

Appendices

EC.1. Appendix A: Proofs

EC.1.1. Proof of Proposition 1

We prove Proposition 1 by construction. If the system (8)-(12) owns a flow solution $\mathbf{F}$, then we assign values to L_{ij} and T satisfying the following conditions:

$$L_{ij} = \left\lfloor \frac{F_{ij}T}{n_{ij}^1} \right\rfloor + 1 \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.1})$$

$$L_{ij}h_{\min} + h_{\min}^s \leq T \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.2})$$

$$(L_{ij_1} + L_{kj_2})h_{\min} + 2h_{\min}^s \leq T \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.3})$$

The above value assignment is guaranteed to have a feasible solution due to the following reasons. By Eqs.(10)-(11), we change the “<” operators to “ $\leq$ ” operators and introduce a factor $1 - \tilde{\rho}$ to the right-hand-side, where $\tilde{\rho} \in (0, 1)$. Then we have:

$$\begin{aligned} L_{ij}h_{\min} + h_{\min}^s &= \left(\left\lfloor \frac{F_{ij}T}{n_{ij}^1} \right\rfloor + 1 \right) h_{\min} + h_{\min}^s \\ &\leq \left(\left\lfloor \frac{(1 - \tilde{\rho})T}{h_{\min}} \right\rfloor + 1 \right) h_{\min} + h_{\min}^s \leq (1 - \tilde{\rho})T + h_{\min} + h_{\min}^s \end{aligned} \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.4})$$

$$\begin{aligned} (L_{ij_1} + L_{kj_2})h_{\min} + 2h_{\min}^s &= \left(\left\lfloor \frac{F_{ij_1}T}{n_{ij_1}^1} \right\rfloor + \left\lfloor \frac{F_{kj_2}T}{n_{kj_2}^1} \right\rfloor + 2 \right) h_{\min} + 2h_{\min}^s \\ &\leq \left(\frac{F_{ij_1}T}{n_{ij_1}^1} + \frac{F_{kj_2}T}{n_{kj_2}^1} + 2 \right) h_{\min} + 2h_{\min}^s \\ &\leq (1 - \tilde{\rho})T + 2h_{\min} + 2h_{\min}^s \end{aligned} \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.5})$$

Thus, when T is sufficiently large, as long as $T \geq \frac{2h_{\min} + 2h_{\min}^s}{\tilde{\rho}}$, the value assignment satisfies Eqs.(EC.1)-(EC.3).

Now, with the value assignment of L_{ij} and T , we show that there always exists R_{ij} satisfying the conflict avoidance constraints (6)-(7). Without loss of generality, assume $R_{j_2} \geq R_{j_1}$ (the symmetric case follows analogously). A feasible assignment can be obtained by setting:

$$R_{j_1} = 0, \quad R_{j_2} = L_{ij_1}h_{\min} + h_{\min}^s \quad \forall (i, j_1) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.6})$$

This construction guarantees that $R_{j_2} - R_{j_1}$ satisfies Eq. (6), and together with condition (EC.3) ensures that Eq. (7) is also satisfied. Hence, the feasibility of $\mathbf{R}$ is always guaranteed once T and $\mathbf{L}$ satisfy the stated conditions.

To summarize, we have now constructed a feasible rhythm design that can accommodate the given demand, and the proof is then completed. $\square$

EC.1.2. Proof of Proposition 2

We prove Proposition 2 by construction. Given $T_{\max} = mh_{\min} + 2h_{\min}^s, m \in \mathbb{N}$, if the system (13)-(17) owns a flow solution $\mathbf{F}$, then we assign values to L_{ij} satisfying the following conditions:

$$L_{ij} = \left\lceil \frac{F_{ij} T_{\max}}{n_{ij}^1} \right\rceil \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.7})$$

Then, for Eq.(15), we have:

$$\begin{aligned} L_{ij} h_{\min} + h_{\min}^s &= \left\lceil \frac{F_{ij} T_{\max}}{n_{ij}^1} \right\rceil h_{\min} + h_{\min}^s \\ &\leq \left\lceil \frac{(m + \lfloor h_{\min}^s / h_{\min} \rfloor) T_{\max}}{mh_{\min} + 2h_{\min}^s} \right\rceil h_{\min} + h_{\min}^s \\ &= (m + \lfloor h_{\min}^s / h_{\min} \rfloor) h_{\min} + h_{\min}^s \\ &\leq mh_{\min} + 2h_{\min}^s = T_{\max} \quad \forall (i, j) \in \mathcal{L} \end{aligned} \quad (\text{EC.8})$$

suggesting that Eq.(5) holds. On the other hand, for Eq.(16), we have:

$$\begin{aligned} (L_{ij_1} + L_{kj_2}) h_{\min} + 2h_{\min}^s &= \left(\left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} \right\rceil + \left\lceil \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil \right) h_{\min} + 2h_{\min}^s \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \end{aligned} \quad (\text{EC.9})$$

Denote $\phi_1 = \left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} \right\rceil - \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1}$, $\phi_2 = \left\lceil \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil - \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1}$. Notice the following facts:

$$\begin{aligned} \left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} + \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil &= \left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} \right\rceil + \left\lceil \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil & \text{if } 0 \leq \phi_1 + \phi_2 < 1 \\ \left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} + \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil &= \left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} \right\rceil + \left\lceil \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil - 1 & \text{if } 1 \leq \phi_1 + \phi_2 < 2 \end{aligned}$$

Thus we further yield:

$$\begin{aligned} (L_{ij_1} + L_{kj_2}) h_{\min} + 2h_{\min}^s &\leq \left(\left\lceil \frac{F_{ij_1} T_{\max}}{n_{ij_1}^1} + \frac{F_{kj_2} T_{\max}}{n_{kj_2}^1} \right\rceil + 1 \right) h_{\min} + 2h_{\min}^s \\ &\leq mh_{\min} + 2h_{\min}^s = T_{\max} \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \end{aligned} \quad (\text{EC.10})$$

Thus, we have shown that the value assignment (EC.7) satisfies Eqs.(15)-(16). Following similar logic as in the proof of Proposition 1, we thus completed the proof. $\square$

EC.1.3. Proof of Lemma 1

When $\mathbf{d} \notin \text{Int}\mathcal{D}$, for any flow pattern $\mathbf{F}$ satisfying Eqs.(8)-(9) and Eq.(12), there must be $(i, j) \in \mathcal{L}$ that $\frac{F_{ij}}{n_{ij}^1} \geq \frac{1}{h_{\min}}$, or $(i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C}$ such that $\frac{F_{ij_1}}{n_{ij_1}^1} + \frac{F_{kj_2}}{n_{kj_2}^1} \geq \frac{1}{h_{\min}}$. For the first case, the arrival rate of link (i, j) is F_{ij} , and if the maximum throughput rate is $\frac{n_{ij}^1}{h_{\min}} < F_{ij}$, then the number of vehicles waiting to pass the link is expected to approach infinity as $t \rightarrow +\infty$, so:

$$\lim_{m \rightarrow +\infty} \frac{1}{m} \sum_{t=1}^m \mathbb{E}[N(t)] = +\infty$$

By Definition 2, no matter what control is adopted, the network traffic cannot be stabilized. The relative tricky point lies in that when $\frac{n_{ij}^l}{h_{\min}} = F_{ij}$; for this scenario, the link becomes a queuing system with its service capability equal to the demand rate. From related queuing theory (e.g., M/D/1 queuing theory), the expected number of entities in the system for such case is infinity, also implying that the network traffic cannot be stabilized.

Similar reasoning can be applied to the second case. Thus, if $\mathbf{d} \notin \text{Int}\mathcal{D}$, no control can stabilize $\mathbf{d}$. Proof completes. $\square$

EC.1.4. Proof of Lemma 2

Eqs.(19)-(21) suggest that for every path $r \in \mathcal{P}^w$, $w \in \mathcal{W}$, there is “reserved room” on every VC it passes through for the vehicles assigned on it in each cycle; such reservation is not affected by vehicles on other paths. Thus, denote $N_r(t)$ as the number of vehicles assigned to path r within the network, we only need to prove that:

$$\frac{1}{m} \sum_{t=1}^m \mathbb{E}[N_r(t)] < +\infty \quad \forall r \in \mathcal{P}^w, w \in \mathcal{W}, m = 1, 2, \dots \quad (\text{EC.11})$$

By the logic of RC, once a vehicle assigned to path r is allowed to join a VC, the duration it will stay in the network becomes known. Since the reserved room in each cycle L_r is finite, the number of vehicles in VC (i.e., allowed to leave from the origin) is finite. Then Eq.(EC.11) is equivalent to:

$$\frac{1}{m} \sum_{t=1}^m \mathbb{E}[\hat{N}_r(t)] < +\infty \quad \forall r \in \mathcal{P}^w, w \in \mathcal{W}, m = 1, 2, \dots \quad (\text{EC.12})$$

where $\hat{N}_r(t)$ is the number of vehicles assigned on path r that are waiting in queue at the origin node. Thus we yield the following updating equation for $\hat{N}_r(t)$:

$$\hat{N}_r(t+1) = \hat{N}_r(t) + \lambda_r(t+1) - \mu_r(t) \quad \forall t = 1, 2, \dots \quad (\text{EC.13})$$

where $\lambda_r(t)$ is the number of vehicles entering the network at the beginning of time t (which is i.i.d. generated), and $\mu_r(t)$ is the number of vehicles allowed to join a VC at t ; $\mu_r(t) = \min(L_r, N_r(t))$. Then we have:

$$[\hat{N}_r(t+1)]^2 - [\hat{N}_r(t)]^2 = [\lambda_r(t+1) - \mu_r(t)]^2 + 2[\lambda_r(t+1) - \mu_r(t)]\hat{N}_r(t) \quad (\text{EC.14})$$

And further:

$$\mathbb{E} \left\{ [\hat{N}_r(t+1)]^2 - [\hat{N}_r(t)]^2 | \hat{N}_r(t) \right\} = \mathbb{E}[\lambda_r(t)]^2 - 2\mu_r(t)\mathbb{E}[\lambda_r(t)] + [\mu_r(t)]^2 + 2\{\mathbb{E}[\lambda_r(t)] - \mu_r(t)\}\hat{N}_r(t) \quad (\text{EC.15})$$

Since $\mu_r(t) \leq L_r$ and both $\mathbb{E}[\lambda_r(t)]$ and $\mathbb{E}[\lambda_r(t)]^2$ are finite, we have $\mathbb{E}[\lambda_r(t)]^2 - 2\mu_r(t)\mathbb{E}[\lambda_r(t)] + [\mu_r(t)]^2 \leq K_0$, where K_0 is a constant. On the other hand, when $\hat{N}_r(t) \leq L_r$, we have:

$$2\{\mathbb{E}[\lambda_r(t)] - \mu_r(t)\}\hat{N}_r(t) \leq 2\mathbb{E}[\lambda_r(t)]L_r = 2F_r L_r T \quad (\text{EC.16})$$

When $\hat{N}_r(t) > L_r$, we have:

$$2\{\mathbb{E}[\lambda_r(t)] - \mu_r(t)\}\hat{N}_r(t) = 2\{\mathbb{E}[\lambda_r(t)] - L_r\}\hat{N}_r(t) = 2(F_r T - L_r)\hat{N}_r(t) \quad (\text{EC.17})$$

Denote $-\kappa = F_r T - L_r < 0$, and combining Eqs.(EC.15)-(EC.17) we yield:

$$\mathbb{E} \left\{ [\hat{N}_r(t+1)]^2 - [\hat{N}_r(t)]^2 | \hat{N}_r(t) \right\} \leq K_0 + 2F_r L_r T + \kappa (L_r - \hat{N}_r(t)) \quad (\text{EC.18})$$

Note that the condition $L_r > F_r T$ must be strict. If $L_r = F_r T$, then $\kappa = 0$ and the last term in Eq.(EC.18) vanishes, eliminating the negative expected growth effect when the queue becomes large. From a queueing perspective, stability is not guaranteed in this case, as the expected number of entities in the system grows without bound.

Now, by taking the expectation to Eq.(EC.18) and summing over $t = 0, 1, \dots, M$, we have:

$$\mathbb{E}[\hat{N}_r(m+1)]^2 - \mathbb{E}[\hat{N}_r(1)]^2 \leq M(K_0 + 2F_r L_r T + \kappa L_r) - \kappa \sum_{t=1}^M \mathbb{E}[\hat{N}_r(t)] \quad (\text{EC.19})$$

Letting $\hat{N}_r(1) = 0$, we finally yield:

$$\frac{1}{m} \sum_{t=1}^m \mathbb{E}[\hat{N}_r(t)] \leq \frac{1}{\kappa} (K_0 + 2F_r L_r T + \kappa L_r) < +\infty \quad (\text{EC.20})$$

which confirms Eq.(EC.12). Proof completes. $\square$

EC.1.5. Proof of Proposition 3

By Lemma 2, it is sufficient to prove that for any $\mathbf{d} \in \text{Int}\mathcal{D}$, we can always design a rhythm satisfying Eqs.(19)-(21) to accommodate it. Similar to the proof of Proposition 1, we assign values to L_r and T satisfying the following conditions:

$$L_r = \lfloor F_r T \rfloor + k_r \quad \forall r \in \mathcal{P}^w, w \in \mathcal{W} \quad (\text{EC.21})$$

$$L_{ij} n_{ij}^1 = \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} L_r \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.22})$$

$$L_{ij} h_{\min} + h_{\min}^s \leq T \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.23})$$

$$(L_{ij_1} + L_{kj_2}) h_{\min} + 2h_{\min}^s \leq T \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.24})$$

$$k_r \in \mathbb{N}^+ \quad \forall r \in \mathcal{P}^w, w \in \mathcal{W} \quad (\text{EC.25})$$

Then the remaining task is to show that there always exists a feasible (L_r, k_r, T) that makes Eqs.(EC.21)-(EC.25) hold. Similar to the proof of Proposition 1, $\mathbf{d} \in \text{Int}\mathcal{D}$ suggests that Eqs.(8)-(12) holds, and we can introduce a factor $\tilde{\rho} \in (0, 1)$ to replace the “<” operators with “ $\leq$ ” operators. Then by Eqs.(EC.21)-(EC.22) we have:

$$\begin{aligned} L_{ij} h_{\min} + h_{\min}^s &= \frac{1}{n_{ij}^1} \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} (\lfloor F_r T \rfloor + k_r) h_{\min} + h_{\min}^s \\ &\leq \frac{1}{n_{ij}^1} (F_{ij} T + k_{ij}) h_{\min} + h_{\min}^s \leq (1 - \tilde{\rho})T + \frac{k_{ij} h_{\min}}{n_{ij}^1} + h_{\min}^s \end{aligned} \quad (\text{EC.26})$$

$$\begin{aligned} (L_{ij_1} + L_{kj_2}) h_{\min} + 2h_{\min}^s &= \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \left[\frac{\delta_{ij_1 r} (\lfloor F_r T \rfloor + k_r)}{n_{ij_1}^1} + \frac{\delta_{kj_2 r} (\lfloor F_r T \rfloor + k_r)}{n_{kj_2}^1} \right] h_{\min} + 2h_{\min}^s \\ &\leq \left[\frac{F_{ij_1} T + k_{ij_1}}{n_{ij_1}^1} + \frac{F_{kj_2} T + k_{kj_2}}{n_{kj_2}^1} \right] h_{\min} + 2h_{\min}^s \\ &\leq (1 - \tilde{\rho})T + \left(\frac{k_{ij_1}}{n_{ij_1}^1} + \frac{k_{kj_2}}{n_{kj_2}^1} \right) h_{\min} + 2h_{\min}^s \end{aligned} \quad (\text{EC.27})$$

where $k_{ij} = \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} k_r$. Thus, as long as $T \geq \frac{1}{\rho} \left[\left(\frac{k_{ij1}}{n_{ij1}^1} + \frac{k_{ij2}}{n_{ij2}^1} \right) h_{\min} + 2h_{\min}^s \right]$, (EC.23)-(EC.24) is straightforwardly guaranteed. Therefore, there always exists a desirable (L_r, k_r, T) . The remaining part of the proof is the same as that of Proposition 1, which we omit for brevity.

After the above construction of rhythm design, we successfully identify a rhythm design that accommodates $\mathbf{d}$ and meanwhile satisfies Eqs.(19)-(21). By Lemma 2, we then conclude that RC can stabilize any $\mathbf{d} \in \text{Int}\mathcal{D}$. Proof completes. $\square$

EC.1.6. Proof of Eq. (22)

From Proposition 3 we know that the set of demands that can be stabilized by RC with knotted intersections can be characterized by Eqs.(8)-(12). On the other hand, for conventional signal control with the NEMA setting, the set of stabilizable demands can be characterized as follows:

$$d^w = \sum_{r \in \mathcal{P}^w} F_r \quad \forall w \in \mathcal{W} \quad (\text{EC.28})$$

$$F_{ij} = \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} F_r \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.29})$$

$$\frac{F_{ij}}{n_{ij}^1} < \frac{1}{h_{\min}} \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.30})$$

$$\sum_{q=1,2,3,4} \max \left\{ \frac{F_{q,1}^k}{n_{q,1}^{1,k}}, \frac{F_{q,2}^k}{n_{q,2}^{1,k}} \right\} < \frac{1}{h_{\min}} \quad \forall k \in \mathcal{I} \quad (\text{EC.31})$$

$$F_r \geq 0 \quad \forall w \in \mathcal{W}, r \in \mathcal{P}^w \quad (\text{EC.32})$$

Here, $F_{q,r}^k$ represents the traffic flow rate associated with the r^{th} movement in the q^{th} phase of intersection $k \in \mathcal{I}$, and $n_{q,r}^{1,k}$ is the associated number of lanes. Eq.(EC.31) suggests that the sum of maximum single-lane flow rates of all four signal phases should be less than the saturation flow rate, i.e., $1/h_{\min}$. With the above preparation, we can now proceed our proof as follows.

1) $\frac{\gamma_{\text{II}}}{\gamma_{\text{I}}} \geq 1$. Proving this result is equivalent to showing that any demand $\mathbf{d}$ that can be stabilized by the signal control can also be stabilized by RC with knotted intersections. We note that for Eq.(11), any conflicting movement pair is associated with such a constraint, and meanwhile the two movements in any conflicting movement pair cannot be assigned with the same phase in signal control. Therefore, any conflicting movement pair satisfying Eq.(EC.31) must also satisfy Eq.(11), which leads to our desired result. The bound cannot be further increased because when we consider an isolated intersection with only two conflicting directions owning positive traffic flows, then $\frac{\gamma_{\text{II}}}{\gamma_{\text{I}}}$ can be arbitrarily close to 1 in this scenario.

2) $\frac{\gamma_{\text{II}}}{\gamma_{\text{I}}} \leq 2$. Proving this result is equivalent to showing that if a demand $\mathbf{d}$ can be stabilized by RC with knotted intersections, then $\frac{1}{2}\mathbf{d}$ can be stabilized by the signal control. We note the following inequality:

$$\sum_{q=1,2,3,4} \max \left\{ \frac{F_{q,1}^k}{n_{q,1}^{1,k}}, \frac{F_{q,2}^k}{n_{q,2}^{1,k}} \right\} \leq 2 \left(\frac{F_X^k}{n_X^{1,k}} + \frac{F_Y^k}{n_Y^{1,k}} \right)$$

Here, the subscript X indicates the movement whose single-lane flow rate is the largest among all movements, and subscript Y indicates the movement belonging to the phase with the second largest single-lane

flow rate, and whose single-lane flow rate is larger than its counterpart in the same phase. Since the demand $\mathbf{d}$ can be stabilized by RC with knotted intersections, then we have:

$$\sum_{q=1,2,3,4} \frac{1}{2} \max \left\{ \frac{F_{q,1}^k}{n_{q,1}^{1,k}}, \frac{F_{q,2}^k}{n_{q,2}^{1,k}} \right\} \leq \frac{F_X^k}{n_X^{1,k}} + \frac{F_Y^k}{n_Y^{1,k}} < \frac{1}{h_{\min}} \quad \forall k \in \mathcal{I}$$

$$\frac{F_{ij}}{2n_{ij}^1} < \frac{1}{2h_{\min}} < \frac{1}{h_{\min}} \quad \forall (i,j) \in \mathcal{L}$$

Thus $\frac{1}{2}\mathbf{d}$ can be stabilized by the signal control. The bound cannot be further reduced because when all directions in this intersection owning the same magnitude of single-lane traffic flow, then all the “ $\leq$ ” operators above become “ $=$ ” operator, suggesting that in this case $\frac{\gamma_{\mathbf{u}}}{\gamma_{\mathbf{l}}}$ can be arbitrarily close to 2.

Summarize the above two cases, we then completed the proof. $\square$

EC.1.7. Proof of Proposition 5: One-way Network Case

For simplification, in the proof we adopt a queuing network to represent the whole network. Each temporary waiting zone and each entrance is associated with a queue. The stability analysis is carried out with respect to these queues, as the VC capacity on links acts as service constraints that limit the number of vehicles released onto each link, and link storage dynamics do not need to be explicitly modeled. Vehicles leaving one queue will join the next queue in the next time slot, so the driving time between consecutive temporary waiting zones is ignored. We denote $a \in \widehat{\mathcal{U}}_b$ if a is an upstream link of b and $a \in \widehat{\mathcal{V}}_b$ if a is a downstream link of b , and denote $F_{a \rightarrow b}$ as the traffic demand assigned on the sub-path constituting the consecutive link pair (a, b) in one cycle, and $r_{a \rightarrow b}$ is the associated routing proportions; the above two terms are treated as positive (otherwise we can delete link b in the proof). Also, we assume all merging and diverging are with two branches for conveniences, and generalizing the proof to cases with more branches is easy. The whole proof is divided into the following parts.

Part I

We first write down the queue updating equations of the whole network. Denote $N_{a \rightarrow b}(t)$ as the number of vehicles on link a at time slot t that head to link b . Then, for links a, a' with $\widehat{\mathcal{V}}_a = \widehat{\mathcal{V}}_{a'} = b$ (i.e., links a, a' form a merging structure), the associated queue updating equations are given by:

$$N_{a \rightarrow b}(t+1) = N_{a \rightarrow b}(t) - A_{a \rightarrow b}(t) + \sum_{k \in \widehat{\mathcal{U}}_a} A_{k \rightarrow a}(t) \quad (\text{EC.33})$$

$$N_{a' \rightarrow b}(t+1) = N_{a' \rightarrow b}(t) - A_{a' \rightarrow b}(t) + \sum_{k \in \widehat{\mathcal{U}}_{a'}} A_{k \rightarrow a'}(t) \quad (\text{EC.34})$$

where $A_{a \rightarrow b}(t)$ is the number of vehicles transferred from a to b at t , and the local control rule is given by:

$$A_{a \rightarrow b}(t) = \min[N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1]$$

$$\text{if } N_{a \rightarrow b}(t) \geq N_{a' \rightarrow b}(t) \text{ and } N_{a \rightarrow b}(t) \geq \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \quad (\text{EC.35})$$

$$A_{a \rightarrow b}(t) = \min[N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1 - A_{a' \rightarrow b}(t)]$$

$$\text{if } N_{a \rightarrow b}(t) < N_{a' \rightarrow b}(t) \text{ and } N_{a \rightarrow b}(t) \geq \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \quad (\text{EC.36})$$

$$A_{a \rightarrow b}(t) = \min [N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1, \lfloor F_{a \rightarrow b} T - \rho \rfloor]$$

$$\text{if } N_{a \rightarrow b}(t) \geq N_{a' \rightarrow b}(t) \text{ and } N_{a \rightarrow b}(t) < \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \quad (\text{EC.37})$$

$$A_{a \rightarrow b}(t) = \min [N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1 - A_{a' \rightarrow b}(t), \lfloor F_{a \rightarrow b} T - \rho \rfloor]$$

$$\text{if } N_{a \rightarrow b}(t) < N_{a' \rightarrow b}(t) \text{ and } N_{a \rightarrow b}(t) < \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \quad (\text{EC.38})$$

where $L_a n_a^1$ means the maximum number of vehicles allowed in the container on link a as in the main body, and ρ is a small positive number. $A_{a' \rightarrow b}(t)$ is symmetric to $A_{a \rightarrow b}(t)$. On the other hand, for links a, a' with $\widehat{\mathcal{U}}_a = \widehat{\mathcal{U}}_{a'} = b$ (i.e., a, a' form a diverging structure), the associated queue updating equations are given by:

$$N_{b \rightarrow a}(t+1) = N_{b \rightarrow a}(t) - A_{b \rightarrow a}(t) + \sum_{i=1}^{L_b n_b^1} X_{b,i}(t) Z_{b,i}(t) \quad (\text{EC.39})$$

$$N_{b \rightarrow a'}(t+1) = N_{b \rightarrow a'}(t) - A_{b \rightarrow a'}(t) + \sum_{i=1}^{L_b n_b^1} X_{b,i}(t) (1 - Z_{b,i}(t)) \quad (\text{EC.40})$$

where:

$$A_{b \rightarrow a}(t) = \min [N_{b \rightarrow a}(t), L_b n_b^1, L_a n_a^1]$$

$$\text{if } N_{b \rightarrow a}(t) \geq N_{b \rightarrow a'}(t) \text{ and } N_{b \rightarrow a}(t) \geq \sum_{k \in \widehat{\mathcal{V}}_a} r_{a \rightarrow k} N_{a \rightarrow k}(t) \quad (\text{EC.41})$$

$$A_{b \rightarrow a}(t) = \min [N_{b \rightarrow a}(t), L_b n_b^1 - A_{b \rightarrow a'}(t), L_a n_a^1]$$

$$\text{if } N_{b \rightarrow a}(t) < N_{b \rightarrow a'}(t) \text{ and } N_{b \rightarrow a}(t) \geq \sum_{k \in \widehat{\mathcal{V}}_a} r_{a \rightarrow k} N_{a \rightarrow k}(t) \quad (\text{EC.42})$$

$$A_{b \rightarrow a}(t) = \min [N_{b \rightarrow a}(t), L_b n_b^1, L_a n_a^1, \lfloor F_{b \rightarrow a} T - \rho \rfloor]$$

$$\text{if } N_{b \rightarrow a}(t) \geq N_{b \rightarrow a'}(t) \text{ and } N_{b \rightarrow a}(t) < \sum_{k \in \widehat{\mathcal{V}}_a} r_{a \rightarrow k} N_{a \rightarrow k}(t) \quad (\text{EC.43})$$

$$A_{b \rightarrow a}(t) = \min [N_{b \rightarrow a}(t), L_b n_b^1 - A_{b \rightarrow a'}(t), L_a n_a^1, \lfloor F_{b \rightarrow a} T - \rho \rfloor]$$

$$\text{if } N_{b \rightarrow a}(t) < N_{b \rightarrow a'}(t) \text{ and } N_{b \rightarrow a}(t) < \sum_{k \in \widehat{\mathcal{V}}_a} r_{a \rightarrow k} N_{a \rightarrow k}(t) \quad (\text{EC.44})$$

$$X_{b,i}(t) = 1 \quad \text{if } \sum_{k \in \widehat{\mathcal{U}}_b} A_{k \rightarrow b}(t) \leq i \quad (\text{EC.45})$$

$$X_{b,i}(t) = 0 \quad \text{if } \sum_{k \in \widehat{\mathcal{U}}_b} A_{k \rightarrow b}(t) > i \quad (\text{EC.46})$$

and $Z_{b,i}(t) \in \{0, 1\}$ is a random variable such that $Z_{b,i}(t) = 1$ with probability equal to the associated pre-given routing proportions acquired by the rhythm design. The formulation of Eqs.(EC.39)-(EC.40) aim to capture the stochastic nature of vehicle turning at a diverging point. $A_{b \rightarrow a'}(t)$ is symmetric to $A_{b \rightarrow a}(t)$.

For entrance links the queue updating equation is:

$$N_{a \rightarrow b}(t+1) = N_{a \rightarrow b}(t) - A_{a \rightarrow b}(t) + \lambda_a(t+1) \quad (\text{EC.47})$$

where $\lambda_a(t)$ is the number of vehicles entering the network at this entrance at the beginning of time slot t (which is i.i.d. generated), and $A_{a \rightarrow b}(t) = \min [N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1]$ if $N_{a \rightarrow b}(t) \geq \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t)$, $A_{a \rightarrow b}(t) = \min [N_{a \rightarrow b}(t), L_a n_a^1, L_b n_b^1, \lfloor F_{a \rightarrow b} T - \rho \rfloor]$ if $N_{a \rightarrow b}(t) < \sum_{k \in \widehat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t)$.

Part II

Now, by aggregating all the above updating formulae, we obtain the following equation:

$$\begin{aligned}
\frac{1}{2} \left(\sum_{a \rightarrow b} [N_{a \rightarrow b}(t+1)]^2 - [N_{a \rightarrow b}(t)]^2 \right) &= \Xi(t) + \sum_{a \in \mathcal{L}_1} \left[\sum_{k \in \hat{\mathcal{U}}_a} A_{k \rightarrow a}(t) - A_{a \rightarrow b}(t) \right] N_{a \rightarrow b}(t) \\
&+ \sum_{a \in \mathcal{L}_2} \left(\left[\sum_{i=1}^{L_a n_a^1} X_{a,i}(t) Z_{a,i}(t) - A_{a \rightarrow b}(t) \right] N_{a \rightarrow b}(t) + \left[\sum_{i=1}^{L_a n_a^1} X_{a,i}(t) (1 - Z_{a,i}(t)) - A_{a \rightarrow b'}(t) \right] N_{a \rightarrow b'}(t) \right) \\
&+ \sum_{a \in \mathcal{L}_3} (\lambda_a(t+1) - A_{a \rightarrow b}(t)) N_{a \rightarrow b}(t)
\end{aligned} \tag{EC.48}$$

where $\mathcal{L}_1$ is the set of links approaching a merging point, $\mathcal{L}_2$ is the set of links approaching a diverging point, and $\mathcal{L}_3$ is the set of entry links. $\Xi(t)$ is a complicated term that does not contain $N_{a \rightarrow b}(t)$, and it is easy to show that this term can be upper bounded by a large constant K . By taking the conditional expectation of Eq.(EC.48) we yield:

$$\begin{aligned}
\frac{1}{2} \mathbb{E} \left(\sum_{a \rightarrow b} [N_{a \rightarrow b}(t+1)]^2 - [N_{a \rightarrow b}(t)]^2 \middle| \mathbf{N}(t) \right) &= \mathbb{E}(\Xi(t)) + \sum_{a \in \mathcal{L}_1} \left[\sum_{k \in \hat{\mathcal{U}}_a} A_{k \rightarrow a}(t) - A_{a \rightarrow b}(t) \right] N_{a \rightarrow b}(t) \\
&+ \sum_{a \in \mathcal{L}_2} \left(\left[r_{a \rightarrow b} \sum_{k \in \hat{\mathcal{U}}_a} A_{k \rightarrow a}(t) - A_{a \rightarrow b}(t) \right] N_{a \rightarrow b}(t) + \left[r_{a \rightarrow b'} \sum_{k \in \hat{\mathcal{U}}_a} A_{k \rightarrow a}(t) - A_{a \rightarrow b'}(t) \right] N_{a \rightarrow b'}(t) \right) \\
&+ \sum_{a \in \mathcal{L}_3} (\mathbb{E}(\lambda_a) - A_{a \rightarrow b}(t)) N_{a \rightarrow b}(t)
\end{aligned} \tag{EC.49}$$

where $r_{a \rightarrow b}$ is the pre-determined routing proportion from a to b in a diverging point. The above equality holds because $\mathbb{E}_{\mathbf{Z}} \left(\sum_{i=1}^{L_a n_a^1} X_{a,i}(t) Z_{a,i}(t) \right) = \sum_{i=1}^{L_a n_a^1} X_{a,i}(t) \mathbb{E}(Z_{a,i}(t)) = r_{a \rightarrow b} \sum_{k \in \hat{\mathcal{U}}_a} A_{k \rightarrow a}(t)$. By rewriting the above equation (EC.49), we obtain the following result:

$$\begin{aligned}
\frac{1}{2} \mathbb{E} \left(\sum_{a \rightarrow b} [N_{a \rightarrow b}(t+1)]^2 - [N_{a \rightarrow b}(t)]^2 \middle| \mathbf{N}(t) \right) \\
= \mathbb{E}(\Xi(t)) + \sum_{a \rightarrow b} A_{a \rightarrow b}(t) \left[-N_{a \rightarrow b}(t) + \sum_{k \in \hat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \right] + \sum_{a \in \mathcal{L}_3} \mathbb{E}(\lambda_a) N_{a \rightarrow b}(t)
\end{aligned} \tag{EC.50}$$

Now, denote $F_{a \rightarrow b}$ as the traffic demand assigned on the sub-path constituting the consecutive link pair (a, b) , we have:

$$\begin{aligned}
&\sum_{a \rightarrow b} F_{a \rightarrow b} T \left[-N_{a \rightarrow b}(t) + \sum_{k \in \hat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \right] + \sum_{a \in \mathcal{L}_3} \mathbb{E}(\lambda_a) N_{a \rightarrow b}(t) \\
&= \sum_{a \rightarrow b} \left[r_{a \rightarrow b} \sum_{k \in \hat{\mathcal{U}}_a} F_{k \rightarrow a} - F_{a \rightarrow b} \right] T N_{a \rightarrow b}(t) + \sum_{a \in \mathcal{L}_3} (\mathbb{E}(\lambda_a) - F_{a \rightarrow b} T) N_{a \rightarrow b}(t) = 0
\end{aligned}$$

The above equality is due to the fact that $\mathbb{E}(\lambda_a) = F_{a \rightarrow b} T$ for all $a \in \mathcal{L}_3$, and $r_{a \rightarrow b} \sum_{k \in \hat{\mathcal{U}}_a} F_{k \rightarrow a} = F_{a \rightarrow b}$ by flow conservation. Plugging the above equation into Eq.(EC.50) we obtain:

$$\begin{aligned}
\frac{1}{2} \mathbb{E} \left(\sum_{a \rightarrow b} [N_{a \rightarrow b}(t+1)]^2 - [N_{a \rightarrow b}(t)]^2 \middle| \mathbf{N}(t) \right) \\
= \mathbb{E}(\Xi(t)) + \sum_{a \rightarrow b} [F_{a \rightarrow b} T - A_{a \rightarrow b}(t)] \left[N_{a \rightarrow b}(t) - \sum_{k \in \hat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t) \right]
\end{aligned} \tag{EC.51}$$

Denote $J_{a \rightarrow b}(t) = N_{a \rightarrow b}(t) - \sum_{k \in \hat{\mathcal{V}}_b} r_{b \rightarrow k} N_{b \rightarrow k}(t)$, we have $\mathbf{J}(t) = \mathbf{U}\mathbf{N}(t)$, where $\mathbf{U}$ is a square matrix. It can be easily shown that $\mathbf{U}$ is a weakly diagonally dominant matrix, and it is invertible by the network topology.

Part III

Next, we focus on a merging point, i.e., $a \rightarrow b$ and $a' \rightarrow b$. We show that:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq -\rho_1 \sum_{x=a, a'} |J_{x \rightarrow b}(t)| + K \quad (\text{EC.52})$$

where ρ_1 is a positive number and K is a constant. Without loss of generality, we assume $N_{a \rightarrow b}(t) \geq N_{a' \rightarrow b}(t)$. We discuss it in the following cases.

Case 1: $J_{a \rightarrow b}(t) < 0$. In this case, by the control rule Eqs.(EC.33)-(EC.34) we have:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq \rho \sum_{x=a, a'} J_{x \rightarrow b}(t) = -\rho \sum_{x=a, a'} |J_{x \rightarrow b}(t)|$$

Case 2: $J_{a \rightarrow b}(t) \geq 0$ and $J_{a' \rightarrow b}(t) < 0$. In this case, for $a' \rightarrow b$ we have $[F_{a' \rightarrow b}T - A_{a' \rightarrow b}(t)] J_{a' \rightarrow b}(t) \leq -\rho |J_{a' \rightarrow b}(t)|$ with similar logic as in Case 1. For $a \rightarrow b$, since by the strict accommodation we have $F_{a \rightarrow b}T \leq (1 - \rho) \min(L_a n_a^1, L_b n_b^1)$ for some $\rho > 0$. Therefore, similar to the proof of Lemma 2, we can show that $[F_{a \rightarrow b}T - A_{a \rightarrow b}(t)] J_{a \rightarrow b}(t) \leq -\rho_2 |J_{a \rightarrow b}(t)| + K_2$ for some $\rho_2 > 0$ and $K_2 > 0$. Therefore, we yield:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq -\min(\rho_2, \rho) \sum_{x=a, a'} |J_{x \rightarrow b}(t)| + K_2$$

Case 3: $J_{a \rightarrow b}(t) \geq J_{a' \rightarrow b}(t) \geq 0$ and $N_{a' \rightarrow b}(t) < \min(L_{a'} n_{a'}^1, \max(L_b n_b^1 - L_a n_a^1, 0))$. In this case, we have $[F_{a' \rightarrow b}T - A_{a' \rightarrow b}(t)] J_{a' \rightarrow b}(t) \leq F_{a' \rightarrow b} L_{a'} n_{a'}^1$, and the analysis of $a \rightarrow b$ is the same as in Case 2. Therefore:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq -\rho_2 \sum_{x=a, a'} |J_{x \rightarrow b}(t)| + (1 + \rho_2) F_{a' \rightarrow b} L_{a'} n_{a'}^1$$

Case 4: $J_{a \rightarrow b}(t) \geq J_{a' \rightarrow b}(t) \geq 0$ and $N_{a' \rightarrow b}(t) \geq \min(L_{a'} n_{a'}^1, \max(L_b n_b^1 - L_a n_a^1, 0))$. In this case, if $N_{a \rightarrow b}(t) + N_{a' \rightarrow b}(t) \leq (1 + \rho)(F_{a \rightarrow b} + F_{a' \rightarrow b})T$, by similar reasoning we know that $\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t)$ is upper bounded by a constant K_4 . On the other hand, if $N_{a \rightarrow b}(t) + N_{a' \rightarrow b}(t) > (1 + \rho)(F_{a \rightarrow b} + F_{a' \rightarrow b})T$, from the control rule Eqs.(EC.33)-(EC.34) it is guaranteed that $\frac{A_{a \rightarrow b}(t)}{F_{a \rightarrow b}} \geq \frac{A_{a' \rightarrow b}(t)}{F_{a' \rightarrow b}}$, therefore we have:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq \sum_{x=a, a'} [F_{x \rightarrow b}T - B_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq -\rho \sum_{x=a, a'} |J_{x \rightarrow b}(t)|$$

where $\frac{B_{a \rightarrow b}(t)}{F_{a \rightarrow b}} = \frac{B_{a' \rightarrow b}(t)}{F_{a' \rightarrow b}}$ and $B_{a \rightarrow b}(t) + B_{a' \rightarrow b}(t) = A_{a \rightarrow b}(t) + A_{a' \rightarrow b}(t)$. The first inequality is due to the fact that $J_{a \rightarrow b}(t) \geq J_{a' \rightarrow b}(t)$. Combining the above reasoning we yield:

$$\sum_{x=a, a'} [F_{x \rightarrow b}T - A_{x \rightarrow b}(t)] J_{x \rightarrow b}(t) \leq -\rho \sum_{x=a, a'} |J_{x \rightarrow b}(t)| + (1 + \rho) K_4$$

Collecting the results of all four cases stated above, we confirm the validity of Eq.(EC.52) with some $\rho_1 > 0$.

Now, we can replicate the above analysis for all diverging points as well as entry links, and then summing them up we yield:

$$\sum_{a \rightarrow b} [F_{a \rightarrow b}T - A_{a \rightarrow b}(t)] J_{a \rightarrow b}(t) \leq -\rho_1 \sum_{a \rightarrow b} |J_{a \rightarrow b}(t)| + K' \quad (\text{EC.53})$$

where K' is a fixed constant.

Part IV

Then, we show that there always exists a positive constant $\eta > 0$ such that:

$$\sum_{a \rightarrow b} |J_{a \rightarrow b}(t)| \geq \eta \sum_{a \rightarrow b} |N_{a \rightarrow b}(t)| \quad (\text{EC.54})$$

We have the following inequalities:

$$\|\mathbf{N}(t)\|_\infty \cdot \|\mathbf{U}\mathbf{N}(t)\|_1 \geq \mathbf{N}(t)^T \mathbf{U} \mathbf{N}(t) \geq e_{\min}[\mathbf{U}] \mathbf{N}(t)^T \mathbf{N}(t)$$

where $e_{\min}[\mathbf{U}]$ is the minimum eigenvalue of $\mathbf{U}$. The second inequality is due to the bounding property of Rayleigh quotient. Since $\mathbf{U}$ is an invertible diagonally dominant matrix, by Gershgorin circle theorem (Horn and Johnson 2012), all eigenvalues of $\mathbf{U}$ have a positive real part, therefore $e_{\min}[\mathbf{U}] > 0$. By Cauchy-Schwartz inequality, we have:

$$e_{\min}[\mathbf{U}] \mathbf{N}(t)^T \mathbf{N}(t) \geq \frac{e_{\min}[\mathbf{U}]}{C_{a \rightarrow b}} \|\mathbf{N}(t)\|_1^2$$

where $C_{a \rightarrow b}$ is the total number of link pairs. Combining the above inequalities we yield:

$$\sum_{a \rightarrow b} |J_{a \rightarrow b}(t)| = \|\mathbf{U}\mathbf{N}(t)\|_1 \geq \frac{e_{\min}[\mathbf{U}]}{C_{a \rightarrow b} \|\mathbf{N}(t)\|_\infty} \|\mathbf{N}(t)\|_1^2 \geq \frac{e_{\min}[\mathbf{U}]}{C_{a \rightarrow b}} \sum_{a \rightarrow b} |N_{a \rightarrow b}(t)| \quad (\text{EC.55})$$

Letting $\eta = \frac{e_{\min}[\mathbf{U}]}{C_{a \rightarrow b}}$ proves Eq.(EC.54).

Part V

Combining the results from Part II to Part IV, we obtain the following inequality:

$$\mathbb{E} \left(\sum_{a \rightarrow b} [N_{a \rightarrow b}(t+1)]^2 - [N_{a \rightarrow b}(t)]^2 \middle| \mathbf{N}(t) \right) \leq -2\rho_1 \eta \sum_{a \rightarrow b} N_{a \rightarrow b}(t) + \overline{K} \quad (\text{EC.56})$$

where $\overline{K}$ is a fixed constant. Then, following similar procedure in the proof of Lemma 2, we can complete the whole proof. $\square$

EC.1.8. Proof of Proposition 5: Two-way Network Case

The proof of this proposition is largely similar to the proof in the one-way network case. Therefore, in this proof, we will skip those identical parts and only present the contents that are different from the case of one-way networks. The notations also follow the notations used in the former proof, unless specified otherwise.

In this proof, we will analyze an intersection as a whole. In the following proof, we use 1, 2, 3, 4 to represent the four links corresponding to the four approaching directions of an intersection, and we use 1', 2', 3', 4' to represent the four links corresponding to the four departure directions of the intersection. Thus, the intersection will own a total of 12 pairs of link-to-link connections as follows: $1 \rightarrow 2'$, $1 \rightarrow 3'$, $1 \rightarrow 4'$, $2 \rightarrow 1'$, $2 \rightarrow 3'$, $2 \rightarrow 4'$, $3 \rightarrow 1'$, $3 \rightarrow 2'$, $3 \rightarrow 4'$, $4 \rightarrow 1'$, $4 \rightarrow 2'$, $4 \rightarrow 3'$; the set of which is denoted by $\mathcal{J}$. It is important to note that, in the directed graph representation of the network, there are actually intersection internal links in the middle of each pair of connections mentioned above. For these links, we will use the symbol $\langle \cdot, \cdot \rangle$ to represent them. For example, for the internal link in the connection $1 \rightarrow 2'$, we use $\langle 1, 2' \rangle$ to express it.

Then we establish the local control rules for each intersection, i.e., $A_{x \rightarrow y}(t), x \rightarrow y \in \mathcal{J}$. Denote $J_{x \rightarrow y}(t) = N_{x \rightarrow y}(t) - \sum_{k \in \widehat{\mathcal{V}}_y} r_{y \rightarrow k} N_{y \rightarrow k}(t)$. In a given time slot t , the assignment rules for $A_{x \rightarrow y}(t)$ are presented in the following algorithm.

Local control rules

Input: Intersection $n \in \mathcal{I}$; $\mathbf{J}(t)$.

Output: $\mathbf{A}(t)$.

Step 1. According to the values of all $J_{x \rightarrow y}(t)$ at the intersection, sort them in a descending order, and then sort all the connections in $\mathcal{J}$ in the acquired order.

Step 2. Initialize $c_x \leftarrow L_x n_x^1$ for all x that is an approaching link or a departure link of this intersection.

Step 3. For all $x \rightarrow y \in \mathcal{J}$, executing as follows:

- 1) If $J_{x \rightarrow y}(t) \geq 0$, then let $A_{x \rightarrow y}(t) \leftarrow \min\{N_{x \rightarrow y}(t), c_x, c_y, c_{\langle x, y \rangle}\}$, and update $c_x \leftarrow c_x - A_{x \rightarrow y}(t)$, $c_y \leftarrow c_y - A_{x \rightarrow y}(t)$, $c_{\langle x, y \rangle} \leftarrow c_{\langle x, y \rangle} - A_{x \rightarrow y}(t)$;
 - 2) Otherwise, if $J_{x \rightarrow y}(t) < 0$, then let $A_{x \rightarrow y}(t) \leftarrow \min\{N_{x \rightarrow y}(t), c_x, c_y, c_{\langle x, y \rangle}, \lfloor F_{x \rightarrow y} T - \rho \rfloor\}$ (where ρ is a small positive number), and update $c_x \leftarrow c_x - A_{x \rightarrow y}(t)$, $c_y \leftarrow c_y - A_{x \rightarrow y}(t)$, $c_{\langle x, y \rangle} \leftarrow c_{\langle x, y \rangle} - A_{x \rightarrow y}(t)$.
-

With the above local control rules, we continue the proof. The remaining part of Part I, along with Part II of the proof, are basically the same as in the proof of the one-way network case, so we skip them here. We directly enter the proof of Part III, i.e., we need to show that there exists $\rho_1 > 0, K' > 0$ such that:

$$\sum_{x \rightarrow y} [F_{x \rightarrow y} T - A_{x \rightarrow y}] J_{x \rightarrow y}(t) \leq -\rho_1 \sum_{x \rightarrow y} |J_{x \rightarrow y}(t)| + K' \quad (\text{EC.57})$$

Let us focus on one knotted intersection in the network. For all $x \rightarrow y \in \mathcal{J}$, we denote $\bar{N}_{x \rightarrow y} = \max\{L_x n_x^1, L_y n_y^1, L_{\langle x, y \rangle} n_{\langle x, y \rangle}^1\}$, and also we denote $K_0 = (1 + \rho_0) \sum_{x \rightarrow y \in \mathcal{J}} F_{x \rightarrow y} T \bar{N}_{x \rightarrow y}$, where ρ_0 is a small positive number. We categorize all 12 movements $x \rightarrow y \in \mathcal{J}$ as the following three types for discussion.

- For those movements with $J_{x \rightarrow y}(t) < 0$, according to the local control rules, there must exist a $\rho > 0$ such that:

$$[F_{x \rightarrow y} T - A_{x \rightarrow y}] J_{x \rightarrow y}(t) \leq \rho J_{x \rightarrow y}(t) = -\rho |J_{x \rightarrow y}(t)| \quad (\text{EC.58})$$

- For those movements satisfying 1) $J_{x \rightarrow y}(t) \geq 0$, and 2) all vehicles are allowed to pass through the intersection in this time slot, we must have:

$$[F_{x \rightarrow y} T - A_{x \rightarrow y}] J_{x \rightarrow y}(t) \leq F_{x \rightarrow y} T J_{x \rightarrow y}(t) \leq -\rho_0 F_{x \rightarrow y} T |J_{x \rightarrow y}(t)| + (1 + \rho_0) F_{x \rightarrow y} T \bar{N}_{x \rightarrow y} \quad (\text{EC.59})$$

- For those movements satisfying 1) $J_{x \rightarrow y}(t) \geq 0$, and 2) at least one vehicle is forced to stay in the waiting zone due to the capacity restrictions, there must exist a link along this movement (i.e., one in $x, y, \langle x, y \rangle$) whose VC capacity is reached. If the VC capacity is fully utilized on link $\langle x, y \rangle$, since VC capacity constraint must be obeyed for demand assignment, by strict accommodation, there exists a $\rho_0 > 0$ such that:

$$[F_{x \rightarrow y} T - A_{x \rightarrow y}] J_{x \rightarrow y}(t) \leq -\rho_0 |J_{x \rightarrow y}(t)| \quad (\text{EC.60})$$

On the other hand, if the VC capacity limit is reached on the approaching link x or the departure link y of the intersection (let us assume that the capacity limit is reached on link x), then for that link, there exists a ρ_0 such that:

$$\sum_{k \in \hat{\mathcal{V}}_x} F_{x \rightarrow k} T - \sum_{k \in \hat{\mathcal{V}}_x} A_{x \rightarrow k}(t) \leq -\rho_0 \quad (\text{EC.61})$$

According to the local control rules, the order of control allocation is determined in a descending order of $J_{x \rightarrow y}(t)$. If the last allocated direction $x \rightarrow y$ has $F_{x \rightarrow y}T - A_{x \rightarrow y} > -\rho_0$, by Eq.(EC.61), There must exist a set of directions $x \rightarrow y'$ such that:

$$\left(F_{x \rightarrow y} + \sum_{y'} F_{x \rightarrow y'}\right)T - A_{x \rightarrow y}(t) - \sum_{y'} A_{x \rightarrow y'}(t) \leq -\rho_0 \quad (\text{EC.62})$$

$$J_{x \rightarrow y'}(t) \geq J_{x \rightarrow y}(t) \quad \forall y' \quad (\text{EC.63})$$

Then, we yield:

$$\begin{aligned} & [F_{x \rightarrow y}T - A_{x \rightarrow y}]J_{x \rightarrow y}(t) + \sum_{y'} [F_{x \rightarrow y'}T - A_{x \rightarrow y'}]J_{x \rightarrow y'}(t) \\ & \leq -\rho_0 \left[J_{x \rightarrow y}(t) + \sum_{y'} J_{x \rightarrow y'}(t) \right] = -\rho_0 \left[|J_{x \rightarrow y}(t)| + \sum_{y'} |J_{x \rightarrow y'}(t)| \right] \end{aligned} \quad (\text{EC.64})$$

After the above discussion, we sum up the inequalities corresponding to all movement directions at the intersection (here, some the terms in inequality (EC.64) could have overlap with Eqs.(EC.59) or (EC.60), in which case the former can cover the latter), and we can acquire the following: for this intersection, there exists $\rho_1 > 0$ such that the following inequality holds:

$$\sum_{x \rightarrow y \in \mathcal{J}} [F_{x \rightarrow y}T - A_{x \rightarrow y}]J_{x \rightarrow y}(t) \leq -\rho_1 \sum_{x \rightarrow y} |J_{x \rightarrow y}(t)| + K_0 \quad (\text{EC.65})$$

Thus, we have completed the proof in Part III. The proof in Part IV and Part V is basically the same as in the one-way network case, which we omit it here. Proof completes. $\square$

EC.1.9. Proof of Proposition 6

By Proposition 2, when $T_{\max} = mh_{\min} + 2h_{\min}^s$, we can always design a rhythm to accommodate a demand $\mathbf{d}$ that satisfies Eqs.(13)-(17). Trivially, when $T_{\max} \geq mh_{\min} + 2h_{\min}^s$, designing the same rhythm that can accommodate a demand $\mathbf{d}$ satisfying Eqs.(13)-(17) is feasible. Denote the set of demands described by Eqs.(13)-(17) as $\mathcal{D}(m)$, and similarly we denote $\text{Int}\mathcal{D}(m)$ as the set acquired by changing the “ $\leq$ ” operators in Eqs.(15)-(16) to “ $<$ ” operators. By Proposition 5, if $\mathbf{d} \in \text{Int}\mathcal{D}(m)$, then there exists a rhythm design with cycle length $T \leq T_{\max}$ that can stabilize this demand.

Next, we show that: if $\alpha^{-1}(m)\mathbf{d} \in \text{Int}\mathcal{D}$, then $\mathbf{d} \in \text{Int}\mathcal{D}(m)$. If $\alpha^{-1}(m)\mathbf{d} \in \text{Int}\mathcal{D}$, then by definition, there exists a flow pattern $\mathbf{F}$ such that:

$$d^w = \sum_{r \in \mathcal{P}^w} F_r \quad \forall w \in \mathcal{W} \quad (\text{EC.66})$$

$$F_{ij} = \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} F_r \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.67})$$

$$\frac{F_{ij}}{n_{ij}^1} < \frac{\alpha(m)}{h_{\min}} = \frac{m-1}{mh_{\min} + 2h_{\min}^s} < \frac{m + \lfloor h_{\min}^s/h_{\min} \rfloor}{mh_{\min} + 2h_{\min}^s} \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.68})$$

$$\frac{F_{ij_1}}{n_{ij_1}^1} + \frac{F_{kj_2}}{n_{kj_2}^1} < \frac{\alpha(m)}{h_{\min}} = \frac{m-1}{mh_{\min} + 2h_{\min}^s} \quad \forall (i, j_1), (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.69})$$

$$F_r \geq 0 \quad \forall w \in \mathcal{W}, r \in \mathcal{P}^w \quad (\text{EC.70})$$

According to Proposition 2, we thus know that $\mathbf{d} \in \text{Int}\mathcal{D}(m)$. An equivalent interpretation of this claim is that: if $\mathbf{d} \notin \text{Int}\mathcal{D}(m)$, then $\alpha^{-1}(m)\mathbf{d} \notin \text{Int}\mathcal{D}$. $\mathbf{d} \notin \text{Int}\mathcal{D}(m)$ suggests that $\mathbf{d}$ cannot be stabilized by a rhythm design with cycle length $T_{\max} < mh_{\min} + 2h_{\min}^s$, and $\alpha^{-1}(m)\mathbf{d} \notin \text{Int}\mathcal{D}$ suggests that $\alpha^{-1}(m)\mathbf{d}$ cannot be stabilized by any control. By Definition 4, we thus verifies that RC via strategic holdings with a cycle length $T_{\max} \geq mh_{\min} + 2h_{\min}^s$ can nearly maximize network throughput by a factor of $\alpha(m)$. Proof completes. $\square$

EC.2. Appendix B: Model for Rhythm Design

This appendix presents the mathematical formulation of rhythm design problem for a grid network.

Conflict Avoidance Constraints. The rhythm design must ensure conflict-free operations at all conflict points and on all links. This requirement can be expressed by the following constraints:

$$R_{j_2} - R_{j_1} + (1 - \theta_{j_1, j_2})T \geq L_{ij_1}h_{\min} + h_{\min}^s, \quad \forall (i, j_1) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.71})$$

$$R_{j_1} - R_{j_2} + \theta_{j_1, j_2}T \geq L_{kj_2}h_{\min} + h_{\min}^s, \quad \forall (k, j_2) \in \mathcal{L}, (j_1, j_2) \in \mathcal{C} \quad (\text{EC.72})$$

$$T \geq L_{ij}h_{\min} + h_{\min}^s, \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.73})$$

where θ_{j_1, j_2} is a binary variable indicating the temporal ordering of the two conflicting movements, with $\theta_{j_1, j_2} = 1$ if $R_{j_1} \geq R_{j_2}$ and $\theta_{j_1, j_2} = 0$ otherwise.

Travel Time Computation. Given the relative arrival times, the designed travel time for VCs on each link can be computed. The travel time must be no shorter than the corresponding free-flow time. The following constraints are imposed:

$$t_{ij} = R_j - R_i + \alpha_{ij}T, \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.74})$$

$$\underline{t}_{ij} \leq t_{ij} < \underline{t}_{ij} + T, \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.75})$$

where t_{ij} denotes the designed travel time on link (i, j) , and $\underline{t}_{ij}$ denotes the free-flow travel time. $\alpha_{ij} \in \mathbb{Z}$ is introduced to ensure that the travel time lies within the interval $[\underline{t}_{ij}, \underline{t}_{ij} + T)$ due to the cyclic operation of the RC.

Traffic Flow Assignment. To establish an efficient RC, OD demand must be assigned to feasible paths, leading to the following constraints:

$$d^w = \sum_{r \in \mathcal{P}^w} F_r, \quad \forall w \in \mathcal{W} \quad (\text{EC.76})$$

$$F_{ij} = \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{P}^w} \delta_{ijr} F_r, \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.77})$$

$$F_{ij}T \leq L_{ij}n_{ij}^1, \quad \forall (i, j) \in \mathcal{L} \quad (\text{EC.78})$$

Objective Function. The objective of the rhythm design problem consists of two components. The first component is the total travel time within the network, given by $\sum_{(i, j) \in \mathcal{L}} F_{ij}t_{ij}$. The second component captures the entrance delay induced by the cyclic operation of the RC, expressed as $\frac{1}{2} \sum_{w \in \mathcal{W}} d^w T$. Thus, the objective function is formulated as:

$$\min_{\mathbf{R}, \mathbf{L}, \theta, \mathbf{t}, \alpha, \mathbf{F}} \sum_{(i, j) \in \mathcal{L}} F_{ij}t_{ij} + \frac{1}{2} \sum_{w \in \mathcal{W}} d^w T \quad (\text{EC.79})$$

The above constraints and objective function together define the complete mathematical formulation of the rhythm design problem for the grid network.

EC.3. Appendix C: Comparative Tests in a Real-World Network

In the last part of numerical studies, we implement RC in a road network with topology acquired from a real-world network in Wangjing district, Beijing, China (Figure EC.1). There are 86 intersections and 17 entrances and exits in the network. The OD pair settings are also shown in Figure EC.1(b), with a total of 12 OD pairs; the demand rate of each OD pair is assumed to be identical throughout this subsection and varies from under-saturated to over-saturated to test the performance of RC under various traffic conditions. The intersection geometry design and the RC plan are obtained using the model proposed in [Chen et al. \(2023\)](#).

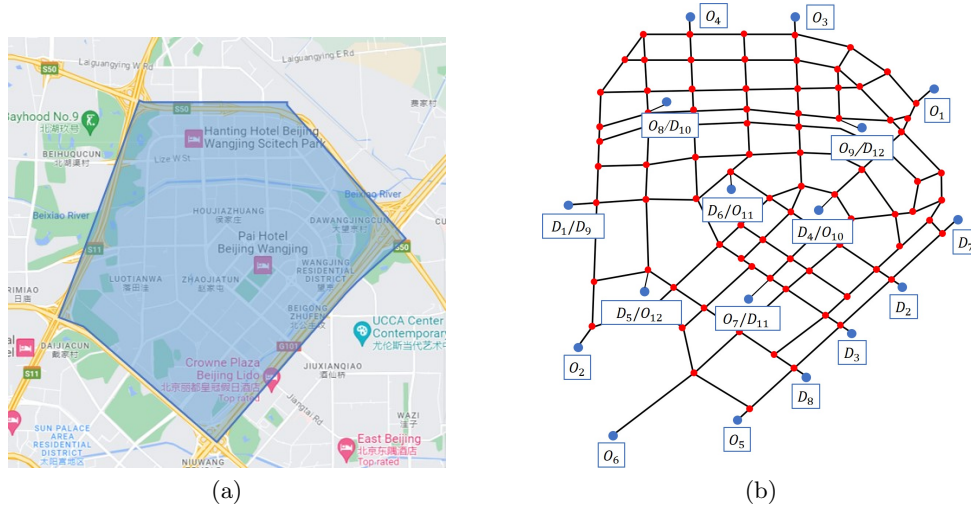


Figure EC.1 Map and topology for the real-world road network. (a) Map. (b) Topology.

In this study, we implement two benchmarks as comparisons. They are elaborated as follows.

- **Traffic signal control (TSC).** We augment conventional TSC with network coordination and NFD-based perimeter control following [Keyvan-Ekbatani et al. \(2012\)](#), where gating is applied with a critical volume of 3,000 vph identified via pre-simulation.

- **First-come-first-serve (FCFS) reservation-based control.** We evaluate the reservation-based intersection control of [Dresner and Stone \(2004\)](#) under an FCFS policy, where approaching vehicles request crossing permission and are accepted or rejected by controller based on the current intersection occupancy to ensure conflict-free operations.

The above benchmarks are implemented on the Simulation of Urban Mobility (SUMO) platform ([Behrisch et al. 2011](#)), using the TraCI4Matlab API ([Acosta et al. 2015](#)) for control implementation in Matlab. In TSC the cycle lengths are 30 and 90 sec, with green times bounded between 4 sec and 64 sec. The perimeter control strategy is re-optimized every 30 sec to adapt to traffic conditions. Vehicle routing under TSC and FCFS follows the same randomized strategy as in RC. Each simulation runs for 1 h, with the first 10 min discarded as a warm-up period to capture steady-state performance.

The simulation results are shown in Figure EC.2, where the x-axis denotes traffic demand, and the y-axis reports the average vehicle travel time and network throughput, respectively. Overall, RC exhibits distinctive advantages over TSC and FCFS in terms of vehicle travel time. For light traffic scenarios, FCFS and RC

generate lower travel time than TSC with longer cycle length, as both strategies flexibly adapt control decisions to real-time vehicle arrivals. As the demand increases, however, vehicle travel time under FCFS and TSC increases rapidly, especially when the demand magnitude exceeds 4,000 vph. In contrast, travel time under RC remains relatively stable throughout the entire demand range, indicating that RC could stabilize the traffic within the tested conditions.

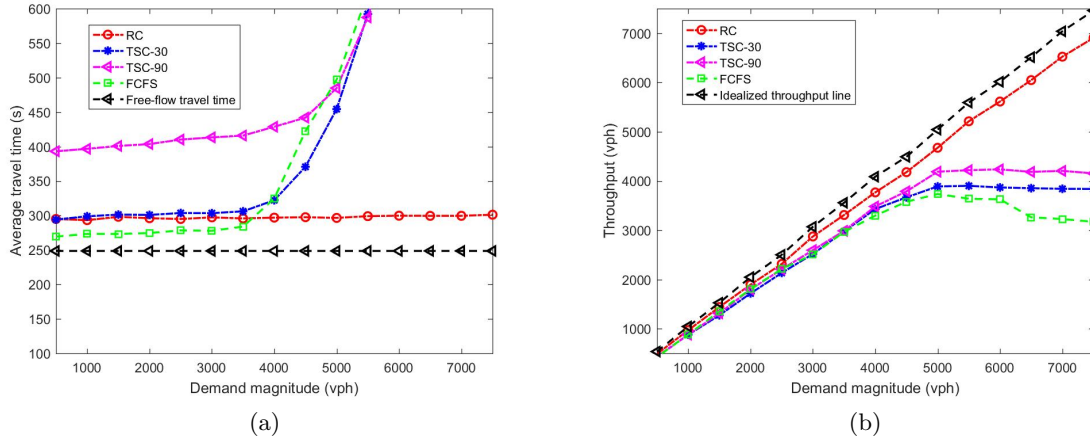


Figure EC.2 Average vehicle delay and vehicle throughput comparisons between RC and benchmarks. (a) Average vehicle delay. (b) Vehicle throughput.

For network throughput, it initially increases with demand under all control strategies when demand is low. As demand continues to rise, however, network throughput under FCFS begins to decrease, reflecting the well-known phenomenon of capacity degradation, whereby network capacity decreases in over-saturated traffic conditions (Daganzo 2007). In contrast, throughput under RC continues to increase with demand, showing that RC effectively eliminates capacity degradation. Although perimeter control in TSC can also eliminate capacity degradation, the maximum throughput achieved by TSC (approximately 4,200 vph) is lower than that in RC (approximately 6,100 vph). These observations demonstrate the remarkable capability of RC in handling heavy traffic in general road networks.

References

- Acosta, A. F., Espinosa, J. E., and Espinosa, J. (2015). TraCI4matlab: Enabling the Integration of the SUMO Road Traffic Simulator and Matlab Through a Software Re-engineering Process. In *Modeling Mobility with Open Data*, pages 155–170. Springer.
- Behrisch, M., Bieker, L., Erdmann, J., and Krajzewicz, D. (2011). Sumo—simulation of urban mobility: an overview. In *Proceedings of SIMUL 2011, The Third International Conference on Advances in System Simulation*. ThinkMind.
- Chen, X., Lin, X., Li, M., He, F., and Meng, Q. (2023). A nearly throughput-maximum knotted intersection design and control for connected and automated vehicles. *Transportation Research Part B: Methodological*, 171:44–79.
- Daganzo, C. F. (2007). Urban gridlock: Macroscopic modeling and mitigation approaches. *Transportation Research Part B: Methodological*, 41(1):49–62.
- Dresner, K. and Stone, P. (2004). Multiagent traffic management: A reservation-based intersection control mechanism. In *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems—Volume 2*, pages 530–537.
- Horn, R. A. and Johnson, C. R. (2012). *Matrix analysis*. Cambridge university press.
- Keyvan-Ekbatani, M., Kouvelas, A., Papamichail, I., and Papageorgiou, M. (2012). Exploiting the fundamental diagram of urban networks for feedback-based gating. *Transportation Research Part B: Methodological*, 46(10):1393–1403.