Appendix A. Model formulation

This section formalizes the complete TnR-3R. In addition to the notation introduced in Section 3.2, we introduce two auxiliary variables that are required for the model formulation. Ensuring that the number of robots at robot depots does not exceed the capacity limit requires continuous tracking of robot movements. For this, we introduce the binary variables β^-_{rji} and β^+_{kji}, which indicate whether a robot tour (r/k to j) starts or ends before the truck arrives at stop i. Using these variables avoids the limitations associated with discrete time steps (time indices). Table A.8 summarizes the complete notation.

Index sets	
R ($\hat{R}$)	Set of distinct (duplicated) robot depots
D ($\hat{D}$)	Set of distinct (duplicated) drop-off points
C	Set of all customers
C^{d}	Set of delivery customers
C^{s}	Set of return customers, i.e., customers requesting a return shipment
C^{t}	Set of customers requesting a return shipment by truck, $C^{\mathrm{t}} \subseteq C^{\mathrm{s}}$
$\hat{L}$	Set of potential truck stop locations, $\hat{L} := \hat{R} \cup \hat{D} \cup C^{\mathrm{s}}$
H_r	Set of duplicate elements of one distinct robot depot $r \in R$, including itself
Parameters	
a_i (b_i)	Start (end) of time window for location $i \in C \cup \hat{L} \cup \{0\}$
d_{ij}	Truck travel distance between locations i and j, $i, j \in \hat{L} \cup \{0, n+1\}$
t^{t}_{ij}	Truck travel time between locations i and j, $i, j \in \hat{L} \cup \{0, n+1\}$
t^{r}_{jk}	Robot travel time between locations j and k, $j, k \in R \cup \hat{L} \cup C^{\mathrm{d}}$
o_r	Initial number of robots available in location $r \in R$
m_r	Robot capacity of robot depot $r \in R$
I^{r} (I^{p})	Initial number of robots (parcels) aboard the truck
Q^{r} (Q^{p})	Robot (parcel volume) capacity of the truck
Cost parameters	
c^{t} (c^{r})	Cost of truck (robot) per time unit
c^{d}	Cost of truck per distance unit
c^{l}	Cost of delayed service per time unit
Decision variables	
x_{ij}	Binary: 1, if truck travels from location i to j, $i, j \in \hat{L} \cup \{0, n+1\}$, 0 otherwise
y_{jk}	Binary: 1, if robot travels from location j to k, $j, k \in R \cup \hat{L} \cup C^{\mathrm{d}}$, 0 otherwise
w_i	Arrival time of truck or robot at location $i \in R \cup \hat{L} \cup C^{\mathrm{d}} \cup \{0, n+1\}$
q^{r}_i	Number of robots aboard the truck after visiting location $i \in \hat{L} \cup \{0\}$
q^{p}_i	Number of parcels aboard the truck after visiting location $i \in \hat{L} \cup \{0\}$
Auxiliary variables	
β^-_{rji}	Binary: 1, if robot departs from location $r \in R$ to location $j \in R \cup \hat{L} \cup C^{\mathrm{d}}$ before truck arrives at location $i \in H_r$, 0 otherwise
β^+_{kji}	Binary: 1, if robots start from location $k \in R \cup \hat{L} \cup C^{\mathrm{d}}$ and arrive at location $j \in H_r$, before truck arrives at location $i \in H_r \setminus \{r\}$, for $r \in R$, 0 otherwise

Table A.8: Notation

$$
\begin{aligned}
\min\ & c^{\mathrm{t}} \cdot (w_{n+1} - w_0) \\
& + \sum_{i \in \hat{L} \cup \{0\}} \sum_{j \in \hat{L} \cup \{n+1\}} c^{\mathrm{d}} \cdot d_{ij} \cdot x_{ij} \\
& + \sum_{j \in R \cup \hat{L} \cup C^{\mathrm{d}}} \sum_{k \in R \cup \hat{L} \cup C^{\mathrm{d}}} c^{\mathrm{r}} \cdot t^{\mathrm{r}}_{jk} \cdot y_{jk} \\
& + \sum_{i \in R \cup \hat{L}} \sum_{c \in C} c^{\mathrm{r}} \cdot \max(0, a_c - w_c) \cdot y_{ic} \\
& + \sum_{c \in C} c^{\mathrm{l}} \cdot \max(0, w_c - b_c) \qquad \text{(A.1)}
\end{aligned}
$$

subject to

$$\sum_{j \in \hat{L}} x_{0j} = 1 \qquad \text{(A.2)}$$

$$\sum_{i \in \hat{L} \cup \{0\}} x_{ij} = \sum_{i \in \hat{L} \cup \{n+1\}} x_{ji} \qquad \forall j \in \hat{L} \qquad \text{(A.3)}$$

$$\sum_{i \in \hat{L} \cup \{0\}} x_{ic} = 1 \qquad \forall c \in C^{\mathrm{t}} \qquad \text{(A.4)}$$

$$\sum_{i \in \hat{L} \cup \{0\}} x_{ic} + \sum_{j \in R \cup \hat{L}} y_{jc} = 1 \qquad \forall c \in C^{\mathrm{s}} \setminus C^{\mathrm{t}} \qquad \text{(A.5)}$$

$$\sum_{j \in R \cup \hat{L}} y_{jc} = \sum_{j \in \hat{R}} y_{cj} \qquad \forall c \in C^{\mathrm{s}} \setminus C^{\mathrm{t}} \qquad \text{(A.6)}$$

$$\sum_{j \in \hat{L}} y_{jc} = 1 = \sum_{j \in R \cup \hat{R}} y_{cj} \qquad \forall c \in C^{\mathrm{d}} \qquad \text{(A.7)}$$

$$\sum_{k \in R \cup \hat{R} \cup C} y_{jk} \leq M \cdot \sum_{i \in \hat{L} \cup \{0\}} x_{ij} \qquad \forall j \in \hat{L} \qquad \text{(A.8)}$$

$$\sum_{k \in R \cup \hat{L} \cup C^{\mathrm{d}}} y_{kj} \leq M \cdot \sum_{i \in \hat{L} \cup \{0\}} x_{ij} \qquad \forall j \in \hat{R} \qquad \text{(A.9)}$$

$$w_j \geq \max(w_i, a_i) + t^{\mathrm{t}}_{ij} - M \cdot (1 - x_{ij}) \qquad \forall i \in \hat{L} \cup \{0\}, j \in \hat{L} \cup \{n+1\} \qquad \text{(A.10)}$$

$$w_c \geq w_i + t^{\mathrm{r}}_{ic} - M \cdot (1 - y_{ic}) \qquad \forall i \in \hat{L}, c \in C \qquad \text{(A.11)}$$

$$w_c \leq (w_j - t^{\mathrm{t}}_{ij}) + t^{\mathrm{r}}_{ic} + M \cdot (2 - x_{ij} - y_{ic}) \qquad \forall i \in \hat{L}, j \in \hat{L} \cup \{n+1\}, c \in C \qquad \text{(A.12)}$$

$$
\begin{aligned}
& w_c + \max(0, a_c - w_c) + t^{\mathrm{r}}_{cj} \leq \\
& w_j + M \cdot (2 - y_{ic} - y_{cj}) \qquad \forall i \in R \cup \hat{L}, j \in R \cup \hat{R}, c \in C \qquad \text{(A.13)}
\end{aligned}
$$

$$w_i - (w_j - t^{\mathrm{r}}_{rj}) \leq M \cdot \beta^{-}_{rji} \qquad \forall r \in R, i \in H_r, j \in \hat{R} \cup C^{\mathrm{s}} \qquad \text{(A.14)}$$

$$w_i - (w_j - t^{\mathrm{r}}_{rj}) \geq -M \cdot (1 - \beta^{-}_{rji}) \qquad \forall r \in R, i \in H_r, j \in \hat{R} \cup C^{\mathrm{s}} \qquad \text{(A.15)}$$

$$w_i - (w_k + t^{\mathrm{r}}_{kj}) \leq M \cdot \beta^{+}_{kji} \qquad \forall r \in R, i, j \in H_r, k \in R \cup \hat{L} \cup C^{\mathrm{d}} \qquad \text{(A.16)}$$

$$w_i - (w_k + t^{\mathrm{r}}_{kj}) \geq -M \cdot (1 - \beta^{+}_{kji}) \qquad \forall r \in R, i, j \in H_r, k \in R \cup \hat{L} \cup C^{\mathrm{d}} \quad (A.17)$$

$$o_r - \sum_{j \in \hat{R} \cup C^{\mathrm{s}}} y_{rj} \cdot \beta^{-}_{rji} + \sum_{h \in R \cup \hat{L} \cup C^{\mathrm{d}}} \sum_{k \in H_r | i \leq k} y_{hk} \cdot \beta^{+}_{hki} \leq m_r \qquad \forall r \in R,\ i \in H_r \quad (A.18)$$

$$\sum_{h \in \hat{L} \cup \{0\}} x_{hi} \geq \sum_{h \in \hat{L} \cup \{0\}} x_{hj} \qquad \forall r \in R, i, j \in H_r \setminus \{r\} \mid i < j \quad (A.19)$$

$$w_i \leq w_j \qquad \forall r \in R, i, j \in H_r \setminus \{r\} \mid i < j \quad (A.20)$$

$$\sum_{j \in \hat{R} \cup C^{\mathrm{s}}} y_{rj} \leq o_r \qquad \forall r \in R \quad (A.21)$$

$$q^{\mathrm{r}}_0 = I^{\mathrm{r}} \quad (A.22)$$

$$q^{\mathrm{r}}_j - q^{\mathrm{r}}_i \geq \sum_{h \in R \cup \hat{L} \cup C^{\mathrm{d}}} y_{hj} - \sum_{k \in R \cup \hat{R} \cup C} y_{jk} - M \cdot (1 - x_{ij}) \qquad \forall i \in \hat{L} \cup \{0\}, j \in \hat{L} \quad (A.23)$$

$$q^{\mathrm{r}}_j - q^{\mathrm{r}}_i \leq \sum_{h \in R \cup \hat{L} \cup C^{\mathrm{d}}} y_{hj} - \sum_{k \in R \cup \hat{R} \cup C} y_{jk} + M \cdot (1 - x_{ij}) \qquad \forall i \in \hat{L} \cup \{0\}, j \in \hat{L} \quad (A.24)$$

$$q^{\mathrm{p}}_0 = I^{\mathrm{p}} \quad (A.25)$$

$$q^{\mathrm{p}}_c \leq q^{\mathrm{p}}_i + 1 - \sum_{k \in C^{\mathrm{d}}} y_{ck} + M \cdot (1 - x_{ic}) \qquad \forall i \in \hat{L} \cup \{0\}, c \in C^{\mathrm{s}} \quad (A.26)$$

$$q^{\mathrm{p}}_j \leq q^{\mathrm{p}}_i + \sum_{c \in C^{\mathrm{s}}} y_{cj} - \sum_{c \in C^{\mathrm{d}}} y_{jc} + M \cdot (1 - x_{ij}) \qquad \forall i \in \hat{L} \cup \{0\}, j \in \hat{L} \setminus C^{\mathrm{s}} \quad (A.27)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in \hat{L} \cup \{0\}, j \in \hat{L} \cup \{n+1\} \quad (A.28)$$

$$y_{jk} \in \mathbb{N}_0 \qquad \forall j \in R \cup \hat{L} \cup C^{\mathrm{d}}, k \in R \cup \hat{R} \cup C \quad (A.29)$$

$$q^{\mathrm{r}}_i \in [0, Q^{\mathrm{r}}] \qquad \forall i \in \hat{L} \cup \{0\} \quad (A.30)$$

$$q^{\mathrm{p}}_i \in [0, Q^{\mathrm{p}}] \qquad \forall i \in \hat{L} \cup \{0\} \quad (A.31)$$

$$w_i \in \mathbb{R}^{+}_0 \qquad \forall i \in R \cup \hat{L} \cup \{0, n+1\} \quad (A.32)$$

$$\beta^{-}_{kji}, \beta^{+}_{kji} \in \{0, 1\} \qquad \forall k, j \in R \cup \hat{L} \cup C^{\mathrm{d}}, i \in R \cup \hat{R} \quad (A.33)$$

Constraints (A.2) ensure that the truck departs from the warehouse. Constraints (A.3) enforce flow conservation for all locations visited by the truck. Constraints (A.4)–(A.7) jointly ensure that all customers are served and that the robots' flow conservation is maintained throughout the network. Specifically, Constraints (A.4) ensure that all mandatory truck customers are visited by the truck. Constraints (A.5) stipulate that each optional customer can be served by either truck or robot. Constraints (A.6) ensure the returns of robots are synchronized with the truck, meaning that the robot's return location must be visited by the truck at a later time. Similarly, Constraints (A.7) address the same conditions for delivery customers, but with no need for synchronization. Constraints (A.8) state that robots may only be dispatched from truck-visited stops, except for those serving return customers, which may also depart autonomously from robot depots (see Constraints (A.5)). Constraints (A.9) allow robots to return to truck-visited stops – mandatory for robot-served return customers (see Constraints (A.6)) – or to their designated final depot. Constraints (A.10)–(A.12) define the arrival and waiting times of the truck (Constraint (A.10)) and the robots (Constraints (A.11) and (A.12)) at their respective locations. Constraints (A.13) prevent robots from moving to locations already visited by the truck, which is necessary for correctly determining the number of robots present at each depot. Constraints (A.14)–(A.17) establish whether the robot's departure time to a truck stop (Constraints (A.14) and (A.15)) or its arrival time at the truck stop (Constraints (A.16) and (A.17)) occurs before the truck's own arrival at one of its duplicates. This information is then used in Constraints (A.18), which ensures that the number of robots at any depot never exceeds its maximum capacity. To support correct determination, the model enforces an ascending order of visits to truck duplicates (Constraints (A.19) and (A.20)), which also act as symmetry-breaking constraints. Constraints (A.21) restrict robot departures from depots to only those initially stationed there. Constraints (A.22)–(A.24) monitor the number of robots aboard the truck, accounting for the initial load upon departure (Constraint (A.22)) and the subsequent robot flow (Constraints (A.23) and (A.24)). These constraints are also essential for tracking the number of robots in robot depots. Similarly, Constraints (A.25)–(A.27) govern parcel flow. Finally, Constraints (A.28)–(A.33) define the variable domains.

Appendix B. Solution framework

Appendix B.1. Detailed process

Figure B.9 details the process steps of the MH solution framework.

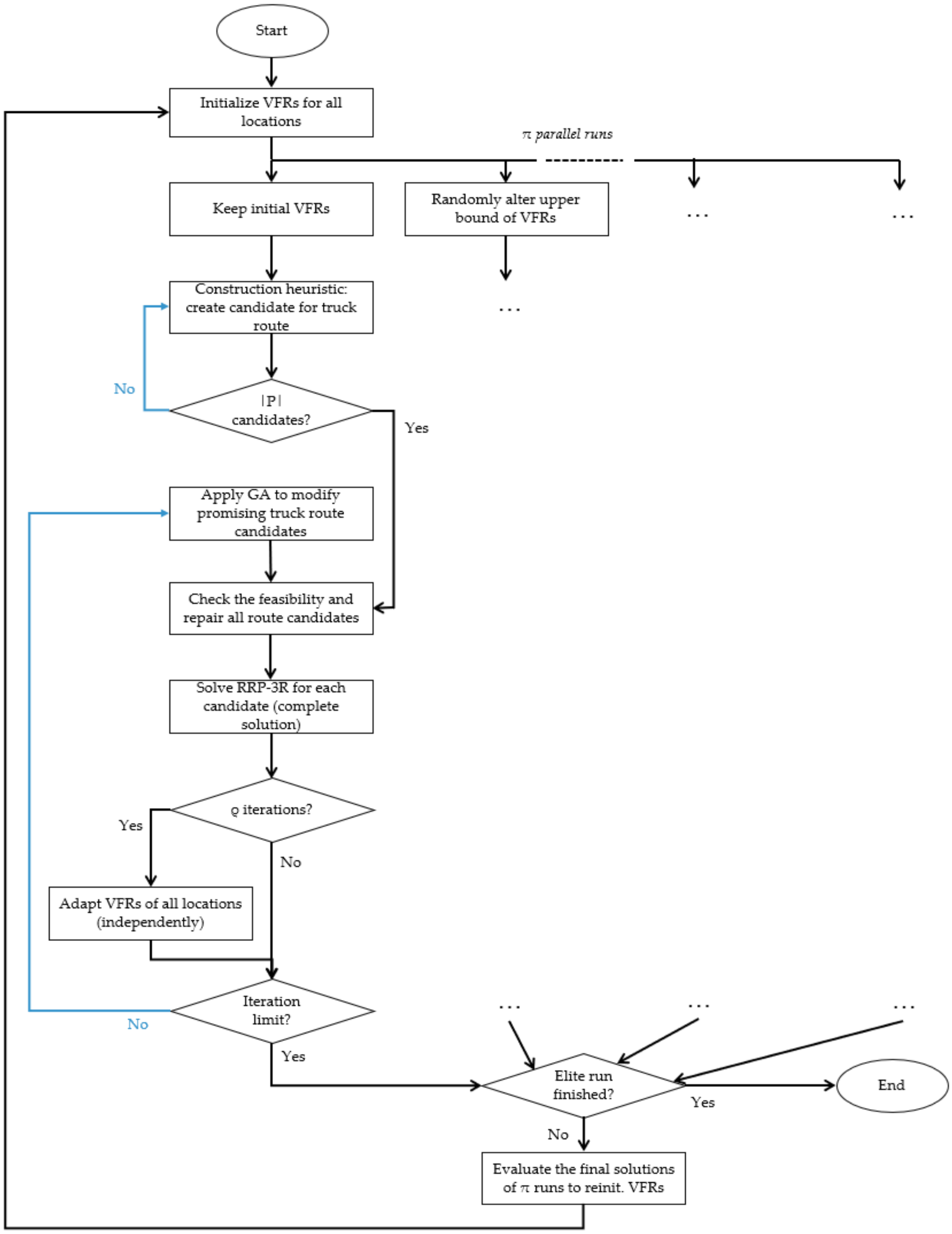

Figure B.9: Illustration of the detailed solution framework process

Appendix B.2. Construction Heuristic

Step i: Selection of locations for a route candidate. Figure B.10 illustrates the initial step of the construction heuristic. The roulette wheel represents the clusters ($\varrho = 1, 2, 3$) according to the locations' upper bounds, namely $ub_{\mathrm{R2}} = 3$, $ub_{\mathrm{R1}} = ub_{\mathrm{R3}} = ub_{\mathrm{D2}} = 2$, and $ub_{\mathrm{D1}} = 1$. For the sake of simplicity, we assume that the acceptance probability of including a location once selected is 100% in the example. We then select one cluster with probability $\mathbb{P}(\varrho) = \frac{ub_\varrho}{\sum_{\varrho=1}^{\overline{\varrho}} ub_\varrho}$, e.g., $\mathbb{P}(1) =$

$\frac{1}{\sum_{\varrho=1}^{3} ub_\varrho} = 0.167$. In the illustrative example, cluster 2 (with $ub_2 = 2$) has been chosen using the defined probabilities, and a random location (R1) is selected. Following this selection, the upper bound of R1 is reduced by one to $ub_{\text{R1}} = 1$ for the subsequent selection process, such that R1 is reassigned to cluster 1 with $ub_1 = 1$. In the next round, location R2 is selected; after its cluster update, no locations remain in cluster $ub_3 = 3$. Consequently, the probabilities of the remaining clusters are updated (33% for cluster 1 and 67% for cluster 2, respectively). The locations selected in this manner are subsequently incorporated into the routing process, as described in Section 4.2.

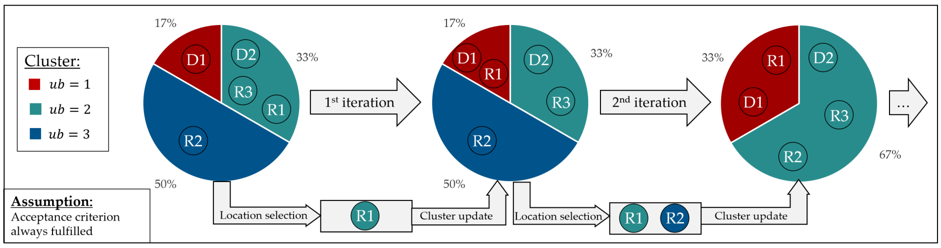

Figure B.10: Illustration of the location selection process (example)

Step ii: Routing of candidate route. A modified nearest-neighborhood search is employed for routing selected locations (see Step i) within the construction heuristic. It accounts for weighted truck travel time and delay costs and is based on the modified edge weights $\bar{c}_{ij}$:

$$\bar{c}_{ij} = t_{ij}^t \cdot c^t \cdot \alpha_p + \max(0, w_j - \tilde{b}_j) \cdot c_j^d \cdot (1 - \alpha_p).$$

Here, t_{ij}^t is the truck travel time between locations i and j, $i, j \in L$, w_j the arrival time at location j, and weight $\alpha_p = \frac{p}{|P|}$, $p = 1, \ldots, |P|$ depends on the current individual p. The first term then reflects the truck travel time costs, and the second term reflects the amount of delay depending on the artificial deadlines ($\tilde{b}_j$). Please note that customer deadlines are not changed such that $\tilde{b}_c = b_c, c \in C$.

Appendix C. RRP-3R: Robot movement examples

Figure C.11 illustrates the implicit robot tracking without explicit time indices within the RRP-3R. The illustrated truck route has eight stops, with stops 0 and 8 being the warehouse. The example shows the stops $j = 0, 1, \ldots, 8$ and the corresponding locations $f(j) \in F$ of the route. As such, the first stop $j = 1$ happens at an optional return shipment customer (C^S), from where a robot is sent to visit the delivery customer C_1. The robot visits C_1 and terminates its tour at the distinct robot depot R1. This depot is also part of the truck route, i.e., there is a corresponding set $G_r = \{3, \text{R1}\}$ with $r = \text{R1}$, and the third stop $f(3) = \text{R1}$. The robot arrives at the customer at time $t = 15$ and has a travel time of 10 minutes to its final destination. As a consequence, the robot can only return to the distinct robot depot location of R1 (green dot) and not stop R1 on the route ($j = 3$), as the truck arrives there at $t = 20$. Further, customer C_2 is visited by robot at time $t = 40$. From here, the robot returns to depot R3, which is visited twice on the truck route (at stops 5 and 7). This corresponds to the set $G_{\text{R3}} = \{5, 7, \text{R3}\}$. Since stop 5 ($f(5) = \text{R3}$) is not feasible as the truck arrives there before $t = 40$, only stop 7 and the distinct depot R3 remain as potential return options. To ensure consistent tracking of robot movements over time, the robot is constrained to select the earliest feasible option (Constraints (19)). Consequently, the robot continues to stop 7 ($f(7) = \text{R3}$), as the temporal condition $40 + 15 \leq 60 = t_7$ holds, and the latest possible arrival time at this stop ($t = 60$) is shorter than the arrival time at the distinct depot ($t_{\text{R3}} = \infty$).

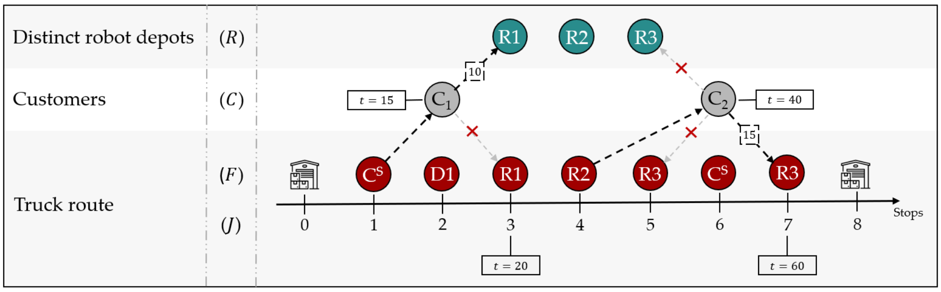

Figure C.11: Illustration of robot movements to track robot availability (example)

We require robots to travel to the earliest possible occurrence of a depot (i.e., truck stop or distinct robot depot) as the model does not include an explicit time index; however, we need to track the number of robots at the depots nonetheless. We achieve this by utilizing the stop nodes $j \in J$, which indicate the state of a location at various times along the truck route.

For illustration, consider Figure C.12. We use the notation $R1'$ and $R1''$ to indicate the two different stops of the truck at the distinct robot depot $R1$. This reflects the state of $R1$ at different times of the truck route. Note that all three states of $R1$ share the same initial robot availability o_r but map the robot availability at this location at different times (see examples of Constraints (21) in Figure C.12). When the truck arrives at $R1'$ at time 50 (third truck stop), exactly three robots are available. In detail, three robots were initially available; one robot left the depot and returned after completing the return shipment collection (service to C_2) before the truck arrived here at time 50. Since we ensure that no robot can approach the stop $R1''$ or the distinct robot depot $R1$ at this time (Constraints (19)), the number of robots at location $R1$ at time 50 is precisely known. When the truck arrives at $R1''$ at time 90, again three robots are available. Two robots departed by this time for serving customers C_2 and C_3, while two others entered $R1''$: one originating from $R1'$ (implying that it remains at the same location) and another arriving from delivery customer C_4. If the latter robot were to travel directly to the distinct location $R1$ instead of $R1''$, it would be unclear whether it arrives before the truck reaches $R1''$ (at time 90), making it impossible to track the number of robots at any given time.

Appendix D. Crossover operators

The crossover operators described in Section 4.4 are illustrated in Figure D.13. The illustration only demonstrates the general functionality of the operators and does not account for problem-specific aspects, such as probability-based inclusion calculations. Accordingly, we adopt the following assumptions: (i) Parent P_1 serves as the reference route, such that all offspring routes

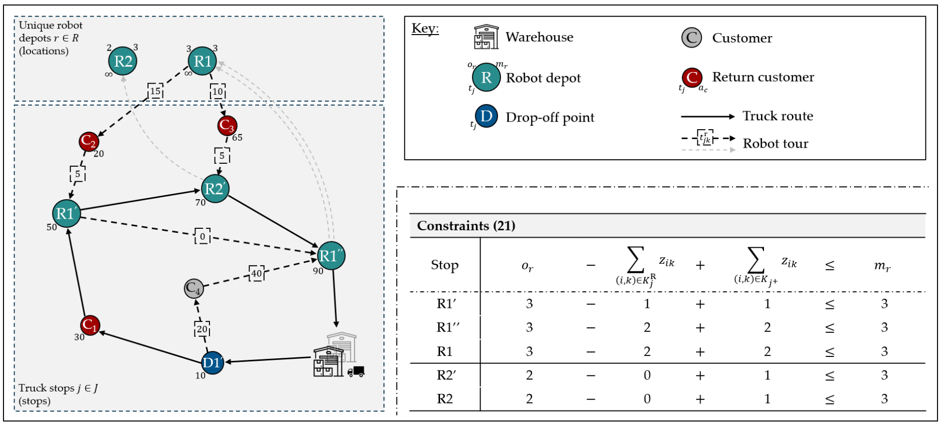

Figure C.12: Illustration of robot movements while ensuring depot capacity adherence (example)

are constrained to the length of P_1, and (ii) for every location i, we assume $\text{VFR}_i = [1; 1]$, which corresponds to a binary yes/no decision regarding the inclusion of the respective location.

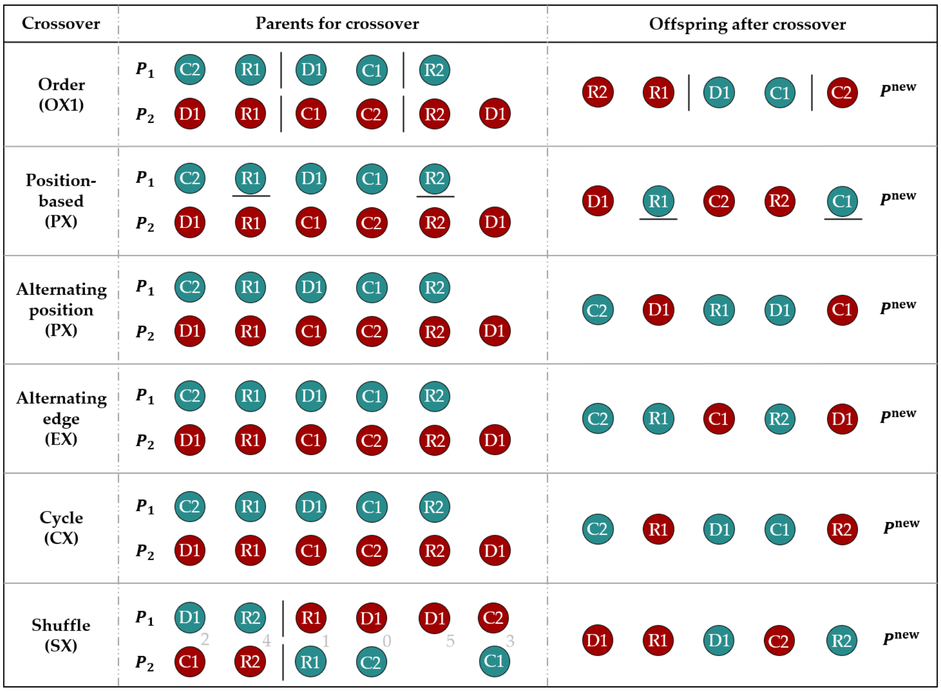

Figure D.13: Illustration of the different crossover processes (example)

Appendix E. Benchmark comparison to alternative exact approach

In addition to the exact comparison using complete enumeration, we compare the performance of the MH with that of Gurobi (version 12.1) as a standard solver. This enables us to validate the model's correctness and compare an alternative setting with an increased network that could not be solved using complete enumeration. However, the increased problem complexity renders the exact solution with Gurobi computationally intractable, even when linearizing the model developed in Appendix A.

We consider a setting with $|R| = 6$ and $|D| = 3$, and a reduced number of customer locations ranging from 6 to 21. The basic configuration of 40% return customers and 20% truck-mandatory customers is maintained. We further set the initial robot availability on the truck and at each robot depot to $I^{\text{r}} = o_r = 3$ for all $r \in R$, with a maximum depot capacity of $m_r = 5$. Finally, we limit the number of visits per location to one and the solver's runtime to 3 hours. Table E.9 illustrates the results.

	Gurobi				MH		Improvement	
$\|C\|$	Costs [CU]	Gap [%]	Time [s]	No. opt.[1]	Costs [CU]	Time [s]	Δ^2[%]	Time [%]
6	18.51	47.27	10,078	2	18.24	39	1.47	99.61
9	25.43	68.22	10,800	0	24.90	51	2.11	99.53
12	28.79	70.41	10,800	0	27.45	64	4.77	99.41
15	32.80	76.23	10,800	0	30.13	74	8.49	99.32
18	37.53	76.50	10,800	0	33.23	89	12.15	99.18
21	42.55	79.43	10,800	0	37.49	102	12.64	99.06
Average	30.94	69.68	10,680	0.33	28.57	65	6.94	99.35

[1] Number of instances solved to optimality.
[2] Gap of the MH solution compared to the solution found by Gurobi.

Table E.9: Performance comparison with exact approach (Gurobi) (*avg. of 25 instances*)

The results show substantial optimality gaps even for the smallest instances: from an average gap of 47% for 6 customers to a gap of almost 80% for 21 customers. Only two instances with 6 customers can be solved to optimality by Gurobi, and the MH achieves the same solution in these cases. The MH solves all instances with well below two minutes while simultaneously improving solution quality.