0	209	209.00	0.00	2.99	209	0	261	261.00	0.00	4.62	261	0
val3A	245	245.00	0.00	710.04	245	-	253	253.00	0.00	2.59	253	0	253	253.00	0.00	2.40	253	0
val3B	186	186.00	0.00	52.08	186	-	227	227.00	0.00	38.23	227	0	237	237.00	0.00	15.30	237	0
val3C	65	65.00	0.00	1.07	65	0	92	92.00	0.00	1.73	92	0	116	116.00	0.00	2.00	116	0
val4A	915	1020.00	10.29	TL	923	-	1152	1242.00	7.25	TL	1211	-	1242	1242.00	0.00	37.59	1242	-
val4B	708	802.08	11.73	TL	718	-	972	1115.00	12.83	TL	978	-	1098	1248.00	12.02	TL	1195	-
val4C	556	581.20	4.34	TL	566	-	785	837.15	6.23	TL	798	-	938	1056.75	11.24	TL	964	-
val4D	300	300.00	0.00	23.52	300	-	432	432.00	0.00	88.11	426	-	555	555.00	0.00	203.09	547	-
val5A	909	1013.43	10.30	TL	948	-	1151	1221.00	5.73	TL	1191	-	1221	1221.00	0.00	66.74	1221	-
val5B	708	811.13	12.71	TL	725	-	929	1092.00	14.93	TL	978	-	1074	1221.00	12.04	TL	1137	-
val5C	585	597.26	2.05	TL	592	-	772	905.84	14.78	TL	773	-	888	1117.86	20.56	TL	925	-
val5D	326	326.00	0.00	34.27	326	-	465	465.00	0.00	290.48	464	-	585	599.89	2.48	TL	588	-
val6A	677	748.00	9.49	TL	700	-	850	891.00	4.60	TL	871	-	891	891.00	0.00	191.89	891	-
val6B	508	508.00	0.00	3081.48	504	-	684	737.80	7.29	TL	684	-	786	861.00	8.71	TL	804	3.41
val6C	207	207.00	0.00	9.52	207	0	291	291.00	0.00	22.35	291	0.82	371	371.00	0.00	68.41	371	0.64
val7A	800	922.50	13.28	TL	848	-	1000	1110.00	9.91	TL	1081	-	1103	1110.00	0.63	TL	1110	-
val7B	621	722.47	14.04	TL	633	-	842	1000.00	15.80	TL	911	-	1019	1118.00	8.86	TL	1082	-
val7C	286	286.00	0.00	43.32	286	0	418	418.00	0.00	71.77	418	0	541	541.00	0.00	649.35	541	0
val8A	831	889.00	6.52	TL	856	-	1033	1099.00	6.01	TL	1074	-	1099	1099.00	0.00	61.03	1099	-
val8B	636	718.25	11.45	TL	665	-	836	966.71	13.52	TL	893	-	1012	1103.00	8.25	TL	1058	-
val8C	270	270.00	0.00	30.29	270	-	389	389.00	0.00	360.65	389	-	493	503.80	2.14	TL	496	-
val9A	922	1044.33	11.71	TL	996	-	1124	1265.00	11.15	TL	1229	-	1265	1265.00	0.00	864.14	1265	-
val9B	742	832.00	10.82	TL	774	-	974	1122.22	13.21	TL	1033	-	1077	1262.00	14.66	TL	1216	-
val9C	608	686.88	11.48	TL	627	-	824	961.33	14.29	TL	876	-	976	1171.67	16.70	TL	1066	-
val9D	298	298.00	0.00	1032.40	296	-	437	437.00	0.00	3007.84	426	-	554	571.25	3.02	TL	555	-
val10A	980	1111.00	11.79	TL	1038	-	1270	1348.00	5.79	TL	1302	-	1348	1348.00	0.00	350.93	1348	-
val10B	803	917.00	12.43	TL	829	-	1032	1209.30	14.66	TL	1104	-	1212	1349.00	10.16	TL	1307	-
val10C	655	760.54	13.88	TL	658	-	848	1033.00	17.91	TL	912	-	1043	1244.00	16.16	TL	1129	-
val10D	327	327.00	0.00	389.94	322	-	465	478.91	2.90	TL	458	-	572	645.82	11.43	TL	591	-

Table 2: The Second Scenario: Performance of Strengthened LBB and B&P

References

C. Archetti, D. Feillet, A. Hertz, and M. G. Speranza. The undirected capacitated arc routing problem with profits. *Computers & Operations Research*, 37(11):1860–1869, 2010.