# Appendix

## A. Algorithm for the overall process of decision-making

Given the definition of an IDN and cost terms presented in subsection 4.2, we present in Algorithm 1 the solution process for a given state $s_\theta$.

**Algorithm 1**

**Require:** $\mu_v^{v'}$ $\forall (v, v') \in \{(v, v') | v \in N, v' \in N\}$; $s_\theta$

1: **for** $k \in K_\theta$ **do**
2:     Enumerate the set $D_\theta^k$ of all nodes that meet the IDN criteria in Definition 1
3: **end for**
4: **for** $k \in K_\theta$ **do**
5:     **for** $(v', t') \in D_\theta^k$ **do**
6:         Calculate the approximated costs $\Omega_{(v',t')}^{k\theta}$
7:     **end for**
8: **end for**
9: Formulate the MILP minimize (21) subject to constraints (9) - (16)
10: Solve the MILP and retrieve the decisions on $\bar{\delta}_\theta = (\bar{x}^\theta, \bar{m}^\theta)$
11: **return** $\bar{\delta}_\theta$

The state contains all relevant information, namely the time point at which the decision epoch starts as well as the known commodities. First it must be searched for all IDN set (line 1 - 3) of every commodity. This searches for all nodes that match the criteria from definition 1. If an IDN set $D_\theta^k$ has been determined for all commodities, the approximation of the costs $\Omega_{(v',t')}^{k\theta}$ can be made for each IDN, which is required for the objective function (line 4 - 8). For this all steps, described in Subsection 4.2, must be completed. After the approximation of the costs is done, the MILP is set up using these cost approximations for the nodes in the IDN set(line 9) and solved (line 10). Finally, the individual decisions about route plans of the commodity and used transport services are extracted and returned as decision $\delta_\theta$ (line 11).

## B. On the size and runtime of MIPs

In this section, we take a closer look at MIPs, their runtimes and performance gaps. We will examine four relevant factors: the average number of linear constraints in every decision epoch, the average number of decision variables in every decision epoch, the overall runtime and the average performance gaps of the MIPs in every decision epoch. However, commodities were not routed in every experiment until the end of the planning horizon at $T = 25$. For this reason, all average values in the following tables (3, 4, 5) were calculated only using decision epochs in which a commodity was routed. The Go-as-fast-as-possible and Wait-as-long-as-possible methods are excluded from this analysis, as they could be solved without a MIP and the runtime was less than one second in all instances.

First, we take a look at the overall runtime of the instances. Table 2 shows the average time each method took to solve the instances for different delivery and size settings. While Slack IDN and Myopic have relatively short runtimes,

| delivery setting | size setting | Slack IDN (in s) | Myopic (in s) | Direct Lookahead (in s) | CFA (in s) |
|---|---|---|---|---|---|
| fast | small | 266 | 2330 | 85,332 | 31,803 |
| fast | medium | 295 | 2471 | 89,269 | 33,904 |
| fast | large | 298 | 3254 | 92,492 | 39,163 |
| fast | extra | 294 | 3322 | 91,870 | 43,688 |
| moderate | small | 305 | 4726 | 114,902 | 57,062 |
| slow | small | 380 | 7270 | 121,374 | 68,837 |

**Table 2 Runtime comparison for different delivery and size settings.**

| delivery setting | size setting | Slack IDN | Myopic | Direct Lookahead | CFA |
|---|---|---|---|---|---|
| fast | small | 9,286 | 14,748 | 43,315 | 12,689 |
| fast | medium | 9,286 | 14,735 | 43,118 | 12,685 |
| fast | large | 9,286 | 14,713 | 42,901 | 12,702 |
| fast | extra | 9,286 | 14,713 | 43,068 | 12,716 |
| moderate | small | 9,369 | 18,573 | 54,304 | 15,700 |
| slow | small | 9,369 | 21,859 | 66,462 | 18,108 |

**Table 3 Comparison of the average number of linear constraints in a decision epoch for different delivery and size settings.**

| delivery setting | size setting | Slack IDN | Myopic | Direct Lookahead | CFA |
|---|---|---|---|---|---|
| fast | small | 43,986 | 90,203 | 308,182 | 73,075 |
| fast | medium | 43,986 | 90,026 | 306,947 | 73,201 |
| fast | large | 43,986 | 90,010 | 304,563 | 73,109 |
| fast | extra | 43,986 | 89,881 | 305,411 | 73,117 |
| moderate | small | 43,980 | 123,186 | 407,267 | 97,431 |
| slow | small | 43,980 | 152,733 | 516,788 | 117,978 |

**Table 4 Comparison of the average number of decision variables in a decision epoch for different delivery and size settings.**

the runtimes for Direct Lookahead and CFA are significantly longer. However, there are also significant differences between these two methods. For example, the average runtime of the CFA across all instances is $45,743$ seconds, while the average of Direct Lookahead is $99,207$ seconds. This means that Direct Lookahead takes, on average, more than twice as long to solve a problem instance.

Tables 3 and 4 illustrate the size of the MIP. They show the average number of linear constraints and decision variables in each decision epoch for various delivery and size settings when using different methods. It is noticeable how much larger the average MIP is in the decision epochs when using Direct Lookahead. This is, of course, due to the fact that a period twice as long is considered. It is also important to note the impact of extending the delivery window: The more time the commodities have, the larger the MIPs generally become, as they are shipped over a longer period. The only exception is Slack IDN, but this can be explained by how the method works. Since the IDN with the highest slack is always selected, routing decisions are generally independent of the due date of the commodities.

Finally, we examine the average performance gaps of the MIPs for the various methods. These are specified for the different settings in Table 5. It can be seen that for all methods except Direct Lookahead, the gaps are less than 2 and in most cases even 0. The large gaps in Direct Lookahead are likely due to the fact that larger problems are being considered, which cannot be fully solved within the two-hour time window we specified.

| delivery setting | size setting | Slack IDN (%) | Myopic (%) | Direct Lookahead (%) | CFA (%) |
|---|---|---|---|---|---|
| fast | small | 0.00 | 0.00 | 4.50 | 0.23 |
| fast | medium | 0.00 | 0.00 | 4.83 | 0.32 |
| fast | large | 0.00 | 0.00 | 5.16 | 0.30 |
| fast | extra | 0.00 | 0.00 | 5.06 | 0.51 |
| moderate | small | 0.00 | 0.00 | 9.16 | 0.92 |
| slow | small | 0.00 | 0.00 | 11.55 | 1.55 |

**Table 5 Comparison of the average MIP Gaps in a decision epoch for different delivery and size settings.**

| delivery setting | size setting | CFA (%) | CFA-A* (%) |
|---|---|---|---|
| fast | small | 100 | 103.175 |
| fast | medium | 100 | 102.746 |
| fast | large | 100 | 102.623 |
| fast | extra | 100 | 102.862 |
| moderate | small | 100 | 103.554 |
| slow | small | 100 | 104.032 |

**Table 6 Relative costs comparison of CFA and CFA-A* under various delivery and size settings.**

| delivery setting | size setting | CFA (in s) | CFA-A* (in s) |
|---|---|---|---|
| fast | small | 31,803 | 20,886 |
| fast | medium | 33,904 | 22,135 |
| fast | large | 39,163 | 27,221 |
| fast | extra | 43,688 | 33,763 |
| moderate | small | 57,062 | 33,686 |
| slow | small | 68,837 | 50,032 |

**Table 7 Runtime comparison of CFA and CFA-A* under various delivery and size settings.**

## C. On the reduction of decision space

In the following, we will explore a way to narrow down the decision space in order to reduce the runtime of the problems without significantly compromising the quality of the solutions. Here, as in Appendix B, the experiments were conducted with $T = 25$; however, for the calculation of the averages, only decision epochs in which commodities were routed (i.e., in which an MIP was generated) were included (see table 8, 9 and 10). Next, we examine the results:

Table 6 compares the costs of the CFA-A* relative to the CFA under various delivery and size settings. On average, the CFA-A* generates 3.166% higher costs. It is notable that the cost difference increases with a slower delivery setting. Although the increase is only small, the reason for this could be that, when given more time, commodities are routed via longer detours, as shown in Table 6. Since not all detours are possible due to the detour restriction, the result deteriorates.

The runtimes of the two methods under various delivery and size settings are shown in Table 7. On average, solving the instances using the CFA takes $45,743$ seconds, while when using CFA-A* solving takes only $31,287$ seconds. The runtime is therefore about 31.6% shorter. These savings are most likely attributable to the smaller number of linear constraints and decision variables in the individual MIPs at the decision epochs. On average, the MIPs at the decision epochs when using CFA-A* have $11,228$ linear constraints, while the MIPs when using CFA have $14,100$ constraints. The situation is similar when considering the average number of decision variables at the decision epochs. When using CFA-A*, the MIPs have an average of $44,026$ decision variables, and when using CFA, $84,652$. Tables 8 and 9 show

| delivery setting | size setting | CFA | CFA-A* |
|---|---|---|---|
| fast | small | 12,689 | 10,201 |
| fast | medium | 12,685 | 10,215 |
| fast | large | 12,702 | 10,261 |
| fast | extra | 12,716 | 10,270 |
| moderate | small | 15,700 | 12,395 |
| slow | small | 18,108 | 14,027 |

**Table 8 Comparison of the average number of linear constraints in every decision epoch when using CFA and CFA-A* under various delivery and size settings.**

| delivery setting | size setting | CFA | CFA-A* |
|---|---|---|---|
| fast | small | 73,075 | 39,465 |
| fast | medium | 73,201 | 39,566 |
| fast | large | 73,109 | 39,769 |
| fast | extra | 73,117 | 39,814 |
| moderate | small | 97,431 | 49,082 |
| slow | small | 117,978 | 117,978 |

**Table 9 Comparison of the average number of decision variables in every decision epoch when using CFA and CFA-A* under various delivery and size settings.**

| delivery setting | size setting | CFA (%) | CFA-A* (%) |
|---|---|---|---|
| fast | small | 0.23 | 0.09 |
| fast | medium | 0.32 | 0.12 |
| fast | large | 0.30 | 0.16 |
| fast | extra | 0.51 | 0.22 |
| moderate | small | 0.92 | 0.22 |
| slow | small | 1.55 | 0.39 |

**Table 10 Comparison of the average MIP gap in every decision epoch when using CFA and CFA-A* under various delivery and size settings.**

the average number of linear constraints and decision variables in the decision epochs for various delivery and size settings when using CFA and CFA-A*.

Finally, Table 10 shows the MIP gaps for the various delivery and size settings. While using CFA-A* the MIP gap is less than 0.4% across all settings, when using CFA the MIP gap rises to as high as 1.55%. This is likely also related to the size of the MIPs, as described earlier. Overall, the CFA-A* offers a way to significantly reduce the runtimes, but this comes at the cost of a slight deterioration in cost performance even though the MIP gaps are smaller.

## D. Analyzing the scalability by varying the number of commodities

All of our experiments were conducted using different settings for $\lambda$. Here, $\lambda$ represents the average number of commodities shipped per day (for more detail see chapter 5.1). In the following, we examine how the CFA and its benchmark methods perform when more or fewer new commodities appear each day. It should be noted that the methods Go-as-fast-as-possible and Wait-as-long-as-possible have been abbreviated as Gafap and Walap.

Table 11 shows the relative costs of the various methods compared to the CFA when considering different values for $\lambda$. Our CFA performs best in every scenario and also outperforms the Direct Lookahead. Particularly with the methods Go-as-fast-as-possible, Wait-as-long-as-possible, Slack IDN and Myopic, performance declines as the number

| $\lambda$ | Gafap | Walap | Slack IDN | Myopic | Direct Lookahead | CFA | CFA-A* |
|---|---|---|---|---|---|---|---|
| 20 | 2.080 | 2.045 | 1.977 | 1.529 | 1.054 | 1 | 1.021 |
| 40 | 2.429 | 2.391 | 2.247 | 1.572 | 1.045 | 1 | 1.034 |
| 60 | 2.728 | 2.690 | 2.480 | 1.605 | 1.047 | 1 | 1.039 |

**Table 11 Relative costs comparison of different values for $\lambda$.**

of commodities per day increases, compared to the CFA. This also applies to CFA-A*. The only exception is Direct Lookahead, where performance is worst for the smallest $\lambda$ of 20.

## E. Proof of Proposition 1

*Proof of Proposition 1* As the values $\bar{x}^{k\theta}_{(v,t),(v',t')}$ satisfy constraints (1), (2), and (3) they define a sequence of nodes in $N^*_\theta$ for commodity $k$ that begins at $(o^k_\theta, a^k_\theta)$ and terminates at $(d^k, e^k)$. Namely, we have the sequence $((u_1,t_1),\ldots,(u_n,t_n))$ with $u_1 = o^k_\theta, t_1 = a^k_\theta$ and $u_n = d^k, t_n = e^k$ and $(u_j,t_j) \in N^*_\theta, j = 1,\ldots,n$. Let $(u_{j*}, t_{j*})$ be such that $t_{j*}$ is the smallest of $t_1,\ldots,t_n$ such that $t_{j*} \geq t_\theta + l$. Because $e^k \geq t_\theta + l$ such a $t_{j*}$ must exist. We will next show that $(u_{j*}, t_{j*})$ satisfies the conditions of Definition 1.

We first observe that the sequence of locations, $(u_1,\ldots,u_{j*})$ implies that the path $p^{u_{j*}}_{o^k_\theta}$ exists and thus $(u_{j*}, t_{j*})$ satisfies condition 1. Second, the choice of $t_{j*}$ ensures that both conditions 2 and 3 are satisfied. Finally, as $t_n = e^k$ we must have $\Delta^{d^k}_{u_{j*}} = \sum_{j=j*}^{n-1} \tau_{jj+1} \leq e^k - t_{j*}$. Or, in words, the length in time points of the path from $u_{j*}$ to $d^k$ is no greater than $e^k - t_{j*}$. Alternately, we have that $t_{j*} + \Delta^{d^k}_{u_{j*}} \leq e^k$. As we must have $\mu^{d^k}_{u_{j*}} \leq \Delta^{d^k}_{u_{j*}}$ by definition, condition 4 is satisfied.

## F. Proof of Proposition 2

*Proof of Proposition 2.* We first show that $proj_{\theta,\theta+1}(P_\theta) \in R_\theta$. Let $(\bar{x}^\theta, \bar{m}^\theta)$ be an element of $proj_{\theta,\theta+1}(P_\theta)$. By definition, for each commodity $k \in K_\theta$ such that $e^k \geq t_\theta + l$ there are values $\tilde{x}^{k\theta}_{(v,t),(v',t')}$ for $((v,t),(v',t')) \in E^*_\theta \setminus E^*_{\theta,\theta+1}$ that enable the construction of a single sequence of nodes for commodity $k$ from $(o^k_\theta, a^k_\theta)$ to $(d^k, e^k)$. As in the proof of Proposition 1, an IDN $(v'_k, t'_k)$ can be identified from this sequence. Let $\bar{z}^{k\theta}_{(v',t')} = 1, \bar{z}^{k\theta}_{(u,t)} = 0 \ \ \forall (u,t) \in D^k_\theta$ such that $(u,t) \neq (v',t')$. As such, one can see that $(\bar{x}, \bar{m}, \bar{z})$ satisfy constraints (11) - (16) and thus $(\bar{x}^\theta, \bar{m}^\theta, \bar{z}^\theta) \in Q_\theta$ and $(\bar{x}^\theta, \bar{m}^\theta) \in R_\theta$.

We next show that $R_\theta \in proj_{\theta,\theta+1}(P_\theta)$. Consider an element $(\bar{x}^\theta, \bar{m}^\theta) \in R_\theta$ and a corresponding set of values $\bar{z}^\theta$ such that $(\bar{x}^\theta, \bar{m}^\theta, \bar{z}^\theta) \in Q_\theta$. For each commodity $k \in K_\theta$ such that $e^k \geq t_\theta + l$, we consider the IDN $(v', 't') \in D^k_\theta$ such that $\bar{z}^{k\theta}_{(v',t')} = 1$. By condition 4 of Definition 1 one can construct a path consisting of arcs from $E^*_\theta \setminus E^*_{\theta,\theta+1}$ such that the first arc departs from $(v', 't')$ and the last arc arrives at $(d^k, e^k)$. Such a sequence of arcs can be used to construct values $\tilde{x}^{k\theta}_{(v,t),(v',t')}$ such that $(\bar{x}, \tilde{x}, \bar{m}) \in P_\theta$ and thus $(\bar{x}, \bar{m}) \in proj_{\theta,\theta+1}(P_\theta)$.