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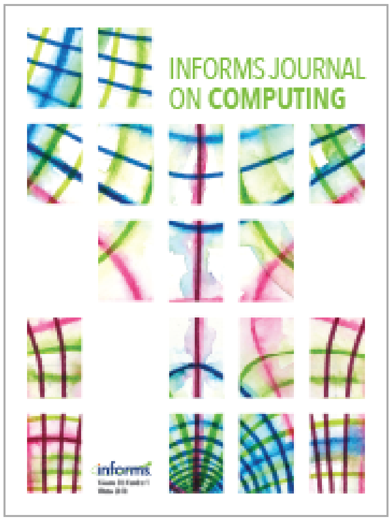

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The Weighted Set Covering Game: A Vaccine Pricing Model for Pediatric Immunization

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The United States pediatric vaccine manufacturing market is analyzed using a static Bertrand oligopoly pricing model that characterizes oligopolistic interactions between asymmetric firms in a homogeneous multiple product market. Firms satisfy demand by appropriately pricing and selling its given set of bundles, where each bundle contains one or more products. In analyzing the pediatric vaccine market, a bundle is a vaccine, where each vaccine contains one or more immunogenic antigens. Consumers seek to purchase at least one of each antigen at an overall minimum cost. Demand is captured by defining a weighted set covering optimization problem, with the weights (prices) controlled by firms engaged in Bertrand competition. A repeated game version of the model enables multiple interactions between firms, allowing examination of tacit collusion. An iterative improvement algorithm is defined that constructs a pure strategy Nash equilibrium (some in the limiting sense) for the static game. Sufficient conditions for the existence of pure strategy Nash equilibria are provided, indicating that this class of games always yields at least one pure strategy equilibrium. Practical results of the pediatric vaccine market analysis follow from the difference in the repeated game equilibrium prices between two combination vaccines, Pediarix® and Pentacel®. Assuming the manufacturers of these vaccines agree to share the market equally with respect to volume, the equilibrium prices from the repeated game indicate a price difference of $0.86, whereas the difference in price between Pediarix® and Pentacel® for contract prices ending March 31, 2010 was $2.74. Interestingly, the subsequent public sector vaccine price list (contract prices ending March 31, 2011) shows a price difference of $0.95, with the price of Pentacel® actually *reduced* from the previous year—an unusual occurrence. The results presented in this paper suggest that a smaller price difference between these two important combination vaccines is appropriate, which is what occurred. In general, such results could serve to inform both manufacturers and purchasers on the appropriate pricing of combination vaccines, given the existence of a reasonable set of collusive agreements.

Key words: game theory; weighted set covering game; immunization; pediatric vaccines
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1. Introduction

Noncooperative game theory provides an appropriate technique for analyzing economic conflict between firms when collusion is forbidden (Vives 1999). A firm's determination of the proper pricing strategy when its profits are affected by the pricing decisions of other firms in the market lends itself well to game theory. Myerson (1999) defines game theory as the study of mathematical models of conflict and cooperation between intelligent and rational decision makers (firms). An *intelligent* firm knows everything there is to know about the game. A *rational* firm acts in a consistent manner pursuant to its own objectives (i.e., maximize its own profit). The motivating principle of this method of analysis is the understanding of fundamental issues underlying the real market of interest.

Models of oligopolistic interaction depict a finite and (typically) small number of firms competing in a homogeneous product market. The strategic variable of interest for each firm depends on the specific model that is implemented. In the classical model set forth by Bertrand (1883), each firm's strategic variable is the price of the homogeneous product. The market reacts to the offered prices by first demanding an attendant quantity and then clearing by some unspecified mechanism. It is assumed that the lowest price firm(s) must supply the entire market demand. In contrast, in the classical model proposed by Cournot (1838), each firm's strategic variable is its quantity produced. The market reacts to the aggregate production level of the firms by first setting a price and then clearing by some unspecified mechanism. Note that market *clearing* refers to the process by which markets gravitate toward prices that balance quantity supplied and quantity demanded so that in the long run, the market is cleared of all surpluses and shortages.

The appropriateness of a model depends on the basic structure of the market of interest. For example, the Bertrand model is well suited to production-to-order markets (e.g., various service industries; Phillips et al. 2001). Conversely, the Cournot model is well suited to production-to-stock markets (e.g., agricultural products, automobiles). Vives (1999) also notes that the industry variable that is more difficult to adjust mid-process because of contractual obligations or other extenuating circumstances, could be the dominant strategic variable, even in industries where the other strategic variable is typically dominant. Indeed, the motivating domain problem of interest in this paper reflects such a situation. In the U.S. pediatric vaccine market, pharmaceutical companies manufacture vaccines on a production-to-stock basis, yet the industry is best modeled as a Bertrand competition since vaccine prices (in the public sector) are fixed for one year periods due to contractual obligations negotiated by the Centers for Disease Control and Prevention (CDC). One important aspect of the vaccine production market is its very long research, development, and production lead times. The analysis presented in this paper treats only the subsequent pricing decision. The premise is that a particular set of pediatric vaccines has arrived at the market and is available for purchase by consumers. Research, development, and production quantity issues are not examined.

A review of the relevant literature suggests a need for a pricing model treating a market exhibiting a demand structure with a highly combinatorial and interdependent nature. The model presented in this paper allows for the analysis of markets where consumers face a weighted set-covering optimization problem (in which a minimum cost set cover is sought) with several competing firms setting the applicable weights in the problem (to maximize individual firm profit). The weighted set covering problem is the following: given S, a set of elements, and B, a set of weighted subsets of S, find a minimum cost collection C of subsets from B such that C covers all elements in S. Given that customers are assumed to be local government public health agencies, a customer can use the solution to a weighted set covering as a guide for purchasing since it is reasonable to assume governments would use more resources to analyze a problem compared to a single individual.

No game has been reported in the literature to account for markets with such a demand system. Although many studies concentrate on pricing behavior in markets with multiproduct firms, studies typically consider markets with differentiated products and demand systems portrayed by smooth (or at least, twice differentiable) functions (Tirole 1988, Vives 1999). Distinguishing characteristics of the results presented in this paper involve the structure of the demand system; a smooth demand function is attributed only to the aggregate cost of the minimum weighted cover (i.e., the group of chosen bundles). An additional combinatorial complication results from the overlapping bundles of products offered by the competing firms. Demand for a bundle is determined by the set of solutions to the defining weighted set covering problem, indicating the discontinuous nature of the market structure.

This paper explores a generalization of the classical Bertrand model. The proposed Bertrand oligopoly pricing model, in the form of a static strategic game, characterizes interactions between asymmetric firms in a homogeneous multiple product market with a combinatorial and interdependent demand structure. A firm controls a given collection of bundles containing one or more products and must determine the price for each of its bundles to maximize profit. The fundamental questions regarding the problem under investigation are the following: How should manufacturers price their set of bundles in an oligopolistic market? Are there equilibrium prices for a game modeling such a market? What if tacit collusion is present? Note that the structure of the game is chosen in part because the combinatorial and interdependent nature of the demand is the primary aspect of the problem under investigation and in part because of tractability concerns. Moreover, it is important to note that in this game formulation, each manufacturer's particular set of bundles is *given* and part of the structure of the game; manufacturers decide only on a price for each bundle, not which bundles are offered to the consumer.

The solution concept employed to analyze the formulated game is the Nash equilibrium (Nash 1951). The classical Bertrand equilibrium is easily seen as a forerunner to the modern game theory solution concept provided by Nash. When examining markets using a Bertrand framework, it is important to consider three critical assumptions. The first assumption relates to production capacity; Bertrand competition assumes that any firm can fully satisfy market demand. The second assumption relates to the temporal aspect of the competition; firms engage in competition only once. The third assumption relates to product differentiation; the firms' products are assumed to be perfectly interchangeable. Together, these assumptions depict an extreme economic situation in which only two firms are required to induce a perfectly competitive market (i.e., prices fall to marginal cost, providing very low economic profit to the competing firms). Moreover, because of the stringency of the assumptions, the resulting Nash equilibrium is often economically naive and unrealistic. To address these concerns, the temporal aspect of the model is relaxed to allow for repeated interaction between the firms. This relaxation provides a noncooperative game theoretic mechanism to expand the set of Nash equilibria to include more realistic behavior.

A pure strategy Nash equilibrium (in the limiting sense) for the static game is sought to analyze the depicted market. The repeated game version of the model enables repeated interaction between firms, allowing examination of tacit collusion. An iterative improvement algorithm is defined that constructs a pure strategy Nash equilibrium (in the limiting sense) for the static game. Sufficient conditions for the existence of pure strategy Nash equilibria (in the limiting sense) are provided, indicating that the proposed class of games always yields at least one pure strategy equilibrium (in the limiting sense). However, the pure strategy Nash equilibria provided are not proved to be unique.

The concept of a pure strategy Nash equilibrium *in the limiting sense* merits further discussion. To illustrate, consider a Bertrand duopoly where two firms face a price competition with asymmetric costs, where $\hat{c}_1 < \hat{c}_2$, over the sale of a single homogeneous good. If the monopoly price is greater than or equal to the second firm's cost, then the optimal price for the first firm does not exist *in a strict sense* because the first firm will always be better off by setting its price ever closer to $\hat{c}_2$ (i.e., charging $\hat{c}_2 - \epsilon$, with ϵ small and positive). In Bertrand pricing games, the sharing rule identifies how demand is split between firms that charge a common, lowest price. The absence of an equilibrium is overcome by adjusting the sharing rule and defining the equilibrium in the limit. An equilibrium in the limiting sense is a price point $(\hat{p}_1 = \hat{c}_2, \hat{p}_2 = \hat{c}_2)$ where the first firm sets its price $\hat{p}_1$ equal to the second firm's cost and receives the entire market demand, earning a unit profit of $\hat{c}_2 - \hat{c}_1$ and where the second firm sets its price $\hat{p}_2$ as low as possible, at its own unit cost, receives no demand, and earns no profit.

The paper is organized as follows: §2 provides a description of the proposed oligopoly pricing models and states the main Nash equilibrium existence results. Section 3 demonstrates the utility of two games by applying them to the analysis of the U.S. public sector pediatric vaccine market. Section 4 provides concluding comments and directions for future research. In the online supplement (available as supplemental material at http://dx.doi.org/10.1287/ijoc.2013.0556), §1 describes the computational difficulty of computing pure strategy Nash equilibria (some in the limiting sense) for the weighted set-covering game and introduces an iterative improvement algorithm devised to compute such equilibria. In the online supplement, §2 provides proofs of the conditions required for the existence of pure strategy Nash equilibria (some in the limiting sense), and §3 provides a table of notations.

2. The Games

This section describes the oligopoly pricing models formulated to characterize oligopolistic interactions between asymmetric firms in a homogeneous multiple product market. Several factors complicate such interactions, most notably the specification of demand from the solution to an integer program (IP). Analysis indicates that a pure strategy Nash equilibrium for the weighted set-covering game always exists. The repeated game version of the model relaxes the static game's temporal assumption of a single interaction, enabling repeated interactions between the firms. We present conditions for the existence of a subgame perfect Nash equilibrium in the repeated game.

2.1. The Weighted Set Covering Game

This section describes the weighted set covering game, a Bertrand oligopoly pricing model. Consider a simultaneous move, single stage, complete information game. The simplest way to portray such a game is in an appended strategic form, consisting of five components: (1) the set of players (firms); (2) the appended game structure (i.e., the weighted set-covering optimization problem); (3) the manner in which players interact with the appended game structure; (4) the set of strategies (prices) available to each player; and (5) the manner in which the players' payoffs (profits) depend on the strategies chosen. Each firm attempts to maximize its profits by independently selecting an appropriate pricing strategy, knowing only the structure of the game (i.e., firms are rational and intelligent). Thus, each firm must consider the pricing strategies that the other firms are likely to select. Moreover, in oligopoly pricing models, such as the one reported here, important elements in the determination of each firm's profits are the relevant market demand structure and its own cost structure (Vives 1999). Several definitions are required to precisely describe the resulting game.

Let N denote the set of firms in the market. Firms produce a collection of bundles containing one or more homogeneous products. Homogeneous products originating from different firms are perceived as identical products by consumers; when appropriate, an additional marginal cost component is included to account for minor product differentiation. The aggregate collection of the firms' bundles allows characterization of market demand.

There is no direct characterization of demand. A weighted set covering optimization problem describes market demand with respect to a single consumer, where S is the set of homogeneous products (i.e., immunogenic antigens) and B, a set of subsets of S, is the set of bundles of products (i.e., pediatric vaccines) available for purchase from the firms in N. A rational consumer seeks to obtain at least one of every homogeneous product in S at the lowest possible overall cost by purchasing bundles in B (i.e., a consumer seeks to find a minimum cost collection C of bundles from B such that C covers all of the elements in S). The minimum weighted cover C ensures $\bigcup_{j\in C}\{j\} \supseteq S$ at overall minimum cost. During algorithm execution, as optimal covers are found they are held in R, and binary cuts (see Balas and Jeroslow 1972) are employed to render those covers infeasible. This is done to enable the identification of alternate optimal solutions. The defining weighted set covering optimization problem is parameterized by $W = (S, B, (w_j)_{j\in B}, (\tilde{c}_j)_{j\in B}, \tau, R)$ and is modeled as a 0–1 IP:

$$\text{WSC}(W) \quad \text{Minimize} \sum_{j\in B} (w_j + \tilde{c}_j + \tau) x_j \tag{O}$$

$$\text{subject to} \sum_{j\in B} a_{ij} x_j \ge 1 \quad \text{for all } i \in S, \tag{1}$$

$$x_j \in \{0, 1\} \quad \text{for all } j \in B, \tag{2}$$

$$\sum_{j\in \hat{C}} x_j - \sum_{j\notin \hat{C}} x_j \le |\hat{C}| - 1 \quad \text{for all } \hat{C} \in R, \tag{3}$$

where

w_j is the price corresponding to bundle $j \in B$,

$\tilde{c}_j \ge 0$ is the product differentiation price adjustment for bundle $j \in B$,

$\tau \ge 0$ is a penalty cost for including a bundle in cover $C \subseteq B$,

$a_{ij} = 1(0)$ if product $i \in S$ is in bundle $j \in B$, $(j \notin B)$, and

$x_j = 1(0)$ is customer's decision variable that indicates bundle $j \in C$, $(j \notin C)$.

WSC(W) is solved for the optimal (i.e., minimum weighted) cover C. The objective function (O) minimizes the weighted sum of the costs to obtain at least one of every product in S. The inclusion of τ and $\tilde{c}_j$ are motivated by the desire to account for the cost of injection and vaccine preparation costs in the analysis of the pediatric vaccine market. See Glazner et al. (2004) for a detailed discussion regarding the costs to healthcare providers for delivering childhood vaccinations. Constraint (1) ensures that at least one of every product in S is obtained. Constraint (2) ensures that the binary requirements of the decision variables are met. Constraint (3) ensures that covers deemed infeasible as a solution are not selected (Balas and Jeroslow 1972).

To capture the relationship between firms and their products, define the set-valued map $g\colon N \to B$, where $g(f) \subseteq B$ is the set of bundles produced by firm $f \in N$. Define

$$K_f = \prod_{j\in g(f)} K_j$$

as the Cartesian product set of prices available to each firm f, where $K_j = \{w_j \in \Re \mid c_j \le w_j \le \beta_j\}$ is the closed interval of available prices for a bundle of products $j \in g(f)$. The weight w_j is the only player decision variable; it represents the price specified by firm f for bundle $j \in g(f)$. Each bundle j has a total unit cost of production c_j and a price upper bound β_j, such that $0 \le c_j \le \beta_j$. A no-bankruptcy assumption prevents the pricing of a bundle under its marginal cost. By design, the set of prices is compact and convex.

When the game is played, each firm f must simultaneously select one of the pricing tuples in the set K_f (i.e., each firm must select a feasible price for each of its bundles). The combination of bundle prices that the firms in N collectively select is referred to as a *price point*, $\mathbf{w} = (w_j)_{j\in B}$. The set of prices available to any firm f may differ from the set of prices available to any firm i because of possibly different cost structures or upper bounds on the set of available prices. When the collection of bundles produced by any two firms differ, or their respective cost structures differ, the game is considered *asymmetric*.

The solution to WSC(W) depicts consumer demand behavior given the market conditions described by W. With the exception of the price point $\mathbf{w}$ and the set of restricted covers R, the market conditions depicted by W remain unchanged. Thus, the market conditions are often described only by the price point $\mathbf{w}$ with R empty, unless otherwise noted. Let the minimum weighted set cover C denote the solution to WSC(W) and let $z(\mathbf{w}) = \sum_{j\in C}(w_j + \tilde{c}_j + \tau)x_j$ denote the associated overall cost of C. Define $X(\mathbf{w})$ as the collection of all minimum weighted covers available at the price point $\mathbf{w}$. The collection $X(\mathbf{w})$ enables specification of the market share for each bundle (i.e., how many of each bundle is purchased per customer). Define $\psi_j(\mathbf{w}) = \sum_{C\in X(\mathbf{w})} I_{C_j}/|X(\mathbf{w})| \in [0,1]$ as the market share of bundle $j \in B$ at the market conditions described by W, where $I_{C_j} = 1(0)$ if bundle $j \in (\notin) C \in X(\mathbf{w})$. Assume that when there is more than one optimal cover, demand is shared equally among the optimal covers.

The aggregate market demand function D: $\Re \to \Re$ specifies the total quantity of bundles purchased as a function of the overall cost of the minimum weighted cover and is of the form

$$D(z(\mathbf{w})) = d - \eta(z(\mathbf{w}))^{\lambda}, \tag{4}$$

where d, η, and λ are such that D is twice continuously differentiable and concave (i.e., $d^2D/dz^2 \le 0$). This form enables specification of conventional demand functions typically found in the literature (Tirole 1988, Vives 1999).

To complete the description of the game, denote the payoff function of each firm $f \in N$ as

$$\pi_f(\mathbf{w}) = D(z(\mathbf{w})) \sum_{j\in g(f)} (\psi_j(\mathbf{w})(w_j - c_j)). \tag{5}$$

Each firm f receives the sum of the unit profits for each bundle $j \in g(f)$ present in each cover $C \in X(\mathbf{w})$ times the number of covers purchased by consumers. In the case where multiple optimal covers are available (i.e., $|X(\mathbf{w})| > 1$), the market share of a particular bundle is determined by the number of optimal covers in $X(\mathbf{w})$ in which it is present. For example, if $X(\mathbf{w}) = \{\{1,2\},\{2,3\}\}$ and $D(z(\mathbf{w})) = 100$, then $\psi_1(\mathbf{w}) = 0.5$, $\psi_2(\mathbf{w}) = 1.0$, and $\psi_3(\mathbf{w}) = 0.5$, resulting in a demand of 50, 100, and 50 for bundles 1, 2, and 3, respectively.

The *weighted set covering game* Γ is given by

$$\Gamma = (N, W, g(N), (K_f)_{f\in N}, (\pi_f)_{f\in N}).$$

The game theoretic solution to Γ is now examined. Analysis reveals insights regarding long-term market profit prospects that are important to both consumers and firms. Solution concepts in the study of games differ mostly with respect to the level of collusion or cooperation allowed between firms (Myerson 1999). In this study, for the pure strategy Nash equilibrium sought in Γ, no cooperation is permitted.

As Vives (1999) notes, in most situations typifying economic conflict, two or more firms make decisions that influence each other's profits, whereas operating in a market environment in which binding legal contracts may not be enforceable (e.g., antitrust laws). In such situations, noncooperative game theory provides an appropriate mathematical framework for analysis because it enables the determination of a rational prediction regarding the outcome of the game.

Nash's concept of equilibrium (Nash 1951) is the central solution concept in noncooperative game theory and consequently to the study of oligopoly pricing models (Vives 1999). A Nash equilibrium (Definition 1) is a set of pricing strategies (i.e., a price point) such that no firm can unilaterally deviate from its strategy to realize a gain in profits.

Definition 1 (Nash Equilibrium). In Γ, a price point $\mathbf{w}^* = (\mathbf{w}^*_f; \mathbf{w}^*_{-f})$ constitutes a pure strategy Nash equilibrium if for any firm $f \in N$, $\pi_f(\mathbf{w}^*_f; \mathbf{w}^*_{-f}) \ge \pi_f(\mathbf{w}_f; \mathbf{w}^*_{-f})$, for all other price points $\mathbf{w}_f$, where $\mathbf{w}^*_f \in K_f$ denotes the set of bundle prices controlled by firm f, and $\mathbf{w}^*_{-f} \in K_{-f}$ denotes the set of bundle prices controlled by firms other than f.

Determining the existence of and efficiently computing Nash equilibria are important areas of research (see, for example, Nisan et al. 2007, Philip 1999, Roughgarden 2010). Although most studies concentrate on mixed strategy Nash equilibria, this effort focuses on pure strategy Nash equilibria. Unfortunately, as Vives (1999, p. 15) notes, "Nonexistence of Nash equilibria in pure strategies is pervasive in oligopoly models." Note that Nash equilibria in this paper refer only to pure strategy Nash equilibria (some in the limiting sense).

Gamma instance Γ is a game with infinite strategy spaces and models a strategic economic situation in which firms may choose from a continuum of prices. As with Bertrand's classic game (Bertrand 1883), Γ exhibits discontinuities in the firms' payoffs. For example, when determining demand for the simple case in which a single homogeneous product is produced by two firms and both products are equally priced, the tie must be broken according to some predetermined sharing rule. This rule ultimately leads to a discontinuity in the firms' payoffs since one firm's slight decrease in the price of the product results in it obtaining full market demand, a discontinuous increase when compared to the demand it received when prices were equal.

As a consequence of the discontinuous nature of Γ, standard theorems found in the literature (see, for example, Dasgupta and Maskin 1986, Debreu 1952, Glicksberg 1952, Nash 1950) cannot be applied to establish the existence of pure strategy Nash equilibria. However, using the algorithm detailed in §1 of the online supplement, pure strategy Nash equilibria (some in the limiting sense) can always be constructed. The following theorem states the main result, proven in §2 of the online supplement.

Theorem 1 (Static Game Equilibrium Existence). *Given an instance of* Γ, *a pure strategy Nash equilibrium always exists.*

The results presented in this paper assume that a pure strategy Nash equilibrium can exist in the limiting sense. In Γ, the concept of an equilibrium in the limiting sense allows a firm with an absolute advantage to increase the total price of a lowest overall cost cover up to the total price of the next lowest cost cover. The next lowest cost cover receives no demand, and the firms whose bundles are present in the next lowest cost cover earn no profit.

2.2. The Repeated Weighted Set Covering Game

This section introduces the repeated weighted set covering game. The resulting analysis addresses the temporal assumption in Γ of a single economic interaction between firms. In reality, firms are likely to interact repeatedly in the market of interest. Adoption of a repeated game structure enables exploration of more sustainable and higher profit price points. As Myerson (1999) notes, firms in the same market may behave quite differently toward one another when there is an expectation of a long-term relationship involving repeated interaction. In the classic symmetric Bertrand game, the Nash equilibrium provides zero economic profit for the competing firms. In such a situation, firms would like to transform the game and extend the set of Nash equilibria to include higher profit results (Myerson 1999, Tirole 1988).

In a repeated game, a firm must consider the effect of its current pricing strategy on the pricing strategies of other firms in the future and the attendant impact on its own future profits. Such considerations almost certainly lead to more cooperative behavior, assuming the firms value future profits highly enough. The temporal extension to Γ enables examination of tacit collusion in the market and its effect on profits and costs for the firms and consumers involved, respectively. The possibility of tacit collusion is entirely enabled by a noncooperative game theoretic mechanism.

Consider the repeated weighted set covering game with standard information, where the exact same instance of Γ is replicated an infinite number of times. Several definitions are required to precisely describe the repeated game. Define $\pi_f(\mathbf{w}^{(t)})$ as firm f's profit for time period t, where $t = 0, 1, \ldots,$ with the bundles of products in B priced at $\mathbf{w}^{(t)}$. The length of time represented by each period depends on the particular situation modeled. In the repeated game, firm f wants to maximize its discounted average profit,

$$(1-\delta_f)\sum_{t=1}^{+\infty}\delta_f^{t-1}\pi_f(\mathbf{w}^{(t)}), \tag{6}$$

where $\pi_f(\mathbf{w}^{(t)})$ is determined according to (5) and $\delta_f \in [0, 1)$ is a discount factor, a measure of the patience or long-term financial perspective of firm f.

At each time period t, the firms in N simultaneously select prices for the current Γ replication. Each firm's pricing strategy may depend on the history of the prices set in replications prior to t, $H^{(t)} = \{\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \ldots, \mathbf{w}^{(t-1)}\}$. Each firm is able to perfectly recall other firms' past pricing decisions. A pricing strategy for firm f, $\sigma_f(H^{(t)}) \in \mathcal{A}_f$, specifies a price $w_j^{(t)}$ for each bundle $j \in g(f)$ for every possible sequence of outcomes $\{\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \ldots, \mathbf{w}^{(t-1)}\}$ of the repeated game, where $\mathcal{A}_f$ is the set of all pricing strategies for firm f. A subgame perfect Nash equilibrium is then sought where for every firm $f \in N$ and any history $H^{(t)}$, the strategy employed by firm f for periods $t, t+1, \ldots$ maximizes (6).

The *repeated weighted set covering game* Γ^r is given by

$$\Gamma^r = \big(\Gamma, (\mathcal{A}_f)_{f\in N}, H^{(t)}, (\delta_f)_{f\in N}\big).$$

The analysis of Γ^r proceeds by formulating a strategy for each firm $f \in N$ that induces subgame perfect equilibria with profits greater than those earned with Nash equilibria in Γ. Denote $\mathbf{w}^*$ and π_f^* as a Nash equilibrium price point and corresponding profit for firm f, respectively, in the static game Γ. Consider the following *grim trigger strategy*: let each firm $f \in N$ tacitly agree to a mutually beneficial price point $\hat{\mathbf{w}}$, where at each time period t, firm f charges $(w_j^{(t)})_{j\in g(f)} = (\hat{w}_j)_{j\in g(f)}$ and produces $\psi_j D(z(\mathbf{w}^{(t)}))$ of each bundle $j \in g(f)$. Firm f maintains this collusive agreement provided $(w_j^{(\hat{t})})_{j\in g(-f)} = (\hat{w}_j)_{j\in g(-f)}$ for all $\hat{t} < t$, where $g(-f)$ is the set of bundles in B not controlled by firm f. Otherwise, firm f reverts to Bertrand behavior by setting prices at $(w_j^*)_{j\in g(f)}$ for all time periods beyond t.

More formally, define the grim trigger strategy $\sigma_f(H^{(t)}) \in \mathcal{A}_f$ as

$$\sigma_f(H^{(t)}) = \begin{cases} (\hat{w}_j)_{j\in g(f)} & \text{if } H^{(t)} \text{ is empty,}\\ (\hat{w}_j)_{j\in g(f)} & \text{if } w_j^{(\hat{t})} = \hat{w}_j,\ j \in g(-f),\\ & \qquad\qquad \text{for all } \hat{t} < t,\\ (w_j^*)_{j\in g(f)} & \text{otherwise.}\end{cases}$$

Under the grim trigger strategy, a firm $f \in N$ selects high prices and receives a higher profit than is achieved should it and the other firms engage in Bertrand behavior. Cooperation is maintained until an opposing firm deviates by undercutting its price. When deviating, the opposing firm receives a (possibly substantial) short-term gain in profit but would receive its lower Nash equilibrium profit in all later periods because firm f responds to the deviation by setting a low Nash equilibrium price ad infinitum. Such unforgiving punishment may appear extreme, yet it is the threat of this punitive action that induces cooperative play. There are other less punitive strategies that may induce equilibria. Myerson (1999) discusses a variety of strategies in the context of repeated games (e.g., *tit-for-tat, getting even, limited punishment,* and *mutual punishment*).

In a standard repeated game like Γ^r, when firms are sufficiently patient, almost any feasible price point can be realized in an equilibrium. In the game theory literature, the theorems proving such results are often referred to as *general feasibility theorems* (see Friedman 1971, Fudenberg and Maskin 1986, Myerson 1999, Rubinstein 1977). Fudenberg and Maskin (1986) provide a general feasibility theorem for subgame perfect Nash equilibria of standard repeated games with discounting. In particular, they provide proof that, given a collusive agreement stipulating an equilibrium price point at which all firms receive a payoff greater than the payoff they can achieve by acting unilaterally, a discount factor exists that sustains the equilibrium.

In the analysis of Γ^r, examining each firm f's optimal deviation from the stipulated equilibrium price point, $\hat{\mathbf{w}}$ provides the desired conditions on each firm's discount factor, δ_f, necessary for sustaining the equilibrium. For the collusive agreement to be rational and the grim trigger strategy equilibrium to be maintained, the short-term gain must be less than or equal to the long-term loss for every firm. The profit stream for maintaining the collusive agreement is $(\pi_f(\hat{\mathbf{w}}), \pi_f(\hat{\mathbf{w}}), \ldots)$, resulting in a discounted average of $\pi_f(\hat{\mathbf{w}})$. The profit stream for deviating for a short term gain is $(\pi_f^d, \pi_f^*, \pi_f^*, \ldots)$, resulting in a discounted average of $(1-\delta_f)(\pi_f^d + (\pi_f^*\delta_f)/(1-\delta_f))$, where π_f^d is the maximum profit attainable by firm f should it deviate from $\hat{\mathbf{w}}$. Conditions necessary for the sustainment of the grim trigger strategy equilibrium is given by

$$\delta_f \geq \frac{\pi_f^d - \pi_f(\hat{\mathbf{w}})}{\pi_f^d - \pi_f^*}, \quad \text{for all } f \in N, \tag{7}$$

which leads to the main existence result for Γ^r.

Theorem 2 (Repeated Game Equilibrium Existence). *If $(\pi_f(\hat{\mathbf{w}}))_{f\in N}$ Pareto dominates the payoffs $(\pi_f^*)_{f\in N}$ of a Nash equilibrium $\mathbf{w}^*$ of the static game Γ, then if $\delta_f \geq (\pi_f^d - \pi_f(\hat{\mathbf{w}}))/(\pi_f^d - \pi_f^*)$ for all $f \in N$, there exists a subgame perfect equilibrium of the infinitely repeated game Γ^r, where the δ-discounted average of profits for firm f is $\pi_f(\hat{\mathbf{w}})$.*

Proof. The result follows from Fudenberg and Maskin (1986) and (7). □

One can argue that the theory of repeated games is too successful in explaining tacit collusion because it can be used to justify nearly any feasible price point as a Nash equilibrium. Indeed, as Tirole (1988, p. 247) notes, the large set of equilibria is an "embarrassment of riches." In some manner, firms must coordinate on a *focal equilibrium*. How this focal equilibrium is selected remains an important issue. Schelling (1960) considered the matter in great detail, arguing that any galvanizing force that focuses the firms' attention on a particular equilibrium point is a *focal effect*, facilitating the selection of that price point (akin to the satisfaction of a self-fulfilling prophecy). Welfare properties of *efficiency* and *equity* may determine the focal equilibria (Myerson 1999). If only one focal equilibrium exists (because of galvanizing, exogenous focal effects), then one should expect to see it realized.

3. The U.S. Pediatric Vaccine Market

This section demonstrates the value of Γ and Γ^r by applying them to the U.S. public sector pediatric vaccine market. Three different scenarios are examined. The first scenario establishes the economic profit of the vaccine manufacturers based on contract prices ending March 31, 2010. The second scenario applies Γ to examine the impact of a Bertrand price competition on the profit levels of the competing vaccine manufacturers. The third scenario applies Γ^r examine the ramifications of tacit collusion on the market. This section begins with a brief description of the market and concludes with a discussion of limitations and general results.

3.1. Market Description

The development of pediatric vaccines is a difficult and costly endeavor. In the U.S. pediatric vaccine market, a relatively small number of pharmaceutical firms engage in the research, development, manufacture, sales, marketing, and distribution of pediatric vaccines (Douglas et al. 2008). All pediatric vaccines distributed in the United States are manufactured privately, with no obligation to sustain or initiate the production of pediatric vaccines, regardless of the importance of such vaccines to public health. Multiple stakeholders influence the development, licensing, production, and distribution of pediatric vaccines. It behooves these stakeholders to be aware of the complexities of the market in which they participate. The economic competition between pharmaceutical firms and the impact of public policies on the market provides a rich domain for analysis.

A number of public health policy studies examine the issue of vaccine pricing in the U.S. vaccine market (Douglas et al. 2008, Hinman 2005, McGuire 2003, Offit 2005, Orenstein et al. 2005, Poland and Marcuse 2004). For example, Hinman (2005) suggests pricing a vaccine in advance of its development based on its estimated social value. McGuire (2003) offers an economic model to facilitate the determination of such prices, reporting that although vaccines have high social value (see Zhou et al. 2005 for a full analysis concerning the economic benefit of vaccines to society), the vaccine manufacturers do not receive appropriate financial incentives for participation in the market. Behzad et al. (2012) report and discuss the set of vaccines with lowest overall cost over three years (2009–2011) for three *cost of injection* values. Freed et al. (2008) study the range of prices paid for the vaccines in private sector using a cross-sectional survey.

Prior operations research studies in this area examine the selection of an optimal set of vaccines for purchase from a consumer perspective (Hall et al. 2008, Jacobson et al. 1999, Weniger et al. 1998) or the determination of optimal vaccine prices from a producers' perspective (Jacobson and Sewell 2003, Jacobson et al. 2005, Robbins et al. 2010, Sewell and Jacobson 2003, Sewell et al. 2001). Weniger et al. (1998) introduce an IP model to aid healthcare policy makers in determining a vaccine formulary that minimizes the cost to fully immunize a child according to a given childhood immunization schedule. Jacobson et al. (1999) present a full technical description of the model introduced by Weniger et al. (1998). Hall et al. (2008) introduce the general vaccine formulary selection problem, providing fundamental insights into the structure of problems concerning minimum cost satisfaction of a childhood immunization schedule. Sewell et al. (2001) adopt a "reverse engineering" scheme involving a bisection algorithm to compute a vaccine's maximum inclusion price (i.e., the maximum price at which a vaccine is selected to be part of the lowest overall cost formulary). Sewell and Jacobson (2003) present a full technical description of the methods in Sewell et al. (2001). Similar efforts are seen in Jacobson and Sewell (2003) and Jacobson et al. (2005). Robbins et al. (2010) present a method to optimally price a pediatric vaccine to maximize a vaccine manufacturer's expected revenue given an uncertain cost parameter.

When investigating the United States pediatric vaccine market, the techniques presented in this paper are a natural game theoretic extension to the work of Robbins et al. (2010), who provide a methodology for analyzing pricing strategies for competing combination vaccines in the U.S. pediatric vaccine market, with the goal of maximizing a pharmaceutical company's expected revenue. Since unit production cost is assumed to be negligible, the methodology effectively seeks to maximize expected profit. The methodology is applied to a single firm and a single combination vaccine (i.e., a bundle of products) and assumes all other vaccine prices remain constant. The proposed approach represents a single price adjustment in a best response dynamics process. Applied systematically, the competing pharmaceutical companies would continually undercut each other in price to achieve higher profits. This market situation is indicative of Bertrand economic competition and clearly lends itself to study via game theory and, more specifically, to study with Γ.

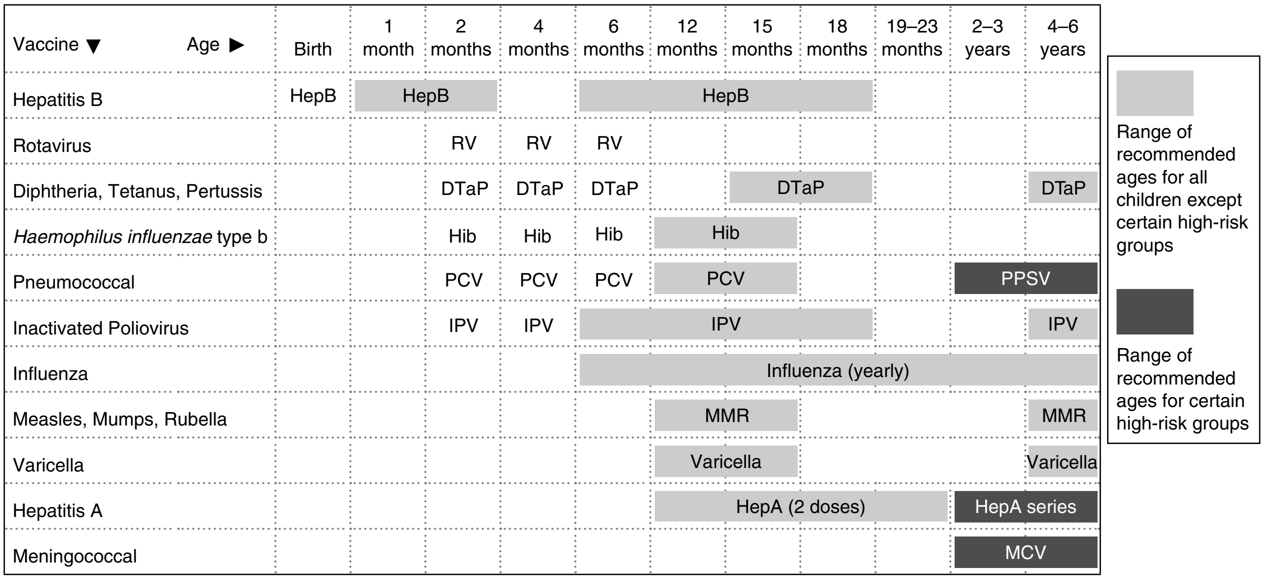
There are numerous stakeholders involved in the U.S. pediatric vaccine market. The pharmaceutical firms GlaxoSmithKline, Merck, and Sanofi Pasteur, manufacture the vaccines of interest in this paper. The Food and Drug Administration (FDA) licenses the use of the vaccines. The CDC, the Advisory Committee on Immunization Practices (ACIP), and the American Academy of Pediatrics (AAP) recommend proper use of the vaccines. The customers (i.e., state and local government public health officials) purchase vaccines for the immunization of the citizens in their administrative areas of responsibility. Federal government public health officials negotiate the vaccine prices for the purchases made by the state and local governments. Pediatric vaccines purchased at the public sector price, as negotiated by the federal government, account for approximately 57% of total pediatric vaccine purchases by volume (Orenstein et al. 2005). The prices negotiated by the government are not necessarily the optimal prices vaccine manufacturers are willing to charge. The federal government intends to negotiate for a lower price and the vaccine manufacturers aim to sell the vaccines at a higher price. For the results presented in this paper, only the public sector of the market is considered, though the methods discussed can be adapted to the private sector.

Vaccine development by pharmaceutical firms requires proficient management of a host of processes, most requiring highly skilled scientists and engineers to successfully produce the products (Douglas et al. 2008). The manufacturing process is expensive and time consuming, requiring vigilant maintenance of stringent FDA regulatory specifications. The estimated total unit production cost of a fully burdened liquid product vaccine (including the costs of filling, vialing, and packaging) is between \$0.70 and \$1.30 (Douglas et al. 2008). In addition to production costs, there is a federal excise tax associated with each vaccine dose; \$0.75 for each antigen the vaccine contains (Centers for Disease Control and Prevention 2010a). Vaccines are slightly differentiated in that they may be packaged in either vials or syringes. This difference in packaging affects costs with respect to preparation time. Vaccine preparations costs for vials and syringes are assumed to be \$0.75 and \$0.25 per dose, respectively (see Jacobson et al. 1999 or Weniger et al. 1998 for detailed descriptions). The vaccines of interest in this paper are those that were licensed in the United States and under federal contract (ending March 31, 2011) for purchase by public sector immunization programs (Centers for Disease Control and Prevention 2010b). Note that monopoly vaccine manufacturers and their products are not included in the analysis. The results presented in this study seek to portray long-term market trends; as such, research and development costs are ignored because the actual cost of producing the vaccine with respect to research and development depreciates over time. However, if research and development costs were of direct interest, one could compare the δ-discounted profit stream generated by an alternative investment vehicle for the research and development costs to the δ-discounted profit stream resulting from a likely tacit collusion equilibrium point in the market.

The FDA's licensing and approval process is a requirement for vaccine use in the United States. Following FDA approval, a positive recommendation is very important to the success of a pediatric vaccine. Changes in recommendations or requirements from the CDC, the ACIP, or the AAP greatly influence the demand for a particular vaccine. These organizations issue numerous guidelines regarding policies to effectively control vaccine-preventable diseases. This includes maintaining a list of acceptable vaccines and publishing an annual schedule concerning the appropriate periodicity and dosages of vaccines, the United States Recommended Childhood Immunization Schedule (RCIS; see Figure 1 from Centers for Disease Control and Prevention 2010a). Public health officials seek to satisfy the RCIS for each child in their administrative area of responsibility to ensure proper immunization coverage and promote public health. The five scheduled immunizations of interest in this study are (1) birth, (2) 2 months, (3) 4 months, (4) 6 months, and (5) 12–18 months.

When formulating Γ instances to model the U.S. pediatric vaccine market, the RCIS defines the weighted set-covering instance that drives customer demand. Indeed, the demand structure reflects a desire by vaccine purchasers to satisfy the RCIS, directly corresponding to finding a minimum cover, where vaccine component antigens cover disease prevention requirements (Hall et al. 2008). There is an assumption of rational purchaser behavior in that a minimum weighted set cover is sought (i.e., a purchaser seeks to satisfy the RCIS at a minimum cost). The analysis presented in this paper focuses on four *competitive* antigens, which provide protection against the following diseases: diphtheria, tetanus, pertussis (DTaP), *Haemophilus influenzae* type b (Hib), hepatitis B (HepB), and polio (IPV). These antigens are said to be *competitive* because more than one firm manufactures a vaccine containing the antigen.

Table 1 provides a summary of the information describing the U.S. public sector pediatric vaccine market. This information is used to construct the three scenarios of interest. Column 1 indicates the set of pharmaceutical firms, N (from Centers for Disease Control and Prevention 2010b, Food and Drug Administration 2009); column 2 indicates the set of pediatric vaccines B, where each vaccine contains a subset of the set of antigens S sold by the firms (Centers for Disease Control and Prevention 2010a, Food and Drug Administration 2009); column 3 indicates the time periods in the RCIS for which the vaccines are licensed to immunize children (from Centers for Disease Control and Prevention 2010b, Food and Drug Administration 2009); and columns 4–8 indicate unit costs per dose. Column 4 indicates the base unit production cost (from Douglas et al. 2008); column 5 indicates the federal excise tax associated with each vaccine (from Centers for Disease Control and Prevention 2010a); and column 6 indicates the total unit cost for manufacturing the vaccine, $(c_j)_{j\in B}$, which is the sum of columns 4 and 5. Column 7 indicates



Vaccine ▼ Age ▶	Birth	1 month	2 months	4 months	6 months	12 months	15 months	18 months	19–23 months	2–3 years	4–6 years
Hepatitis B	HepB	HepB			HepB						
Rotavirus			RV	RV	RV						
Diphtheria, Tetanus, Pertussis			DTaP	DTaP	DTaP		DTaP				DTaP
Haemophilus influenzae type b			Hib	Hib	Hib	Hib					
Pneumococcal			PCV	PCV	PCV	PCV				PPSV	
Inactivated Poliovirus			IPV	IPV	IPV						IPV
Influenza					Influenza (yearly)						
Measles, Mumps, Rubella						MMR					MMR
Varicella						Varicella					Varicella
Hepatitis A						HepA (2 doses)				HepA series	
Meningococcal										MCV	

▨ Range of recommended ages for all children except certain high-risk groups

■ Range of recommended ages for certain high-risk groups

Figure 1 **2011 U.S. Recommended Childhood Immunization Schedule (Through Age 6)**

Table 1 Vaccine Information

(1) Firm	(2) Vaccine		(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	DTaP	Infanrix®	2, 3, 4, 5	\$0.90	\$2.25	\$3.15	\$0.25	\$13.75
	Hib	Hiberix®	2, 3, 4, 5	\$0.70	\$0.75	\$1.45	\$0.75	\$8.66
	HepB	ENGERIX B®	1, 2, 4	\$0.70	\$0.75	\$1.45	\$0.25	\$9.75
	DTaP-HepB-IPV	Pediarix®	2, 3, 4	\$1.30	\$3.75	\$5.05	\$0.25	\$48.75
Merck	Hib	PedvaxHIB®	2, 3, 4, 5	\$0.70	\$0.75	\$1.45	\$0.75	\$11.29
	HepB	RECOMBIVAX HB®	1, 2, 4	\$0.70	\$0.75	\$1.45	\$0.75	\$10.00
	Hib-HepB	COMVAX®	2, 3, 4	\$0.80	\$1.50	\$2.30	\$0.75	\$28.80
Sanofi Pasteur	DTaP	Tripedia®	2, 3, 4, 5	\$0.90	\$2.25	\$3.15	\$0.75	\$13.25
	Hib	ActHIB®	2, 3, 4, 5	\$0.70	\$0.75	\$1.45	\$0.75	\$8.66
	IPV	IPOL®	2, 3, 4	\$0.70	\$0.75	\$1.45	\$0.25	\$11.51
	DTaP/Hib	TriHIBit®	5	\$1.00	\$3.00	\$4.00	\$0.75	\$27.31
	DTaP-IPV/Hib	Pentacel®	2, 3, 4	\$1.30	\$3.75	\$5.05	\$0.75	\$51.49

the product differentiation cost vector, $(\tilde{c}_j)_{j\in B}$, which is the preparation cost of each vaccine based on its packaging. Column 8 indicates the maximum allowable price of a vaccine $(\beta_j)_{j\in B}$, (assumed to be the current public sector price; Centers for Disease Control and Prevention 2010b). The reason that the maximum allowable prices are set to be the current public sector prices is because the federal government is not willing to pay more than the current public sector prices. These prices are the result of the negotiations between federal government and vaccine manufacturers. In each of the scenarios, there is an assumed cost of \$10.00 associated with each injection (i.e., $\tau = 10$) that the consumer considers. See Glazner et al. (2004) for a detailed discussion regarding the costs to healthcare providers for delivering childhood vaccinations.

To characterize the demand function for the Γ instances, three different population and healthcare statistics are required: the number of children completing an RCIS on an annual basis, the vaccine coverage rate among those children completing an RCIS, and the proportion of those children for which the vaccines were purchased at the public sector price. According to a recent National Vital Statistics Report (Martin et al. 2009), approximately 4.3 million births were registered in the United States in 2006. These children represent the maximum potential set of consumers of the vaccines. The most recent National Immunization Survey (NIS) results provide estimated vaccine coverage rates for children ages 19–35 months (Centers for Disease Control and Prevention 2009). With respect to the four diseases of interest in this paper, the NIS provides the proportion of children completing the full schedule (0.782) and the proportion of children completing none of the schedule (0.006). The estimated proportion of children for which full schedules were purchased at public sector prices lies in the interval $(0.782, 0.994)$ and must be estimated using the NIS data. Unfortunately, the NIS results do not specify the exact nature of the reduction from full coverage to no coverage. For the results presented in this paper, a slow decay of coverage is assumed and an equivalent full coverage rate of 0.917 is estimated based on this assumption. The equivalent full coverage rate accounts for vaccines purchased and used to satisfy doses in which full immunization schedules were not completed. Note that the estimated equivalent full coverage rate of 0.917 is arbitrary; further examination of vaccine coverage rates is warranted.

The market demand function used in the Γ instances reflects the *relative inelasticity* (see Mankiw 2007) inherent in the U.S. public sector pediatric vaccine market. The following constant demand function is used: $D(z(\mathbf{w}))$ where $d = (4{,}300{,}00)(0.917)(0.57) \approx 2{,}200{,}000$, $\eta = 1$, and $\lambda = 1$. Regardless of the price of the vaccines and the overall cost of the minimum cost cover, the demand remains the same. Naturally, this market situation is untenable unless price is bounded in some manner. An exogenous government entity (i.e., Congress) provides funding for the purchase of the vaccines, and as one would expect, another government entity (i.e., the CDC) effectively caps prices by exercising its monopsonistic leverage with vaccine manufacturers (see Table 1, column 8). Moreover, the prices of monovalent vaccines, when purchased using federal funds, are capped by law; the newer combination vaccines are not price controlled.

3.2. Annual Firm Profits at March 2010 Vaccine Prices

In the first scenario, given the information in Table 1, the annual profits of the pharmaceutical firms are determined using the techniques discussed by Robbins et al. (2010). Table 2 presents the resulting firm profits. GlaxoSmithKline fares well because of the slight price advantage of the formulary containing three doses of Pediarix® compared to the formulary containing three doses of Pentacel®. Note that the current price of the pediatric vaccines and the attendant

Table 2 **Annual Firm Profits at Contract Prices Ending March 31, 2010**

Firm	Revenue ($M)	Cost ($M)	Profit ($M)
GlaxoSmithKline	$343	$36	$307
Merck	$49	$6	$43
Sanofi Pasteur	$60	$9	$50

profit levels are not in equilibrium. Indeed, by lowering the price of Pentacel, Sanofi Pasteur could obtain a profit as large as that earned by GlaxoSmithKline. GlaxoSmithKline could then follow suit by decreasing prices again. This repeated undercutting in price leads to an unacceptable result for all firms in the market, as shown in the second scenario. Note that the pressure on the price of pediatric vaccines is due in part to the assumption of perfect substitutability among the competing vaccines with respect to satisfying the RCIS.

3.3. Equilibrium Firm Profits in the Static Game

In the second scenario, three Γ instances are formulated: one for vaccines sought in the first period, one for vaccines sought in the fifth period, and one for vaccines sought in the second through fourth periods. The demand for vaccines in the second through fourth periods is consolidated into a single weighted set cover to address the special attribute of Merck's Hib vaccine; if PedvaxHIB® or Comvax® is administered in the second and third time periods, a Hib dose in the fourth period is not required. Together, the three Γ instances provide insight as to what would occur should the pharmaceutical companies engage in Bertrand price competition, continually undercutting one another in price. Tables 3–5 give the pertinent information for the three Γ instances, where the columns are defined as in Table 1.

The inherent difficulty in computing a Nash equilibrium is as expected, given that the computation of payoffs in Γ involves finding a solution to an intractable problem (i.e., weighted set covering). The following theorem (with its proof given in the online supplement) shows this difficulty.

Theorem 3. *Given an instance of* Γ, *computing a pure strategy Nash equilibrium is NP-hard.*

Table 3 **Γ Instance 1 Information**

(1) Firm	(2) Vaccine		(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	HepB	ENGERIX B®	1	$0.70	$0.75	$1.45	$0.25	$9.75
Merck	HepB	RECOMBIVAX HB®	1	$0.70	$0.75	$1.45	$0.75	$10.00

Table 4 **Γ Instance 2 Information**

(1) Firm	(2) Vaccine		(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	DTaP	Infanrix®	2, 3, 4	$0.90	$2.25	$3.15	$0.25	$13.75
	Hib	Hiberix®	2, 3, 4	$0.70	$0.75	$1.45	$0.75	$8.66
	HepB	ENGERIX B®	2, 4	$0.70	$0.75	$1.45	$0.25	$9.75
	DTaP-HepB-IPV	Pediarix®	2, 3, 4	$1.30	$3.75	$5.05	$0.25	$48.75
Merck	Hib	PedvaxHIB®	2, 3, 4	$0.70	$0.75	$1.45	$0.75	$11.29
	HepB	RECOMBIVAX HB®	2, 4	$0.70	$0.75	$1.45	$0.75	$10.00
	Hib-HepB	COMVAX®	2, 3, 4	$0.80	$1.50	$2.30	$0.75	$28.80
Sanofi Pasteur	DTaP	Tripedia®	2, 3, 4	$0.90	$2.25	$3.15	$0.75	$13.25
	Hib	ActHIB®	2, 3, 4	$0.70	$0.75	$1.45	$0.75	$8.66
	IPV	IPOL®	2, 3, 4	$0.70	$0.75	$1.45	$0.25	$11.51
	DTaP-IPV/Hib	Pentacel®	2, 3, 4	$1.30	$3.75	$5.05	$0.75	$51.49

Table 5 **Γ Instance 3 Information**

(1) Firm	(2) Vaccine		(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	DTaP	Infanrix®	5	$0.90	$2.25	$3.15	$0.25	$13.75
	Hib	Hiberix®	5	$0.70	$0.75	$1.45	$0.75	$8.66
Merck	Hib	PedvaxHIB®	5	$0.70	$0.75	$1.45	$0.75	$11.29
Sanofi Pasteur	DTaP	Tripedia®	5	$0.90	$2.25	$3.15	$0.75	$13.25
	Hib	ActHIB®	5	$0.70	$0.75	$1.45	$0.75	$8.66
	DTaP/Hib	TriHIBit®	5	$1.00	$3.00	$4.00	$0.75	$27.31

Table 6 Equilibrium Prices for the Γ Instances

		Price				
Firm	Vaccine	Current	Inst. 1 Nash equilibrium	Inst. 2 Nash equilibrium	Inst. 3 Nash equilibrium	Scenario Nash equilibrium
GlaxoSmithKline	DTaP	$13.75	Free	Free	$3.15	$3.15
	Hib	$8.66	Free	$1.45	$1.45	$1.45
	HepB	$9.75	$1.95	$1.95	Free	$1.95
	DTaP-HepB-IPV	$48.75	Free	$5.55	Free	$5.55
Merck	Hib	$11.29	Free	$1.45	Free	$1.45
	HepB	$10.00	$1.45	$1.45	Free	$1.45
	Hib-HepB	$28.80	Free	Free	Free	Free
Sanofi Pasteur	DTaP	$13.25	Free	Free	Free	Free
	Hib	$8.66	Free	$1.45	Free	$1.45
	IPV	$11.51	Free	Free	Free	Free
	DTaP/Hib	$27.31	Free	Free	$14.85	$14.85
	DTaP-IPV/Hib	$51.49	Free	$5.05	Free	$5.05

Note. Prices from the Centers for Disease Control and Prevention (vaccine price list ending in March 2010).

To find a Nash equilibrium, an iterative improvement algorithm (IIA) (see online supplement §1) is introduced, which seeks a Nash equilibrium price point via a best response scheme, iteratively choosing a firm with the ability to increase its profit, then adjusting prices to achieve the greatest increase in profit (subject to the profit level indicated by the inter-bundle Cournot equilibrium, detailed in Step 13 of IIA (see online supplement §1). By design, IIA constructs a sequence of price points that must terminate under one of three conditions.

IIA finds a pure strategy Nash equilibrium for each of the three Γ instances (see Table 6). The vaccine prices computed by IIA for each of the problem instances are *consistent*, in that for a particular vaccine there is no price difference in the equilibrium price points. The amalgamation of the Nash equilibria provides a consistent, common pure strategy Nash equilibrium for the entire schedule. GlaxoSmithKline's HepB vaccine increases in price by $0.50 to match the HepB vaccine offered by Merck. This price change provides an advantage for GlaxoSmithKline in the second through fourth periods. In the second Γ instance, the formulary consisting of three doses of Pediarix plus two doses of PedvaxHIB provides the best value to the consumer. The next best, lowest cost formulary consists of three doses of Pentacel and two doses of ENGERIX B®. These two formularies provide the best economic value to a purchaser because they cover the RCIS requirements in five doses. Given the relatively high cost of an injection (with respect to the cost of the vaccines), a rational purchaser greatly values a reduction in the number of injections administered. Another reason that these two formularies are so competitive is the Merck Hib advantage. The use of three doses of Pediarix results in over-immunization with respect to HepB. However, this loss in value is made up for by Pediarix's formulary partner, PedvaxHIB, whereby a third dose of Hib is unnecessary. With an equivalent number of doses and equal marginal costs, the only difference between the two formularies results from packaging. GlaxoSmithKline packages its vaccine products in prefilled syringes, which takes less preparation time than vaccines packaged in vials and hence provides a small economic advantage. The GlaxoSmithKline $0.50 price increase for its HepB vaccine provides a $1.50 slack in cost that can be exploited. In the last period, Sanofi Pasteur's TriHIBit® provides a one-dose savings to its closest competing formulary. Its price is increased to match its competitor. Table 6 depicts the Nash equilibrium prices.

The attendant annual profits and costs attributed to the pharmaceutical firms are indicated for the Nash equilibrium price (see Table 7). When comparing the market profits using actual March 2010 vaccine prices (see Table 2) with a market that has engaged in a Bertrand price competition, it is seen that GlaxoSmithKline loses nearly all of its annual profit, dropping from more than $306 million to more than $4 million; Merck drops to a zero profit level margin; and Sanofi Pasteur loses the least, down from more than $51 million to nearly $25 million per year.

The Nash equilibrium payoffs shown in Table 7 indicate that engaging in Bertrand price competition results in a substantial loss of profit for all of the pharmaceutical firms competing in the U.S. public sector

Table 7 Firm Profits at Static Game Equilibrium Price Point

Firm	Revenue ($M)	Cost ($M)	Profit ($M)
GlaxoSmithKline	$40	$36	$4
Merck	$6	$6	$—
Sanofi Pasteur	$34	$9	$25

pediatric vaccine market. Certainly, pharmaceutical firms are aware that systematic price reductions negatively impact future profits, especially considering the price inelasticity in this market. The current vaccine prices and their corresponding adjustments in recent years reflect this understanding. Moreover, the Γ results are economically naive, as discussed in §1. Recall that Γ is a static game with no mechanism to model ongoing or repeated interaction between the competing firms. As such, there is no incentive for firms to cooperate in any manner; collusion in any form is not permitted. Therefore, Γ^r enables a more realistic analysis.

3.4. Equilibrium Firm Profits in the Repeated Game

In the third scenario, three Γ^r instances model the pediatric vaccine market, where a pharmaceutical firm must consider the effect of its current pricing strategy on the pricing strategies of other firms in the future and the attendant impact on its own future profits. The Nash equilibria of vaccine prices determined for each of the Γ^r instances are consistent, in that there are no pricing conflicts. The amalgamation of the three equilibria provides a single, consistent equilibrium. Assume that current prices reflect a price limit, indicating firms can only decrease prices to reach an amicable arrangement.

The focal equilibrium price point is selected based on the current component prices within the two most competitive formularies (i.e., the Pediarix dominant formulary and the Pentacel dominant formulary; see Table 8) and the assumption that Sanofi Pasteur and Merck would reduce the price of the vaccines in the more expensive of the two formularies, the Pentacel dominant formulary, so that the two dominant formularies are equal in cost from a vaccine purchaser's perspective. The equilibrium result holds assuming each pharmaceutical firm values future profits sufficiently high.

Table 9 shows the Nash equilibrium prices for Γ^r. To obtain the Nash equilibrium prices for the repeated game, Merck reduces the price of its HepB from \$10.00 to \$9.25 per dose and Sanofi Pasteur reduces the price of Pentacel from \$51.49 to \$49.61 per dose. The attendant market shares of these two formularies are evenly split, where the firms produce pediatric vaccines so that 1.1 million schedules are satisfied using the Pediarix dominant formulary and 1.1 million schedules are satisfied using the Pentacel dominant formulary.

Table 8 Focal Equilibrium Vaccine Formularies

Firm	Vaccine	Pediarix® dominant formulary	Pentacel® dominant formulary
GlaxoSmithKline	HepB	1	0
	DTaP-HepB-IPV	3	0
Merck	Hib	2	0
	HepB	0	3
Sanofi Pasteur	DTaP/Hib	1	1
	DTaP-IPV/Hib	0	3

Table 9 Equilibrium Prices for Γ^r

		Price[a]	
Firm	Vaccine	2010	Nash equilibrium
GlaxoSmithKline	DTaP	\$13.75	\$13.75
	Hib	\$8.66	\$8.66
	HepB	\$9.75	\$9.75
	DTaP-HepB-IPV	**\$48.75**	**\$48.75**
Merck	Hib	\$11.29	\$11.29
	HepB	\$10.00	\$9.25
	Hib-HepB	\$28.80	\$28.80
Sanofi Pasteur	DTaP	\$13.25	\$13.25
	Hib	\$8.66	\$8.66
	IPV	\$11.51	\$11.51
	DTaP/Hib	\$27.31	\$27.31
	DTaP-IPV/Hib	**\$51.49**	**\$49.61**

[a] Centers for Disease Control and Prevention (2010b). Boldface indicates the two combination vaccines, Pediarix and Pentacel, and their prices.

The Nash equilibrium payoffs shown in Table 10 indicate that in the long run, the firms benefit greatly from maintaining the pediatric vaccines at the focal equilibrium price point indicated in Table 9. In comparing the market where the firms tacitly collude with one that has engaged in Bertrand price competition, on an annual basis GlaxoSmithKline earns nearly \$149 million more, Merck earns more than \$47 million more, and Sanofi Pasteur earns more than \$173 million more. Assuming that the firms continue to tacitly collude by employing grim trigger strategies and that each firm's condition on its discount factor is met (see rightmost column in Table 10), these higher profit levels can be sustained. If any firm breaks the arrangement, Bertrand behavior results, ultimately leading to the Nash equilibrium outcome shown in Table 7. Although these profit levels may seem high, one should recall that past and current research and development costs, and sunk costs with respect to making pricing decisions, are still on the manufacturers' balance sheets and the high profits must be considered in this context.

Table 10 Firm Profits at Repeated Game Equilibrium Price Point 1

Firm	Revenue (\$M)	Cost (\$M)	Profit (\$M)	$\delta_f \geq$
GlaxoSmithKline	\$171	\$18	\$153	0.507
Merck	\$55	\$8	\$47	0.079
Sanofi Pasteur	\$223	\$25	\$198	0.459

Table 11 Firm Profits at Repeated Game Equilibrium Price Point 2

Firm	Revenue ($M)	Cost ($M)	Profit ($M)	$\delta_f \geq$
GlaxoSmithKline	$197	$21	$176	0.431
Merck	$55	$8	$47	0.091
Sanofi Pasteur	$199	$23	$176	0.528

Note that should a desire for a different profit allocation among the firms motivate a requirement for a new focal equilibrium point, an alternative means for attaining the new allocation could be reached by stipulating production limits as part of the collusive agreement. The equilibrium price point would not change; instead, production levels for the pertinent vaccines would adjust to reflect a self-imposed capacity constraint. For example, if GlaxoSmithKline desires profits approximately equal to those earned by Sanofi Pasteur, yet still desires the price point indicated in Table 9, then it becomes a matter of producing the appropriate quantity to meet the required profit target levels. Tacitly agreeing to limit production to induce market shares of 57.5% and 42.5% for the Pediarix dominant formulary and Pentacel dominant formulary, respectively, results in the profit levels seen in Table 11.

The IIA was implemented in MATLAB, and other than the problem analyzed in this paper regarding the U.S. pediatric vaccine market, several variations of the weighted set-covering optimization problem with different number of homogeneous products (antigens) and manufacturing firms were tested. The running times are reported in seconds, and do not include the MATLAB initializing time. The IIA implementation was run on a Intel® Core™ i5 CPU 750 with a 2.67 GHz Processor with 8 GB of available memory. The algorithm running time for the problem analyzed in this paper regarding the U.S. pediatric vaccine market with four antigens and three manufacturing firms is 0.078 seconds on average. By increasing the number of antigens to 10 and 14 (with three manufacturing firms), the algorithm running time increases to 37 seconds and 655 seconds on average, respectively. Increasing the number of manufacturing firms will increase the algorithm running time (e.g., the running time of the algorithm with four antigens and six manufacturing companies is 1.67 seconds on average). There are memory problems concerning large instances of the algorithm. For instances in which the number of homogeneous products is greater than or equal to 16, the memory limitations prevent execution of the algorithm as it is currently written in MATLAB.

3.5. Discussion and Limitations

The practical feature of the results presented in this paper is the difference in the repeated game equilibrium prices between Pediarix and Pentacel (see Table 9). Assuming GlaxoSmithKline and Sanofi Pasteur wish to share the market equally with respect to volume, the equilibrium prices from the repeated game indicate a difference of $0.86. The difference in price between Pediarix and Pentacel for contract prices ending March 31, 2010, was $2.74 (Centers for Disease Control and Prevention 2010b). Interestingly, the public sector vaccine price list for contract prices ending March 31, 2011, (Centers for Disease Control and Prevention 2011) showed a price difference of $0.95 between Pediarix and Pentacel, with the price of Pentacel actually *reduced* from the previous year (contract prices ending March 31, 2010; Centers for Disease Control and Prevention 2010b), an unusual occurrence. The results suggest a smaller price difference between these two important combination vaccines is appropriate, which is what occurred. These results could serve to inform both manufacturers and purchasers on the appropriate pricing of vaccines, given the existence of an assumed set of collusive agreements.

One should note that there are several factors that are not included in this analysis, including important economic factors that could impact the payoffs of the firms in the market of interest. The exclusion of such factors is because of the lack of data or economic models regarding them. These include factors that further differentiate between manufacturer products (e.g., safety and efficacy) as well as costs associated with reduced cold storage handling that result from reductions in the number of separate vaccines necessary to satisfy the RCIS. Other factors not included are brand loyalty, volume discounting, and formulary inertia because of the difficulty in quantifying economic model parameters describing them. Moreover, the risk of vaccine shortages may also impact the analysis, but is not explicitly included in the analysis. For example, the formulary for the two-month time period, consisting of one dose of Pediarix and one dose of PedvaxHIB, is very cost effective; however, if the risk of shortage for Merck's PedvaxHIB is considered too high (or indeed, if PedvaxHIB is currently unavailable; Centers for Disease Control and Prevention 2008), then a risk-averse vaccine purchaser may select the next best formulary despite its higher cost so as to avoid the possibility of not satisfying the RCIS. Such concerns regarding the risk of shortage is not explicitly modeled in Γ, although by modifying the set of available vaccines, such concerns could be examined.

The model presented in this paper assumes that children either receive the full vaccination schedule (all recommended antigens) or receive no coverage. The analysis does not cover the case where the children receive part (but not all) of the vaccines in the formulary. Government agencies are natural purchasers of pediatric vaccines and it is reasonable to assume that authorities follow the RCIS when planning the purchase of pediatric vaccines. However, there are certainly children who do not adhere to the schedule. The treatment of catch-up and high-risk immunization groups is not considered and would impact the desirability of vaccines. For example, monovalent vaccines may be more desirable in catch-up situations because of wastage concerns.

4. Conclusions

The static game Γ is a generalization of Bertrand price competition that provides a mathematical framework for the analysis of markets where a consumer makes purchasing decisions based on the outcome of an associated weighted set covering problem. It is important to note that application of the model should be restricted to such cases where a consumer purchases bundles according to the solution to this weighted set covering problem (i.e., a government agency, not a single consumer). The Nash equilibrium solution concept provides a consistent mechanism by which rational and intelligent pricing behavior of the firms in Γ can be examined and described. Theorem 1 indicates that a pure strategy Nash equilibrium (some in the limiting sense) of the static game Γ always exists. The algorithm described in the online supplement enables computation of a Nash equilibrium and also provides the means for constructing a theory for the existence of an equilibrium. Development of Γ^r addresses the temporal assumption of a single economic interaction between firms. Indeed, firms are likely to interact repeatedly in the market of interest; the repeated game structure enables examination of more realistic market equilibria. Theorem 2 provides conditions under which a subgame perfect Nash equilibrium of the repeated game Γ^r exists.

The proposed static game provides an appropriate mathematical framework by which to analyze oligopolistic interactions in markets such as the U.S. public sector pediatric vaccine market. The results presented here should interest those within the pediatric healthcare community seeking information regarding the economic value, effective pricing strategies, and impact of pediatric vaccines on market conditions. A meaningful understanding of important issues affecting the market of interest is gained, particularly with respect to the important combination vaccines, Pediarix and Pentacel, currently seen as the backbone of most pediatric vaccine formularies. Stakeholders more thoroughly comprehend the consequences of their own pricing decisions as well as the pricing decisions of other stakeholders. Moreover, the holistic impact of rational and intelligent individual stakeholders on the market provides valuable insight. Such information can be leveraged to improve a single stakeholder's position or influence policy decisions that affect the market in its entirety for the betterment of all parties involved.

In future research, a number of variations to Γ could be examined to include the incorporation of fixed charge costs and capacity constraints. Production lead times could also be explicitly modeled when considering capacity. Examining the simultaneous treatment of multiple market segments may provide an appropriate analytical perspective for certain markets of interest (e.g., considering both the public and private sector of the pediatric vaccine market). In the context of the repeated game Γ^r it may be worthwhile to develop a methodology for determining efficient market sharing allocations. Once the Pareto frontier of efficient allocations (i.e., price points and production levels) are identified, application of Nash bargaining theory (Nash 1950) could assist in the determination of an equitable allocation among the participating firms. Uncertainty could be introduced into the model to account for the risk of production interruptions. Often, when determining suppliers, purchasers must consider the risk of shortages and delay. Lastly, this model assumes a no-bankruptcy condition to prevent the pricing of a bundle under its marginal cost. Consideration of the more general case would invalidate the presented methodology. One would need a different approach to address the relaxed formulation. Dropping the no-bankruptcy constraint would be a fruitful area of future research.

Supplemental Material

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